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Evaluating Measure of Modified Rotatability for Second Degree Polynomial Using Symmetrical Unequal Block Arrangements with two unequal block sizes

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ABSTRACT
In this paper we studied evaluating measure of modified rotatability for second degree polynomial using symmetrical unequal block arrangements with two unequal block sizes is suggested, which enables us to assess the degree of modified rotatability for a given second degree polynomial is studied. It is found that the new method sometimes leads to designs with fewer number of design points than those available in the literature. Comparison of different methods of evaluating measure of modified rotatability for second degree polynomial using central composite design, balanced incomplete block design and symmetrical unequal block arrangements with two unequal block sizes is also studied.
***KEYWORDS AND PHRASES**: Response surface designs, measure, degree of modified rotatability.*

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I. INTRODUCTION

Investigation of input-output relationship is a useful activity in many situations. Fitting input-output relationship to unorganised data involves complex computations and control of precision of estimates of response at desired points is not possible. An alternative is to use for fitting planned data obtainable through appropriate designs. There are some series of such designs in literature. Data from symmetrical factorial experiments with quantitative and equispaced factor levels can be used for fitting such relations conveniently. Box and Hunter (1957) introduced a series of response at points equidistant from the centre of the design are all equal. They called these designs Rotatable designs when the relationship between the response variable and several input variables is a quadratic or cubic polynomial. Considerable research activities followed the introduction of these designs though mainly for construction of these designs. Das and Narasimham (1962) developed rotatable designs through balanced incomplete block designs (BIBD). Raghavarao (1963) constructed second order rotatable design (SORD) using incomplete block designs (symmetrical unequal block arrangements (SUBA) with two unequal block sizes). Das et al. (1999) studied response surface designs, symmetrical and asymmetrical, rotatable and modified. Victorbabu and Vasundharadevi (2008) developed modified second order response surface designs, rotatable designs using SUBA with two unequal block sizes. If the circumstances are such that exact rotatability is unattainable, it is still a good idea to make the design nearly rotatable. Thus, it is important of know if a particular design is rotatable. Park et al. (1993) introduced measure of rotatability for response surface designs and illustrated for 3^k factorial and central composite designs (CCD). On second order rotatable designs –a review was contributed by Victorbabu (2007). A lot of research work carried out by Victorbabu and some other authors in the area measure of rotatability and modified rotatability (Victorbabu and Surekha (2012, 2013, 2014, 2015), Victorbabu et al (2016, 2017), Victorbabu, Chiranjeevi and Surekha (2017), Victorbabu and Chiranjeevi (2018), Victorbabu et al (2008)) Jyostna et al (2021, 2020a, 2020b) suggested measure of modified rotatability for second order response surface designs using CCD, BIBD and pairwise balanced designs etc.,

In this paper following Park et al (1993), Das et al (1999), Victorbabu and Vasundharadevi (2008) and Victorbabu and Surekha (2015), a new method of evaluating measure of modified rotatability for second degree polynomial using SUBA with two unequal block sizes is suggested.

II. SORD – CONDITIONS:

Suppose we want to use the second degree polynomial model D= $((x_{iu}))$ to fit the surface,

$$Y_u = b_0 + \sum_{i=1}^{v} b_i x_{iu} + \sum_{i=1}^{v} b_{ii} x_{iu}^2 + \sum\sum_{i<j} b_{ij} x_{iu} x_{ju} + e_u \quad (1)$$

where x_{iu} denotes the level of the i^{th} factor (i =1,2,…,v) in the u^{th} run (u=1,2,…,N) of the experiment, e_u's are uncorrelated random errors with mean zero and variance σ^2 is said to be rotatable design of second order, if the variance of the estimated response of $\hat{Y}_u$ from the fitted surface is only a function of the distance ($d^2=\sum_{i=1}^{v} x_i^2$) of the point $(x_1,x_2, \ldots,x_v)$ from the origin (centre) of the design. Such a spherical variance function for estimation of second degree polynomial is achieved if the design points satisfy the following conditions [cf. Box and Hunter (1957), Das and Narasimham (1962)].

