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A New Investigation on Heterogeneous Bulk Service Queueing Model

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***ABSTRACT**: This investigation is concerned with the study of M/M(a,b)/(2,1)queueing system of two heterogeneous servers with different service rates. In this model it is assumed that the arrival pattern is Poisson style with parameter λ and the service times are assumed to be mutually independent and exponentially distributed with parameters $\mu_1$and μ_2 for the fast and slow servers respectively. The arrivals are served in batches according to FCFS discipline. In this model, the fast server is always retained in the system and a delayed vacation policy for slow server is discussed. The steady state solutions and the system characteristics are derived and analyzed for this model. The analytical results are numerically exemplified for different values of the parameters and levels also.*

***KEY WORDS:** Heterogeneous servers, Bulk queue, Delayed vacation*

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I. INTRODUCTION

The queuing theory is a useful statistical technique for solving peculiar problems. The last two decades have perceived a marvelous growth in the applications of bulk queues to many cramming situations. The primary reason for this is that bulk queueing models are often encountered in real life systems such as shipping systems, computer system, telecommunication, airline arrangement as well as industrial progressions such as production/ inventory systems etc.

Queueing theory is a set of mathematical tools for the analysis of probabilistic systems of customers and servers. It is also known as the theory of overloading, the branch of operational research that discovers the relationship between demand on a service system and the delays grieved by the users of that system. It is measured by one of the standard methodologies with linear programming, simulation and management science because the outcomes are often used when making business judgments about the resources needed to deliver service.

The considerable attention was paid to analyze the queueing system with vacation. Many researchers have analyzed vacation queueing problems for both Markovian and Non-Markovian queueing models. The Markovian models may be more appropriate in practice and have certain advantages. For a detailed comprehensive survey on queueing systems with server vacations and priority systems, one can refer to [3], [4], [5] and [7].The study of bulk queues was originated by [6]. [2] and [5] are highlighted the applications of bulk queues. The queueing systems with bulk arrival or bulk service or both bulk arrival and bulk service have been extensively studied by many authors. Its wide range of practical applications in a variety of situations makes it attractive to many researchers to work on various queueing models and to obtain closed form solutions.

[7] and [8] considers that units are served in batches of not more than '*b'* (say). If, immediately after the completion of a service, the server finds more than '*b'* units waiting, he takes a batch of '*b'* units for service while others wait; if he finds '*n'* units ($0 \leq n \leq b$), he takes all the '*n'* units in a batch for service. [3] Considered the same rule with the restriction that $n \neq 0$ ($1\leq n \leq b$), that is the service facility stops until a unit arrives. This rule will be called a usual bulk service rule, while Bailey; rule will be its modified type called as bulk service rule with intermittently available server and points out that the distribution of the queue length for the modified rule can be obtained from that of the usual rule.

The rule with a fixed batch size '*n'* has been considered by [9] and others. In this case the server waits until there are '*n'* units, and then serves all the '*n'* units in a batch. If there are more than n units waiting when the server becomes free, he takes a batch of '*n'* for service, while others wait.[11] considers this rule: if immediately after the completion of a service, the server finds less than '*a*' units present, he waits until there are '*a*' units, whereupon he takes the batch of '*a'* units for service; if he finds '*a'* or more but at most '*b'*, he takes them all in the batch and if he finds more than '*b'*, he takes in the batch for service '*b'* units, while others wait. The batch takes a minimum of '*a'* units and a maximum of '*b'* units. This rule will be called general bulk service rule.

[10] Introduced the concept of heterogeneity in service. He discussed the situation of certain hyper exponential distributions of service times with parallel channels. [12] Further discussed more problems by assigning the different service rates μ_1 and μ_2 to the two branches respectively. [13] Extended this problem by assigning customers to the two servers with probabilities π_1 and π_2 respectively.

