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A simple algorithm for finding out the convolution sum of two discrete sequences

Ben-Qing GAO
School of Information and Electronics, Beijing Institute of Technology, Beijing 100081，China

ABSTRACT: *A simple algorithm for finding out the convolytion sum of two discrete sequences is presented. It is based on the priciple of two sequence element multiply without advanced digit. It can be used both for finding the convolution sum of two real number sequences, and two complex number sequences or a real number sequences and an imaginary sequence. It also can take off "element convolution sum" respectively from calculative process. By program and the calucation of the examples show that the algorithm in the paper being right and simple and easily implement.*

Key words: *discrete time domain analysis, sequence, algebraical algorithm, convolution sum, element convolution sum.*

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I. INTRODUCTION

In the time-domain analysis of signals and linear system, the convolution operation is a useful and very important calculation method. It is basic tool to calculate zero-state response for liner tine-in variat system. The convolution integrel is used in the continuous time domain analysis of a continuous time-variation system. And the convolution sum is often used a discrete time domain analysis, the both are equity [1] [2]. The convolution calculation is not only used in time domain analysis, also in frequency domain and z domainn, where the random appearing is almost same. The given methods for finding convolution integral or convolution sum are analysis techinc, graphical interpretation, mutiply without advance digit, using FFT as convolation, and so on[1][2][3]. These are often in the library of university.

In the paper we research a method on finding convolution sum from discrete sequence in time domain. Starting from the multipy without advance digit** of two sequence element, and work out a method for finding out the convolution sum of two discrete time domain sequences. It main procees are as follows:

The first, suppose that there are two sequence h and a, which have sequence lengt h LH and la respectively. To find out their convolution sum by the multipy without advance digit of two group sequence element, taking sequence a as multiping sequence and h as the multiped sequence, and incresing h sequence length to LCS (=la+lh-1) with supling 0 after h sequence.

Next, with lift first element a1 of a sequence multipy respectively each element of h to get s1 sequence, and with second element a2 of a multipy respectively each h to get s2 sequence, With similar process we obtain totally s sequence of la row.

At last, perform shift digit and plus operation to the resultants sequence to get cs sequence with n=la row, it just is the convolution sum of h and a sequence. Further, the paper exhibit that the convolution sums of different an element with h sequence, called "element convolution sum" (see next session). The proceess to find out convolution sum is available to find out convolution sum for two complex sequence, etc.

** Note: The '(1) multiply with advance digit' are a great difference with '(2) multiply without advance digit'. For example sequence (2 6) multipy (4), based on (1) the sum digit is (10 4), but based on (2) the sum digit is (8 24).

The program and calculation have been made, all calculation and examples show: the algorithm in the paper is right, simple and easy implimentant.

II. REAL NUMBER ALGORITHM MODE

Suppose that there are two real number sequences a: (a1 a2 a3 a4); h (h1 h2 h3 h4), in order to find out their convolution sum , it can be obtained by their element multily without advance digit[1][2]. Here two sequence element number: la=LH =4. It is well known: the convolution sum element number of two sequences is LCS=LH+la-1, LCS=7. Before implementing element multiply, it is needed with supling 0 after h element to make LH=LCS=7.

Next, for a and h sequence to perform their element multiply without advance digit as in Fig.1

H	h1	h2	h3	h4	0	0			
An	a1	a2	a3	a4					
s1	a1h1	a1h2	a1h3	a1h4			0		
s2	0	a2h1	a2h2	a2h3	a2h4		0		
s3	0		a3h1	a3h2	a3h3	a3h4	0		
s4	0			a4h1	a4h2	a4h3	a4h4		
cs1	a1h1	a1h2	a1h3	a1h4			0	=s1	1element convolution sum
	+	+	+	+	+	+	+		
cs2	0	a2h1	a2h2	a2h3	a2h4		0	=cs1+s2	2 element convolution sum
	+	+	+	+	+	+	+		
cs3	0		a3h1	a3h2	a3h3	a3h4	0	=cs2+s3	3 element convolution sum
	+	+	+	+	+	+	+		
cs4	0			a4h1	a4h2	a4h3	a4h4	=cs3+s4	4 element convolution sum

Fig 1 the process for two real number sequence to find out their convolution sum

By the element multiply without advance digit
From Fig.1 we can seen to find out convolution sum process as follows
(1) Taking a sequence first element a1 (located at lift side first position) to mutiply each element of h sequence for getting s1 sequence (a1h1 a1h2 . . .a1hm), and a sequence second element a2 to mutiply each element of h sequence for getting s2 sequence(a2h1 a2h2 . . .a2hm), and in similar way for getting sn sequence (anh1 anh2 . . .anhm). We sum up to obtain Sla row cs sequence (vector).
(2) Define new sequence (vector) cs in this way: let cs1=s1. For s2, rightward shifting one digit whole element and performing algebraical plus element by element with cs1 to obtain cs2 sequence (vector), etc., i.e.
cs1=s1, cs2=cs1+s2, cs3=cs2+s3, csn=csn-1+Sn,
Totally, it has n (=la) row cs sequence.

It is notice; cs4 in Fig.1 is conmom convolution sum. Cs1 is a convolution sum of the first element of a sequence with h sequence, called "one element convolution sum"; cs2 is a convolution sum of the 1'Th and 2'th element of a sequence with h sequence, called "two element convolution sum"; and so on. Here by the calculation at a time we can obtain some "element convolution sum", it is a property in the paper.