1. $\sum x_{iu}=0$, $\sum x_{iu}x_{ju}=0$, $\sum x_{iu}x_{ju}^2=0$, $\sum x_{iu}x_{ju}x_{ku}=0$, $\sum x_{iu}^3=0$, $\sum x_{iu}x_{ju}^3=0$, $\sum x_{iu}x_{ju}x_{ku}^2=0$, $\sum x_{iu}x_{ju}x_{ku}x_{lu}=0$; for $i \neq j \neq k \neq l$; (2)

2. (i) $\sum x_{iu}^2=$ constant $= N\lambda_2$;

   (ii) $\sum x_{iu}^4=$ constant $= cN\lambda_4$; for all i (3)

3. $\sum x_{iu}^2 x_{ju}^2=$ constant $= N\lambda_4$; for $i \neq j$ (4)

4. $\sum x_{iu}^4 = c\sum x_{iu}^2 x_{ju}^2$ (5)

5. $\frac{\lambda_4}{\lambda_2^2} > \frac{v}{(c+v-1)}$ (6)

where c, λ_2 and λ_4 are constants and the summation is over the design points.

If the above mentioned conditions are satisfied, the variances and covariances of the estimated parameters become,

$$V(\hat{b}_0) = \frac{\lambda_4(c+v-1)\sigma^2}{N\left[\lambda_4(c+v-1)-v\lambda_2^2\right]},$$

$$V(\hat{b}_i)=\frac{\sigma^2}{N\lambda_2},$$

$$V(\hat{b}_{ij})=\frac{\sigma^2}{N\lambda_4},$$

$$V(\hat{b}_{ii})=\frac{\sigma^2}{(c-1)N\lambda_4}\left[\frac{\lambda_4(c+v-2)-(v-1)\lambda_2^2}{\lambda_4(c+v-1)-v\lambda_2^2}\right],$$

$$Cov(\hat{b}_0,\hat{b}_{ii})=\frac{-\lambda_2\sigma^2}{N[\lambda_4(c+v-1)-v\lambda_2^2]},$$

$$Cov(\hat{b}_{ii},\hat{b}_{jj})=\frac{(\lambda_2^2-\lambda_4)\sigma^2}{(c-1)N\lambda_4[\lambda_4(c+v-1)-v\lambda_2^2]} \quad (7)$$

and other covariances are zero.

III. MODIFIED SORD - CONDITIONS

Let $(v, b, r, k_1, k_2, b_1, b_2, \lambda)$, $k = Sup.(k_1, k_2)$, and $b_1+b_2=b$ be a SUBA with two unequal block sizes. Let $2^{t(k)}$ denotes a resolution V fractional factorial of 2^k in ±1 levels, such that no interaction with less than five factors is confounded and n_0 denotes the number of central points in the design. $[\alpha\text{-}(v, b, r, k_1, k_2, b_1, b_2, \lambda)]$ denote the design points generated from the transpose of incidence matrix of SUBA with two unequal block sizes, $[\alpha\text{-}(v, b, r, k_1, k_2, b_1, b_2, \lambda)]2^{t(k)}$ are the $b2^{t(k)}$ design points generated from SUBA with two unequal block sizes by 'multiplication'(Raghavarao, 1971), Let $(\beta,0,...,0)2^1$ denote the design points generated from $(\beta,0,...,0)$ point set. n_0 be the number of central points. The usual method of construction of rotatable designs using SUBA with two unequal block sizes is to take combinations with unknown constants, associate a $2^{t(k)}$ factorial combinations or a suitable fraction of it with factors each at ± 1 levels to make the level codes equidistant. All such combinations form a design. Generally, rotatable designs of second order need at least five levels (suitably coded) at $0,\pm\alpha,\pm\beta$ for all factors ($(0,0,...0)$)-chosen centre of the design, unknown level 'α' and 'β' are to be chosen suitably to satisfy the conditions of the rotatability) generation of design points this way ensures satisfaction of all the conditions even though the design points contain unknown levels.

Alternatively, by putting some restrictions indicating some relation among $\sum x_{iu}^2, \sum x_{iu}^4$ and $\sum x_{iu}^2 x_{ju}^2$ some equations involving the unknowns are obtained and their solution gives the unknown levels. In SORD the restriction used is $\sum x_{iu}^4 = 3\sum x_{iu}^2 x_{ju}^2$, i.e., $c=3$. Other restriction is also possible. The restriction $(\sum x_{iu}^2)^2 = N\sum x_{iu}^2 x_{ju}^2$ ie., $\lambda_2^2=\lambda_4$ to get another series of symmetrical second order response surface designs, which provide more precise estimates of response at specific points of interest than what is available from the corresponding existing designs (cf. Das et al 1999). On simplification of (7) using the above condition $\lambda_2^2=\lambda_4$ the variances and covariances of the estimated parameters are,