[9] have discussed the model *M/M(a,b)/2* with heterogeneous servers and obtained the steady state probabilities and busy period distributions. [12] has studied the same model with balking. A bulk queueing model *M/M(a,b)/(2,1)* for non-identical servers with vacation is studied by [13]. In the literature described above, customer inter-arrival times and customer service times are required to follow certain probability distributions with fixed parameters.

The present investigation, an attempt has been made to analyze two heterogeneous *M/M(a,b)/(2,1)*queueing system with different service rate in which the fast server is always available in the system for service and the slow server can avail the vacation if there are less than '*a-1'* customers in the queue. That is, delayed vacation for slow server id discussed. The study of this queueing model is organized as follows. The model is described in Section 2. Queueing model is formulated mathematically along with notations in Section 3. The steady state solutions have been obtained in Section 4. The performance measures and mean queue length are derived in Section 5.The numerical results and graphical illustrations are discussed to facilitate the sensitivity analysis in Section 6 and 7.Concluding remarks and notable features of investigation done are highlighted in Section 8.

II. PROBLEM DESCRIPTION

In this model it is assumed that the arrival pattern is Poisson with parameter λ. Service is done in batches according to the general bulk service rule introduced by Neuts (1967). The late arrivals are not allowed to join the ongoing service. The successive service times are assumed to be mutually independent and exponentially distributed with parameters μ_i $(i = 1, 2)$ for each of the two servers (fast and slow) and $\mu_1 > \mu_2$.

On completing the service if the slow server finds less than '*a*' or '*a – 1*' customers in the queue and the fast server is busy, he wait in the system for a random period of time before going for vacation, which is called delayed vacation which is exponentially distributed with parameter θ and on returning from vacation, if the slow server finds less than '*a*' waiting customers and the fast server is busy or idle in the system, he stay in the system until he finds at least '*a*' customers. i.e., in this system single vacation for slow server is also considered.

The model leads to the state space

$\{(i, j, n);\ i, j = 0,1,\ i+j \neq 2,\ 0 \leq n \leq a - 1\}\ U\ \{(1, 1, n);\ n \geq 0\}\ U\ \{(1, 0, n);\ n \geq a\}$

In the state (i, j, n), the index $i=0$ refers to the situation that the fast server is idle and $i=1$ means that he is busy and the index $j=0$ refers to the state that the slow server is on vacation and $j=1$ that he is busy. The index $n \geq 0$ refers to the number of waiting customers in the queue.

III. STEADY STATE EQUATIONS

Defining $p_{i,j,n}(t)$ as the probability that the system is in the state (i, j, n), $i, j = 0,1$ and $n \geq 0$ and assuming that the steady state probabilities exists, the balance equation in the steady state are given by

$$(\lambda + \theta)\, P_{100} = \lambda P_{idle\ 0\ a-1} + \mu_2 P_{110} + \mu_1 \sum_{i=a}^{b} P_{10\ idle} \quad (1)$$

$$(\lambda + \mu_1)\, P_{11n} = \lambda P_{11n-1} + \mu_1 P_{11n+b} + \mu_2 P_{11n+b} + \theta P_{10n+b} \quad (n \geq 1) \quad (2)$$

$$(\lambda + \mu_1 + \theta)\, P_{10n} = \lambda P_{10n-1} + \mu_1 P_{10n+b} \quad (n \geq a) \quad (3)$$

$(\lambda + \mu_1 + \mu_2)\, P_{110} = (\mu_1 + \mu_2) \sum_{i=a}^{b} P_{11\,idle} + \theta \sum_{i=a}^{b} P_{10\,idle} + \lambda P_{1\,idlea-1}$ (4)

$(\lambda + \mu_1)\, P_{1\,idlen} = \lambda\, P_{1\,idlen-1} + \theta\, P_{1\,0\,n} \quad (0 \le n \le a\text{-}1)$ (5)

$(\lambda + \mu_1)\, P_{1\,idle\,0} = \theta\, P_{100} + \lambda\, P_{1\,idlea-1}$ (6)

