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Mahgoub Adomian Decomposition Method For Solving Newell-Whitehead-Segel Equation

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ABSTRACT: *Mahgoub Adomian decomposition method (MADM) is used to find the solution of Newell-Whitehead- Segel equation (NWSE). This method is studied for some applications from NWSE. As the results, the method is efficient and easy to use.*

KEYWORDS: *Mahgoub Adomian decomposition method, Mahgoub transform, Adomian decomposition method (ADM), Differential equations (DEs) and Newell-Whitehead- Segel equation.*

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I. INTRODUCTION

DEs play a prominent role in mathematics, engineering, physics and biology (*see* [1-3]). NWSE is one of the most important of DEs. Over the years, several methods for solution of NWSE have been proposed (*see* [4-8]).

Now, let us consider the NWSE in the form [4]

$$u_t(x,t) = ku_{xx}(x,t) + au(x,t) - bu^m(x,t) \qquad (1)$$

Subject to

$$u(x,0) = f(x)$$

where $a, b \in \mathbb{R}$ and $m, k \in \mathbb{Z}^+$.

Recently, [9] introduced the function A as

$$A = \left\{ u(t): \exists M, k_1, k_2 > 0, |u(t)| < Me^{\frac{|t|}{k_j}} \right\},\ t \geq 0 \qquad (2)$$

where M, k_1, k_2 are fixed and M is a finite.

The operator $M\{u(t)\}$ may be expanded as

$$M\{u(t)\} = H(v) = v\int_0^\infty u(t)e^{-vt}\,dt, \quad k_1 \leq v \leq k_2. \qquad (3)$$

The main objective of this paper is to introduce a new method (MADM) for finding the solution of NWSE.

II. APPLICATION OF THE METHOD

Let us rewrite (1) as

$$u_t(x,t) = ku_{xx}(x,t) + au(x,t) - bu^m(x,t) \qquad (4)$$

Subject to

$$u(x,0) = f(x)\,.$$

Using Mahgoub transform to both sides, we obtain

$$M\{u_t(x,t)\} = M\{ku_{xx}(x,t) + au(x,t) - bu^m(x,t)\} \qquad (5)$$

Linearity of $M\{.\}$ yields

$$M\{u_t(x,t)\} = kM\{u_{xx}(x,t)\} + aM\{u(x,t)\} - bM\{u^m(x,t)\} \qquad (6)$$

Solving for (6), we have

$$vM\{u(x,t)\} - vu_t(x,0) = kM\{u_{xx}(x,t)\} + aM\{u(x,t)\} - bM\{u^m(x,t)\} \qquad (7)$$

By substituting $u(x,0)$ into (7), then equation becomes

$$vM\{u(x,t)\} - vf(x) = kM\{u_{xx}(x,t)\} + aM\{u(x,t)\} - bM\{u^m(x,t)\} \qquad (8)$$

And

$$M\{u(x,t)\} = \frac{v}{v-a}f(x) + \frac{k}{v-a}M\{u_{xx}(x,t)\} - \frac{b}{v-a}M\{u^m(x,t)\} \qquad (9)$$

Replacing $u(x,t) = \sum_{n=0}^{\infty} u_n(x,t)$ and $Nu(x,t) = \sum_{n=0}^{\infty} A_n$ in (9), It can be written as

$$M\{\sum_{n=0}^{\infty} u_n(x,t))\} = \frac{v}{v-a}f(x) + \frac{k}{v-a}M\{\sum_{n=0}^{\infty} u_{n_{xx}}(x,t)\} - \frac{b}{v-a}M\{\sum_{n=0}^{\infty} A_n\} \qquad (10)$$

where

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[\sum_{i=0}^{\infty}\lambda^n N u_i\right]_{\lambda=0}, \quad m = 0,1,\dots \tag{11}$$

This gives

$$M\{u_0(x,t)\} = \frac{v}{v-a}f(x)$$

$$M\{u_{n+1}(x,t)\} = \frac{k}{v-a}M\left\{\sum_{n=0}^{\infty}u_{n_{xx}}(x,t)\right\} - \frac{b}{v-a}M\left\{\sum_{n=0}^{\infty}A_n\right\},\ n \geq 0 \tag{12}$$

Take M^{-1} to both sides, thus

$$u_0(x,t) = M^{-1}\left\{\frac{v}{v-a}f(x)\right\}$$

$$u_{n+1}(x,t) = M^{-1}\left\{\frac{k}{v-a}M\left\{\sum_{n=0}^{\infty}u_{n_{xx}}(x,t)\right\} - \frac{b}{v-a}M\left\{\sum_{n=0}^{\infty}A_n\right\}\right\},\ n \geq 0 \tag{13}$$