$$V(\hat{b}_0) = \frac{(c+v-1)\sigma^2}{N(c-1)}$$

$$V(\hat{b}_i) = \frac{\sigma^2}{N\sqrt{\lambda_4}}$$

$$V(\hat{b}_{ij}) = \frac{\sigma^2}{N\lambda_4}$$

$$V(\hat{b}_{ii}) = \frac{\sigma^2}{(c-1)N\lambda_4}$$

$$cov(\hat{b}_0, \hat{b}_{ii}) = \frac{-\sigma^2}{N\sqrt{\lambda_4}(c-1)} \qquad (8)$$

and other covariances are zero. These modifications of the variances and covariances affect the variance of the estimated response at specific points considerably. Using these variances and covariances, variance of estimated response at any point can be obtained. Let $\hat{Y}_u$ denote the estimated response at the point $(x_{1u}, x_{2u}, ... x_{vu})$. Then,

$$V(\hat{Y}_u) = V(\hat{b}_0) + d^2[V(\hat{b}_i) + 2cov(\hat{b}_0, \hat{b}_{ii})] + d^4 V(\hat{b}_{ii}) + (\sum x_{iu}^2 x_{ju}^2)[(c-3)\sigma^2/(c-1)N\lambda_4]$$

The study of modified response surface designs is the same as for SORD except that instead of taking $c=3$ the restriction $(\sum x_{iu}^2)^2 = N\sum x_{iu}^2 x_{ju}^2$ is to be used and this condition will provide different values of the unknowns involved (cf. Das et al. 1999).

IV. CONDITIONS FOR MEASURE OF ROTATABILITY FOR SECOND DEGREE POLYNOMIAL

Following Box and Hunter (1957), Das and Narasimham (1962), Park et al (1993), conditions (2) to (6) and (7) provide the necessary and sufficient conditions for measure of rotatability for any general second degree polynomial. Further we have,

$V(b_i)$ are equal for i,

$V(b_{ii})$ are equal for i,

$V(b_{ij})$ are equal for i, j, where $i \neq j$,

$Cov(b_i, b_{ii}) = Cov(b_i, b_{ij}) = Cov(b_{ii}, b_{ij}) = Cov(b_{ij}, b_{il}) = 0$ for all $i \neq j, j \neq l, l \neq i$. (9)

Park et al. (1993) suggested that if the conditions in (2) to (6) together along with (7) and (9) are satisfied, then the following measure $(P_v(D))$ can be used to assess the degree of rotatability for any general second degree polynomial (cf. Park et al., 1993).

$$P_v(D) = \frac{1}{1+R_v(D)}, \quad (10)$$

Here

$$R_v(D) = \left[\frac{N}{\sigma^2}\right]^2 \frac{6v\left[V(\hat{b}_{ij}) + 2cov(\hat{b}_{ii}, \hat{b}_{jj}) - 2V(\hat{b}_{ii})\right]^2 (v-1)}{(v+2)^2(v+4)(v+6)(v+8)g^8} \quad (11)$$

and g is the scaling factor.

On simplification, numerator of (11), $[V(\hat{b}_{ij}) + 2cov(\hat{b}_{ii}, \hat{b}_{jj}) - 2V(\hat{b}_{ii})]$ using (7) becomes $(c-3)\sigma^2/(c-1)N\lambda_4$. Thus $R_v(D)$ becomes

$$R_v(D) = \left[\frac{N}{\sigma^2}\right]^2 \left(\frac{6v[(c-3)\sigma^2]^2(v-1)}{[(c-1)N\lambda_4]^2(v+2)^2(v+4)(v+6)(v+8)g^8}\right) \quad (12)$$

Note: For SORD, we take $c = 3$. Substituting the value of $'c'$ and on simplification of (12) we get $R_v(D)$ is zero. Hence from (10), we get $P_v(D)$ is one if and only if a design is rotatable and less than one then it is nearly rotatable design.