$(\lambda + \mu_2)\, P_{idle\;10} = \mu_1 P_{110}$ (7)

$(\lambda + \mu_2)\, P_{idle\;1n} = \lambda\; P_{idle\;1n-1} + \mu_1 P_{11n} \quad (0 \le n \le a\text{-}1)$ (8)

$(\lambda + \theta)\, P_{idle\;0n} = \mu_1 P_{10n} + \mu_2 P_{idle\;1n} + \lambda\; P_{idle\;1\,n-1} (0 \le n \le a\text{-}1)$ (9)

$\lambda\, P_{idleidle\;0} = \mu_1 P_{1\,idle\;0}$ (10)

IV. COMPUTATION OF STEADY STATE SOLUTIONS

Let E denote the forward shifting operator defined by $E(P_{10n}) = P_{10n+1}$.

from equation (3) implies ($\mu_1 E^{b+1}$- $(\lambda + \mu_1 + \theta)E+\lambda)\, P_{10n} = 0 \;\; (n \ge a-1)$.

The characteristic equation of has only one real root inside the circle $|Z| = 1$ by Rouche's theorem when $\rho = \frac{\lambda+\theta}{b\mu_1}$ is less than 1. If r_0 (say) is the root of the above characteristic equation with $|r_0|<1$,

Then $p_{10n} = A_1 r_0^{\,n}$, $(n \ge a)$, is the solution for the homogeneous difference equation (3),

we have $P_{10n} = r_0^{n-a+1} P_{10a-1} (n \ge a)$ (11)

Using equation (2), $(\mu_1 + \mu_2)\, E^{b+1} - (\lambda + \mu_1)E + \lambda)\, P_{11n} = -\, \theta P_{10n+b+1} (n \ge 0)$

The characteristic equation of this equation has only one real root r_1 by Rouche's theorem which lies in the interval (0,1) , $\rho< 1$, where $\rho = \frac{\lambda}{b(\mu_1+\mu_2)}$ and after simplification $P_{11n} = (A_2 r_1^n + k_1 r_0^n) P_{10a-1} (n \ge 0)$(12)

Where A_2 is a constant and $k_1 = \frac{-\theta\mu_1 r_0^{b-a+2}}{\mu_2[(\lambda+\theta)r_0-\lambda]+\theta r_0 \mu_1}$

From equation (5),

$(\mu_1\, E^{b+1} - (\lambda + \mu_1)E + \lambda)\, P_{1idelen} = 0$ for $(0 \le n \le a-2)$

The above equation has only one real root r_2 , by Rouche's theorem which lies in the interval (0,1) , $\rho< 1$, where $\rho = \frac{\lambda+\theta}{b\mu_1}$ and after simplification,

$P_{1idlen} = (A_3 r_2^n + k_2 r_1^n + k_3 r_0^n)\, P_{10a-1} (1 \le n \le a-1)$ (13)

Where $k_2 = \frac{(\mu_1+\mu_2)A_2 r_1}{((\lambda-\mu_1-\theta)r_1)\mu_2+\theta r_1}$ and $k_3 = -\,\mu_2 k_1$

Solving equation (13),

$((\lambda + \mu_2)E - \lambda)\, P_{Idle\;1n} = \mu_1 P_{11n+1} (1 \le n \le a-1)$

$P_{Idle\;1n} = (A_4 r_3^n + A_2 k(r_1) r_1^n + k_1 k(r_0) r_0^n)\, P_{10a-1} (1 \le n \le a-1)$ (14)

here $r_3 = \frac{\lambda}{\lambda+\mu_1}$and $k(x) = \frac{\mu_1 x}{(\lambda+\mu_2)x-\lambda}$

Solving equation (8)