III. COMPLEX NUMBER ALGORITHM MODE

There are two complex sequences (ch: 2.0+j0.5 1.0+j0.0 0.0+j1.0), (ca: 1.0+j2.0 0.0+j0.5). To find out their convolution sum, the process for seting up the algorithm mode is similar to real number algorithn mode. But the complex property for complex calculation should be noticed. For example, the multiply of two complex number that consists of four time multiply and three time plus or minus, but the multiply of two real number is only multipling in a time.
Before the process for convolution sum, it is needed with suppling 0 elements after ch elements. The main procees to find out convolution sum are exhibited in Fig.2

Ch	2.0+j0.5	1.0+j0.0	0.0+j1.0	0	
ca	1.0 +j2.0	0.0+j0.5			
Cs1	1.0+j1.5	1.0+j2.0	-2.0+j1.0	0	~~cc1~~ ~~one~~ element convolution sum
cs2	0	-0.25+j1.0	0.0+j0.5	-0.5+j0.0	
	1.0+j4.5	0.75+j3.0	-2.0+j1.5	-0.5+j0.0	~~cc2~~ ~~two~~ element convolution sum

Fig.2 The process for two complex number sequence to find out their convolution
Sum by the element multiply without advance digit

From Fig.2 we can see to find out convolution sum process as follows
(1) Taking ca sequence first element ca1 to multiply each element of ch sequence to obtain cs1 sequence, and ca

sequence second element ca2 to multiply each element of ch sequence to obtain cs2, and so on. We sum up to obtain resultant csla sequence (vector).
(2) Similar process to real numeber algorithm, we have

cc1=cs1, cc2=cc1+cs2, cc3=cc2+s3, ccn=csn-1+Sn

Totally, it has n=la (row).

Notice: cc1 is a convolution sum for the first element of ca sequence with ch sequence, called " one element convolution sum" ; cc2 is a convolution sum of the 1'th and 2'th element of ca sequence with ch sequence, called " two element convolution sum"; and so on.
Here by the calculation at a time we can obtain some "element convolution sum", it is a property in the paper.

IV. PROGRAM AND CALCULATION

(4.1) Calculation schematic diagram
Above processes to find out convolution sum for two real number or complex number can be shown in Fig.3

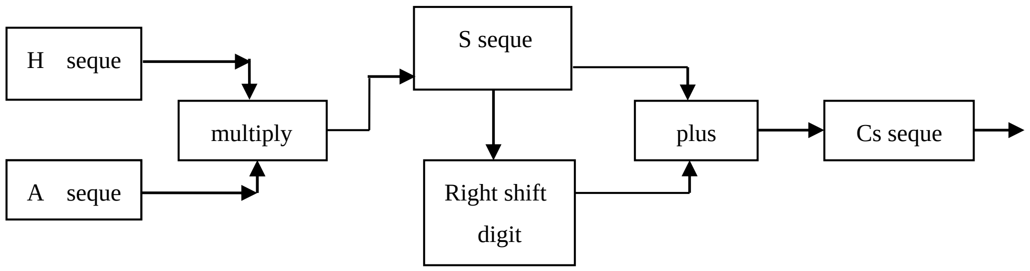

Fig.3 calculation schematic diagram

According to fig.3 a simple program for calculation has be made. Some calculation examples have be exhibted in fllowing.

(4.2) Calculation examples

Example 1 Convolution sum of two real number sequence

Two real number sequence are: a = [1 0 2 1], h = [4 3 2 1 3] , a element number is la=4, h element number is lh=5, and lcs=8. By suppling 0 element to make h sequence length is LCS=8. Using above program each element convolution sum has be obtained in the following

One element convolution sum: [4 3 2 1 3]
Two element convolution sum: [4 3 2 1 3]
Three element convolution sum: [4 3 10 7 7 2 6]
Four element convolution sum: [4 3 10 11 10 4 7 3]

Above "four element convolution sum" just is common convolution sum, but one ~ three element convolution sum are the resultants from calculation procees, and how to use them depond on the practice requiments. By the results from two sequence element multiply without advance digit, it can be seen that above results are all right.

Example 2 Convolution sum of two complex number sequence

After supling 0 elements, two complex sequences are:

Ch: [1. +j.5 1. +j.1 .2+j.1 0]
ca: [1. +1. .5+j.5 .4+j1.2]

By above program, the element convolution sum is obtained:

One element convolution sum: [. 5+j1.5 .9 +j1.1 -.8+j1.2]
Two element convolution sum: [0.5+j1.5 1.15+j1.85 -.35+j1.75 -.4+j0.6]
Three element convolution sum: [0.5+j1.5 1.15+j1.85 -.55+j3.15 -.12+j1.84 -1.12+j0.64]

Comparison above results with that from element multiply without advance digit method, two group results are agreement.

Example 3 Convolution sum of real number sequence with complex number sequence

For the real number sequence ca, we re-write as complex number with 0 imagennary part:

Ch: [1. +j.5 0.5+j1. 1. +j1. 0 0]
ca : [1.+j 0 2.0+j 0 1.+j 0]

By above program, the element convolution sum is obtained:

One element convolution sum: [1. +j.5 .5+j1. 1. +j1.]

Two element convolution sum: [1. +j.5 2.5+j2. 2. +j3. 2. +j2.]
Three element convolution sum: [1. +j.5 2.5+j2. 3.+j3.5 2.5+j3. 1. +j1.]

Comparison above results with that from element multiply without advance digit method, two group results are agreement.

V. CONCLUTION

(1) The algorithm in the paper has advantanges: simple, easily implement.
(2) This algorithm can output normal convolution sum, also can output 'element convolution sum', and it is a property (or advantenge) in the paper.
(3) This algorithm useful region is wide, it can find convolution sum both for two real elements, two complex elements, and for real element and complex element.

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