V. MODIFIED ROTATABILITY FOR SECOND DEGREE POLYNOMIAL USING SUBA WITH TWO UNEQUAL BLOCK SIZES

SUBA with two unequal block sizes: The arrangement of v treatments in b blocks where b_1 blocks of size k_1, and b_2 blocks of size k_2 is said to be a SUBA with two unequal block sizes, if

(i) every treatment occurs $\frac{b_i k_i}{v}$ blocks of size k_i (i = 1, 2), and

(ii) every pair of first associate treatments occurs together in u blocks of size k_1 and in $(\lambda\text{-}u)$ blocks of size k_2 while every pair of second associate treatments occurs together in λ blocks of size k_2.

From (i) each treatment occurs in $(\frac{b_1 k_1}{v}) + (\frac{b_2 k_2}{v}) = r$ blocks in the whole design. $(v, b, r, k_1, k_2, b_1, b_2, \lambda)$ are known as the parameters of the SUBA with two unequal block sizes.

Let $(v, b, r, k_1, k_2, b_1, b_2, \lambda)$, be a SUBA with two unequal block sizes. $2^{t(k)}$ denotes a resolution V fractional factorial of 2^k with +1 or -1 levels, such that no interaction with less than five factors is confounded. $[\alpha\text{-}(v, b, r, k_1, k_2, b_1, b_2, \lambda)]$ denote the design points generated from the transpose of incidence matrix of SUBA with two unequal block sizes, $[\alpha\text{-}(v, b, r, k_1, k_2, b_1, b_2, \lambda)]2^{t(k)}$ are the $b2^{t(k)}$ design points generated from SUBA with two unequal block sizes by 'multiplication' (cf. Raghavarao, 1971). Repeat these design points y_1 times. $(\beta, 0, 0, ..., 0)2^1$ denotes the design points generated from $(\beta, 0, 0, ..., 0)$ point set, and repeat this set of additional design points say y_2 times and n_0 be the number of central points. The method of construction of modified SORD using SUBA with two unequal block sizes is established as follows.

The design points, $y_1[\alpha\text{-}(v, b, r, k_1, k_2, b_1, b_2, \lambda)]2^{t(k)}$ U $y_2(\beta,0,0,....0)2^1$ U (n_0) will give a v-dimensional modified SORD in $N = y_1 b2^{t(k)} + y_2 2v + n_0$ design points, where

$$\left(\frac{\beta}{\alpha}\right)^4 = \frac{y_1(3\lambda\text{-}r)2^{t(k)-1}}{y_2},$$

$$n_0 = \frac{(y_1 r 2^{t(k)}\alpha^2 + y_2 2\beta^2)^2}{y_1 \lambda 2^{t(k)}\alpha^4} - [y_1 b 2^{t(k)} + y_2 2v]$$ and n_0 turns out to be an integer

(cf. Victorbabu and Vasundharadevi (2008)).

VI. MEASURE OF ROTATABILITY FOR SECOND DEGREE POLYNOMIAL USING SUBA WITH TWO UNEQUAL BLOCK SIZES

Here we suggest the result of measure of rotatability for second degree polynomial using SUBA with two unequal block sizes (cf. Victorbabu and Surekha 2013).

Let $(v, b, r, k_1, k_2, b_1, b_2, \lambda)$ denote a SUBA with two unequal block sizes. The design points $y_1[\alpha\text{-}(v, b, r, k_1, k_2, b_1, b_2, \lambda)]2^{t(k)}\ U\ y_2(\beta,0,0,....0)2^1\ U\ (n_0)$ will give a measure of rotatability for second degree polynomial using SUBA with two unequal block sizes in $N = y_1 b 2^{t(k)} + y_2 2v + n_0$ design points with level 'α' and 'β' are prefixed and $c = \frac{y_1 r 2^{t(k)}\alpha^4 + y_2 2\beta^4}{y_1 \lambda 2^{t(k)}\alpha^4}$ (we take $y_1 = 1, y_2 = 1$).

We can obtain the measure of rotatability values for second degree polynomial using SUBA with two unequal block sizes. We have

$$R_v(D) = \left[\frac{(c\text{-}3)}{(c\text{-}1)}\right]^2 \frac{6v(v\text{-}1)}{\lambda_4^2 (v+2)^2 (v+4)(v+6)(v+8) g^8}$$

where

$$g = \begin{cases} \frac{1}{\beta}, & \text{if } \beta < \sqrt{\frac{y_1(b\text{-}r)2^{t(k)-1}}{y_2} + v} \\ \frac{1}{\sqrt{\frac{y_1(b\text{-}r)2^{t(k)-1}}{y_2} + v}}, & \text{if } \beta > \sqrt{\frac{y_1(b\text{-}r)2^{t(k)-1}}{y_2} + v} \end{cases}$$

$$P_v(D) = \frac{1}{1 + R_v(D)}$$

If $P_v(D)$ is 1 if and only if the design is rotatable, and it is smaller than one for a nearly rotatable designs.