$P_{Idle\;0n}$=$(A_5 r_5^n + A_4 G(r_3) r_3^n + A_3 G(r_2) r_2^n]\, r_1^n + [k(r_0)G(r_0) + B(r_0)] r_0^n\,)\, P_{10a-1} (1 \le n \le a-1)$ (15)

here $r_5 = \frac{\lambda}{\lambda+\theta}$, $G(x) = \frac{x\mu_2}{(\lambda+\theta)x-\lambda}$, $B(x) = \frac{x\mu_1}{(\lambda+\theta)x-\lambda}$

Substituting the values of $P_{1\,Idlen}$, $P_{Idle\;0n}$, P_{10n} and P_{11n} in equation (9),

$(\lambda+\mu_1+\mu_2)(A_2)P_{10a-1} = (\mu_1+\mu_2) \left[A_2 \frac{r_1^a - r_1^{b+1}}{1-r_1} \right] + \theta\, [A_3 \frac{r_2^a - r_2^{b+1}}{1-r_2} + k_2 \frac{r_1^a - r_1^{b+1}}{1-r_1} + k_3 \frac{r_0^a - r_0^{b+1}}{1-r_0} \left.\right] P_{10a-1}$ (16)

After simplification, we obtain $A_3 = A_2 L(r_1) + L(r_0)$, (17) where $L(x) = \frac{1-r_1}{1-r_1^a}\left[\frac{\theta x}{(\mu_1)(1-x)\lambda} - \frac{B(x)(\lambda+\mu_1)(1-x^a)}{(1-x)}\right]$

The value of A_4 can be determined using the equation (14)

$$A_4 = \frac{1}{r_3^{a-1}}\left[A_2(1-k(r_1)r_1^{a-1}) - k_1k(r_0)r_0^{a-1}\right] \qquad (18)$$

Also, we obtain the value of A_5

$$A_5 = \frac{1}{r_4^{a-1}}\left[A_3(1-B(r_2)r_2^{a-1})+\lambda - k_2B(r_1)r_1^{a-1} - k_3B(r_0)r_0^{a-1}\right] \qquad (19)$$

The value of $A_2$is. $A_2 = \dfrac{B(r_2)\left(\frac{1-r_2^{a-1}}{1-r_2}\right)+k(r_1)\left(\frac{1-r_1^{a-1}}{1-r_1}\right)-\frac{\theta\rho\, r_0\lambda}{(\lambda+\theta)(1-r_0)}}{\frac{\theta\rho\, r_1\lambda}{(\lambda+\theta)(1-r_1)}-B(r_1)\left(\frac{1-r_1^{a-1}}{1-r_1}\right)-k(r_0)\left(\frac{1-r_0^{a-1}}{1-r_0}\right)}$, where $\rho = \dfrac{\lambda\theta}{\mu_1+\mu_2}$ (20)

Using the value of A_2, we can calculate the values of A_3, A_4 and A_5 .

All the steady state probabilities P_{ijn} for *n ≥0, i, j = 0,1* are obtained in terms of P_{10a-1}.

The value of P_{10a-1} can be determined using normalizing condition,

$$\sum_{n=0}^{a-1}(P_{10n}+P_{Idle\ 1\ n}+P_{1\ idlen}+P_{Idle\ 0\ n})+\sum_{n=a}^{\infty}P_{10n}+\sum_{n=0}^{\infty}P_{11n}=1 \qquad (21)$$

we get the solution after some algebraic simplification as follows,

$$\begin{aligned}\sum_{n=0}^{a-1}(P_{10n}+P_{Idle\ 1\ n}+P_{1\ idlen}+P_{Idle\ 0\ n}) = [&A_5M(r_4)+A_4[G(r_3)M(r_3)+\mathrm{K}(r_3)N(r_3)]\\ &+A_3[G(r_2)M(r_2)+\mathrm{K}(r_2)N(r_2)]\\ &+A_2[k_1G(r_1)M(r_1)+k_3\mathrm{B}(r_1)K(r_1)]\\ &+k_1K(r_0))+k_2\,\mathrm{B}(r_0)]\,P_{10a-1}\end{aligned} \qquad (22)$$

where, $M(x) = \left(\frac{1-x^a}{1-x}\right)+\frac{\mu_1}{(1-\lambda)}\left(\frac{a}{1-x}-\frac{x(1-x^a)}{(1-x)^2}\right)$,