III. EXAMPLE

Consider the NWSE [4]

$$u_t = u_{xx} - 3u\,, \tag{14}$$

with

$$u(x,0) = e^{2x}.$$

Using $M\{.\}$ to both sides, yields

$$M\{u_t\} = M\{u_{xx} - 3u\} \tag{15}$$

This leads to

$$vM\{u(x,t)\} - u(x,0)v = M\{u_{xx}\} - 3M\{u(x,t)\} \tag{16}$$

Using initial condition, we have

$$M\{u(x,t)\} = e^{2x}\frac{v}{v+3} + \frac{1}{v+3}M\{u_{xx}\} \tag{17}$$

Replacing $u(x,t) = \sum_{n=0}^{\infty}u_n(x,t)$ in (17), yields

$$M\left\{\sum_{n=0}^{\infty}u_n(x,t)\right\} = e^{2x}\frac{v}{v+3} + \frac{1}{v+3}M\left\{\sum_{n=0}^{\infty}u_{n_{xx}}(x,t)\right\},\ n \geq 0 \tag{18}$$

Thus, we have

$$M\{u_0(x,t)\} = e^{2x}\frac{v}{v+3}$$

$$M\{u_{n+1}(x,t)\} = \frac{1}{v+3}M\left\{\sum_{n=0}^{\infty}u_{n_{xx}}(x,t)\right\},\ n \geq 0 \tag{19}$$

Using M^{-1} to both sides, then

$$u_0(x,t) = e^{2x-3t}$$

$$u_{n+1}(x,t) = M^{-1}\left\{\frac{1}{v+3}M\left\{\sum_{n=0}^{\infty}u_{n_{xx}}(x,t)\right\}\right\},\ n \geq 0 \tag{20}$$

We can find the solutions as

$$u_0(x,t) = e^{2x-3t}$$

$$u_1(x,t) = M^{-1}\left\{\left\{\frac{1}{v+3}M\left\{u_{0_{xx}}(x,t)\right\}\right\},\ \right\} = 4te^{2x-3t}$$

$$u_2(x,t) = M^{-1}\left\{\frac{1}{v+3}M\left\{u_{1_{xx}}(x,t)\right\}\right\} = 8t^2e^{2x-3t}$$

$$u_3(x,t) = M^{-1}\left\{\frac{1}{v+3}M\left\{u_{2_{xx}}(x,t)\right\}\right\} = \frac{32}{3}t^3e^{2x-3t}$$

Thus

$$u(x,t) = \sum_{n=0}^{\infty}u_n(x,t) = e^{2x-3t} + 4te^{2x-3t} + 8t^2e^{2x-3t} + \frac{32}{3}t^3e^{2x-3t} + \cdots$$

$$= e^{2x-3t}\left(1 + 4t + 8t^2 + \frac{32}{3}t^3 + \cdots\right) = e^{2x+t} \tag{21}$$

IV. CONCLUSION

We have successfully used MADM to find the solution of NWSE. The method is efficient and easy to use.

REFERENCES

[1]. M Almousa, Solution of Second Order Initial- Boundary Value Problems of Partial Integro-Differential Equations by using a New Transform: Mahgoub Transform, European Journal of Advances in Engineering and Technology 5 (10), 2018, 802-805.

[2]. M. Almousa, I. Al-zuhairi, Y. Al-Qudah, H. Qoqazeh. Solution of Heat and Wave Equations using Mahgoub Adomian Decomposition Method, 3(1), 2020, 7-11.

[3]. R. I. Nuruddeen and A. M Nass, Exact Solutions of Wave-Type Equations by the Aboodh Decomposition Method, Stochastic Modelling and Applications, 21(1), 217, 23-30.

[4]. P. Pue-on, Laplace Adomian Decomposition Method for Solving Newell-Whitehead-Segel Equation, Applied Mathematical Sciences, 7(132), 2013, 6593 – 6600.

[5]. M. M. A. Mahgoub, Homotopy Perturbation Method for Solving Newell-Whitehead-Segel Equation, Advances in Theoretical and Applied Mathematics, 11(4), 2016, 399-406.

[6]. B. Latif , M. S. Selamat , A. N. Rosli , A. I. Yusoff , N. M. Hasan, The Semi Analytics Iterative Method for Solving Newell-Whitehead-Segel Equation, Mathematics and Statistics, 8(2), 2020, 87 – 94.

[7]. A. Prakash, M. Goyal, S.Gupta, Fractional variational iteration method for solving time-fractional Newell-Whitehead-Segel equation, Nonlinear Engineering, 8(1), 2019,164-171.

[8]. Kuo-Hsiung Wang G. O. Akinlabi, S. O. Edeki, Perturbation Iteration Transform Method for the Solution of Newell-Whitehead-Segel Model Equations, Journal of Mathematics and Statistics, 13(1), 2017,24-29.

[9]. M. Mahgoub, The New Integral Transform "Mahgoub Transform", Advances in Theoretical and Applied Mathematics, 11(4), 2016, 391-398.

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