VII. EVALUATING MEASURE OF MODIFIED ROTATABILITY FOR SECOND DEGREE POLYNOMIAL DESIGNS USING SUBA WITH TWO UNEQUAL BLOCK SIZES

The proposed method for evaluating measure of modified rotatability for second degree polynomial designs using SUBA with two unequal block sizes is suggested as follows.

Let $(v, b, r, k_1, k_2, b_1, b_2, \lambda), k = \text{Sup.}(k_1, k_2), \text{and } b_1 + b_2 = b$ be a SUBA with two unequal block sizes. Let $2^{t(k)}$ denotes a resolution V fractional factorial of 2^k in ±1 levels, such that no interaction with less than five factors is confounded and n_0 denotes the number of central points in the design. $[1\text{-}(v, b, r, k_1, k_2, b_1, b_2, \lambda)]$ denote the design points generated from the transpose of incidence matrix of SUBA with two unequal block sizes, $[1\text{-}(v, b, r, k_1, k_2, b_1, b_2, \lambda)]2^{t(k)}$ are the $b2^{t(k)}$ design points generated from SUBA with two unequal block sizes by 'multiplication'(Raghavarao, 1971), Repeat these $b2^{t(k)}$ design points y_1 times. Let $(\pm\alpha, 0,...,0)2^1$ denote the design points generated from $(\pm\alpha, 0,...,0)$ point set. Repeat this set of additional design points say y_2 times and n_0 be the number of central points. The

method of evaluating measure of modified rotatability for second degree polynomial designs using SUBA with two unequal block sizes is suggested as follows.
Let us consider the design points,

$y_1[\alpha\text{-}(v, b, r, k_1, k_2, b_1, b_2, \lambda)]2^{t(k)}$ U $y_2(\beta,0,0,....0)2^1$ U (n_0) generated from SUBA with two unequal block sizes. From (3) and (4) we have,

$$\sum x_{iu}^2 = y_1 r 2^{t(k)}\alpha^2 + y_2 2\beta^2 = N\lambda_2 \quad (13)$$

$$\sum x_{iu}^4 = y_1 r 2^{t(k)}\alpha^4 + y_2 2\beta^4 = cN\lambda_4 \quad (14)$$

$$\sum x_{iu}^2 x_{ju}^2 = y_1\lambda 2^{t(k)}\alpha^4 = N\lambda_4 \quad (15)$$

from (14) and (15), we get

$$\left(\frac{\beta}{\alpha}\right)^4 = \frac{y_1(3\lambda\text{-}r)2^{t(k)-1}}{y_2}$$

The modified condition $(\sum x_{iu}^2)^2 = N\sum x_{iu}^2 x_{ju}^2$ leads to N which is given by

$N = \frac{(y_1 r 2^{t(k)} + 2y_2\alpha^2)^2}{y_1\lambda 2^{t(k)}}$ alternatively N may be obtained directly as $y_1 b 2^{t(k)} + y_2 2v + n_0$, where n_0 is given by $n_0 = \frac{(y_1 r 2^{t(k)} + 2y_2\alpha^2)^2}{y_1\lambda 2^{t(k)}} - [y_1 b 2^{t(k)} + y_2 2v]$ and n_0 turns out to be an integer. From equations (13) and (15) and on simplification we get $\lambda_2 = \frac{y_1 r 2^{t(k)} + y_2 2\beta^2}{N}$ and $\lambda_4 = \frac{y_1\lambda 2^{t(k)}}{N}$.