$N(x) = \left(\frac{1-x^a}{1-x}\right)+\frac{\mu_2}{(1-\lambda)}\left(\frac{a}{1-x}-\frac{x(1-x^a)}{(1-x)^2}\right)$

$$\begin{aligned}\text{Then } {P_{10a-1}}^{-1} = [&A_5M(r_4)+A_4[G(r_3)M(r_3)+\mathrm{K}(r_3)N(r_3)]\\ &+A_3[G(r_2)M(r_2)+\mathrm{K}(r_2)N(r_2)]+A_2[k_1G(r_1)M(r_1)+k_3\mathrm{B}(r_1)K(r_1)]+\frac{r_1}{1-r_1}\\ &+\frac{B(r_1)}{1-r_1}+k_1K(r_0)+k_2\,B(r_0)]+\frac{r_0}{1-r_0}+\frac{B(r_0)}{1-r_0}\end{aligned} \qquad (23)$$

V. PERFORMANCE MEASURES

The efficiency of the queueing system can be proved by finding the performance measures of the queueing systems under consideration. As the steady-state probabilities are known, various performance measures of the queue can be easily obtained.

(a) Mean Queue Length

Let L_q be the expected number of customers in the queue then

$$L_q=\sum_{n=0}^{a-1}n(P_{10n}+P_{Idle\ 1\ n}+P_{1\ idlen}+P_{Idle\ 0\ n})+\sum_{n=a}^{\infty}nP_{10n}+\sum_{n=1}^{\infty}nP_{11n} \qquad (24)$$

(b) Probability that both the servers are busy (P_{2B})

$$P_{2B} = (A_2\frac{1}{1-r_1}+k_1\frac{1}{1-r_0})P_{10a-1} \qquad (25)$$

(c)Probability that one server is busy (P_{1B})

$$P_{1B}=[A_5\frac{r_4}{1-r_4}+A_3\frac{r_3}{1-r_3}+k_2\frac{r_1}{1-r_1}+k_3\frac{r_0}{1-r_0}]P_{10a-1} \qquad (26)$$

(d) Probability that the servers either on vacation or idle (P_{idleB})

$$P_{IdleB} = [A_4\frac{r_3}{1-r_3}+A_3\frac{1-r_2^n}{r_2}+k_1k(r_0)+k_3\frac{r_0}{1-r_0}]P_{10a-1} \qquad (27)$$

VI. NUMERICAL ANALYSIS

The numerical values of the expected queue length and various performance measures for different values of parameters *a,b,* λ, μ_1, μ_2 and θ which are given in the following tables

Table 1: Mean Queue Length for various values of *a, b,* μ_1=1, μ_2= 2

λ	θ	Lq				P_{1B}			
		a=9,b=22	a=12,b=22	a=18,b=22	a=25,b=40	a=9,b=22	a=12,b=22	a=18,b=22	a=25,b=40
5	0.10	2.1079	6.1225	8.0051	10.0051	0.2313	0.2787	0.2107	0.1676
10		3.7575	7.0459	9.0003	11.1007	0.5985	0.47031	0.3806	0.3141
15		8.9091	10.0420	11.0015	11.9045	0.6237	0.6161	0.5036	0.4466
20		15.1134	16.4518	16.1068	12.9051	0.7979	0.7132	0.6522	0.5253
5	0.25	2.9945	5.0009	8.0078	10.5001	0.2927	0.2639	0.20885	0.1526
10		4.7395	6.1266	9.0006	12.493	0.5960	0.4335	0.3698	0.2968
15		9.4595	9.9163	11.2063	14.5556	0.6824	0.6543	0.4125	0.4100
20		14.0095	14.1299	14.801	15.5412	0.7109	0.6584	0.5827	0.4812
5	0.5	2.0012	5.0049	8.0047	11.5091	0.3199	0.2569	0.1121	0.1426
10		5.1492	6.1295	9.9027	13.9198	0.5185	0.4720	0.3014	0.2884
15		7.0076	8.4509	10.0000	15.905	0.6238	0.5746	0.4072	0.4035
20		10.6389	11.7172	13.3497	17.0077	0.7180	0.6960	0.4127	0.4120
5	0.75	2.5830	5.9152	7.5024	12.9007	0.3128	0.2001	0.0072	0.1426
10		5.2606	6.3768	8.6536	15.5805	0.5121	0.4561	0.2559	0.2882
15		7.8562	8.8035	10.0030	18.6853	0.6245	0.4949	0.3936	0.3129
20		11.1817	12.6001	14.9334	20.2383	0.7537	0.7171	0.4527	0.4923

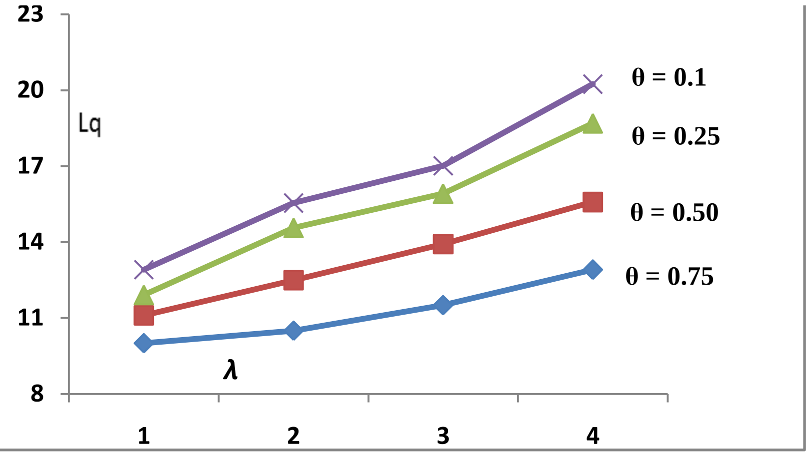
Table 2: The performance measures when μ_1= 1.5, μ_2= 2.5, *a* = 9 and *b*=25

λ	θ	L_q	P_{2B}	P_{1B}	P_{IdleB}
6	0.2	3.0176	0.5017	0.3905	0.0001
12		6.9023	0.4323	0.5793	0.0015
18		13.9011	0.3048	0.7446	0.1568
24		25.0654	0.2098	0.8100	0.0934
6	2	3.0915	0.6074	0.4701	0.0008
12		7.1983	0.5340	0.5245	0.0097
18		12.1485	0.3100	0.6345	0.0678
24		20.2221	0.2908	0.8256	0.0400
6	5	2.1248	0.7124	0.3121	0.0006
12		6.9609	0.5406	0.5576	0.0021
18		11.0041	0.3123	0.6476	0.0524
24		18.3546	0.1177	0.9176	0.0877

From the above Table 2 it is noted that for various values of λ and θ the normalized condition total probability is approximately equal to 1. Also, when the arrival rate λ is increased, the queue length also increased.

VII. PICTORIAL REPRESENTATION

Figure1: L_q Vs λ various values of θ when $a = 25$, $b = 40$, $\mu_1 = 1$ and $\mu_2 = 2$



VIII. CONCLUSION

In this present study, a queueing model *M/M(a,b)/(2,1)* with heterogeneous servers with delayed vacation for slow server depends on batch size are considered. The two main objectives of queueing theory as the whole are to reduce the waiting time of customers and the queue length. This present study satisfies the purpose because, the fast server is always retained in the system and slow server can go only for a delayed vacation. The model proposed here is applied for many real world problems.

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