To obtain evaluating measure of modified rotatability for second degree polynomial using SUBA with two unequal block sizes, we have

$$P_v(D) = \frac{1}{1+R_v(D)}$$

$$R_v(D) = \left[\frac{(c\text{-}3)}{(c\text{-}1)}\right]^2 \frac{6v(v\text{-}1)}{\lambda_4^2(v+2)^2(v+4)(v+6)(v+8)g^8}$$

Here g is a scaling factor,

$$g = \begin{cases} \frac{1}{\beta}, & \text{if } \beta < \sqrt{\frac{y_1(b\text{-}r)2^{t(k)-1}}{y_2} + v} \\ \frac{1}{\sqrt{\frac{y_1(b\text{-}r)2^{t(k)-1}}{y_2} + v}}, & \text{if } \beta > \sqrt{\frac{y_1(b\text{-}r)2^{t(k)-1}}{y_2} + v} \end{cases}$$

The following table gives the values of a evaluating measure of modified rotatability for second degree polynomial using SUBA with two unequal block sizes. It can be verified that $P_v(D)$ is 1 if and only if the design is modified rotatable, and it is smaller than one for nearly modified rotatable designs.

Example: We illustrate the evaluating measure of modified rotatability for second degree polynomial for $v=9$ factors with the help of a SUBA with two unequal block sizes $(v=9, b=15, r=7, k_1=3, k_2=5, b_1=6, b_2=9, \lambda=3)$. The design points,

$y_1[\alpha\text{-}(v=9, b=15, r=7, k_1=3, k_2=5, b_1=6, b_2=9, \lambda=3)]2^4$ U $y_2(\pm\beta,0,0,....0)2^1$ U (n_0) will give a measure of modified rotatability for second degree polynomial in N=300 design points. From (13), (14) and (15), we have

$$\sum x_{iu}^2 = y_1 112\alpha^2 + y_2 2\beta^2 = N\lambda_2 \quad (16)$$

$$\sum x_{iu}^4 = y_1 112\alpha^4 + y_2 2\beta^4 = cN\lambda_4 \quad (17)$$

$$\sum x_{iu}^2 x_{iu}^2 = y_1 48\alpha^4 = N\lambda_4 \quad (18)$$

From equations (17) and (18) with rotatability value $c=3$, $y_1=1$ and $y_2=1$, we get $\beta^4=16 \Rightarrow \beta^2=4 \Rightarrow \beta=2$. From equations (16) and (18) using the modified condition with $(\lambda_2^2=\lambda_4)$ along with $\beta^2=4$, $y_1=1$ and $y_2=1$, we get $N=300$, $n_0=42$. For modified SORD we get $P_v(D)=1$ by taking $\beta=2$ and scaling factor $g=0.5$. Then the design is modified SORD using BIBD.

Instead of taking $\beta=2$ if we take $\beta=2.8$ for the above SUBA with two unequal block sizes $(v=9,b=15,r=7,k_1=3,k_2=5,b_1=6,b_2=9,\lambda=3)$ from equations (17) and (18), we get $c=4.8944$. The scaling factor $g=0.35714$, $R_v(D)=37.6099$ and $P_v(D)=0.0259$. Here $P_v(D)$ becomes smaller it deviates from modified rotatability.

Here, it is observed that, evaluating measure of modified rotatability for second degree polynomial applying SUBA with two unequal block sizes for v=9 and v=10 factors has only 300 and 242 design points respectively whereas the corresponding evaluating measure of modified rotatability for second degree polynomial using BIBD obtained by Jyostna and Victorbabu (2021) needs 726 and 441 design points respectively. Thus the new method leads to 9-factor and 10-factor measure of modified rotatability for second degree polynomial applying SUBA with two unequal block sizes in less number of design points than the corresponding measure of modified rotatability for second degree polynomial using BIBD.

Table gives the values of evaluating measure of modified rotatability $P_v(D)$ for second degree polynomial using SUBA with two unequal block sizes, at different values of 'β' for $6 \le v \le 12$. It can be verified that $P_v(D)$ is one, if and only if a design 'D' is modified rotatable. $P_v(D)$ becomes smaller as 'D' deviates from a modified slope rotatable design. Comparison of different methods of evaluating measure of modified rotatability for second degree polynomial using central composite design, balanced incomplete block design and symmetrical unequal block arrangements with two unequal block sizes is also studied is given in Table 2.

VIII. CONCLUSION:

The evaluating measure of modified rotatability for second degree polynomial designs using SUBA with two unequal block sizes, at different values of 'β' for $6 \le v \le 12$. It can be verified that $P_v(D)$ is one if and only if the design is modified rotatable design and it is less than one for a nearly modified rotatable design. Comparison of different methods of evaluating measure of modified rotatability for second degree polynomial using central composite design, balanced incomplete block design and symmetrical unequal block arrangements with two unequal block sizes is also studied.

Table I: Evaluating measure of modified rotatability for second degree polynomial using SUBA with two unequal block sizes.

$(6,11,7,3,4,2,9,4)$, $N=361$, $n_0=65, y_1=1, y_2=10, \beta=1.414214$				
β	c	g	$R_v(D)$	$P_v(D)$
1.0	2.0625	1	0.0415	0.9602
1.3	2.6425	0.7692	0.0404	0.9612
*1.414214	3	0.7071	0	1
1.6	3.798	0.625	0.1861	0.8431
1.9	5.8225	0.5263	3.0988	0.244
2.2	9.0705	0.4545	16.5375	0.057
2.5	13.957	0.4	58.1209	0.0169

2.8	20.958	0.3571	162.9234	0.0061
3.1	30.6100	0.3297	331.7764	0.003

(9,15,7,3,5,6,9,3), N=300, n_0=42,y_1=1,y_2=1,β=2				
β	c	g	$R_v(D)$	$P_v(D)$
1.0	2.375	1	7.2435	0.9993
1.3	2.4523	0.7692	0.0488	0.9535
1.6	2.6064	0.625	0.1085	0.9022
1.9	2.8763	0.5263	0.0311	0.9699
2.2	3.3094	0.4545	0.41438	0.7071
*2	3	0.5	0	1
2.5	3.9609	0.4	6.7609	0.1289
2.8	4.8944	0.3571	37.6099	0.0259
3.1	6.1813	0.3226	135.2697	0.0073

(10,11,5,4,5,5,6,2), N=242, n_0=26,y_1=1,y_2=2,β=1.414214				
β	c	g	$R_v(D)$	$P_v(D)$
1.0	2.625	1	0.0028	0.9972
1.3	2.857	0.7692	0.0026	0.9974
*1.414214	3	0.7071	0	1
1.6	3.3192	0.625	0.0433	0.9585
1.9	4.129	0.5263	1.1761	0.4595
2.2	5.4282	0.4545	8.7768	0.1023
2.5	7.3828	0.4	38.2687	0.0254
2.8	10.1832	0.3571	122.9595	0.0081
3.1	14.044	0.3226	325.2091	0.0031

(12,15,7,4,6,3,12,3), N=600, n_0=72,y_1=1,y_2=2,β=2				
β	c	g	$R_v(D)$	$P_v(D)$
1.0	2.375	1	0.0057	0.9944
1.3	2.4523	0.7692	0.0318	0.9692
1.6	2.6064	0.625	0.0707	0.9340
1.9	2.8763	0.5263	0.0202	0.9802
2.2	3.3094	0.4545	0.2699	0.7875
*2	3	0.5	0	1
2.5	3.9609	0.4	4.4041	0.185
2.8	4.8944	0.3571	24.4986	0.0392

3.1	6.1813	0.3226	88.1115	0.0112

*indicates modified rotatability value using SUBA with two unequal block sizes. (cf. Victorbabu and Vasundharadevi (2008))

Table II: Comparison of CCD, BIBD, PBD and SUBA with two unequal block sizes methods of construction of evaluating measure of modified rotatability for second degree polynomial

No.of factors (v)	Measure of modified SORD using CCD (N)	Measure of modified SORD using BIBD (N)	Measure of modified SORD using PBD (N)	Measure of modified SORD using SUBA with two unequal block sizes (N)
2	16	-	-	-
3	32	50 (3,3,2,2,1)	-	-
4	36	81 (4,4,3,3,2)	-	-
5	36	150 (5,10,6,3,3)	-	-
6	72	121 (6,10,5,3,2)	-	361 (6,11,7,3,4,2,9,4)
7	100	162 (7,7,4,4,2)	-	-
8	100	300 (8,14,7,4,3)	-	-
9	200	726 (9,18,8,4,3)	242 (9,11,5,5,4,3,2)	300 (9,15,7,3,5,6,9,3)
10	200	441 (10,18,9,5,4)	242 (10,11,5,5,4,	242 (10,11,5,4,5,5,6,2)
11	200	242 (11,11,5,5,2)	-	-
12	324	-	-	600 (12,15,7,4,6,3,12,3)
13	324	-	3364 (13,15,7,7,6,5	-
14	324	-	3364 (14,15,7,7,6,3	-
15	324	-	-	-
16	324	-	-	-
17	324	-	-	-

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