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Mathematical Model of the Drying Process of Leather

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ABSTRACT:*The drying behavior of leather was described using a mathematical model for convective drying process mostly used in the leather industries. The computed transient leather temperature agrees with the theoretical and experimental values. Temperature and moisture variations are computed using the numerical finite-difference method.The model was used to examine the effects of the following parameters, temperature and humidity in the dryer, initial moisture content of the sample, and heat and mass transfer coefficients. The model can be used to predict the variation of these parameters with reasonable accuracy.*

KEYWORDS:*Finite-difference, Convective drying, Leather, transient temperature, moisture content.*

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I. INTRODUCTION

Convective drying plays an essential role,among the many processes performed in the leather industry [2]. Leather fabrication is presently an important industrial development, similar to other technologically advanced processes [5]. Some of the unit operations involved in these industries, especially the drying processes, is still based on empiricism and tradition, with very little use of scientific principles [6].

Researches have been conducted on the drying process of leather through a variety of mathematical models [1], [3] and [4]. Beard [7] assumed that leather can consist of two layers, one dry and one wet layer. The analysis of Beard did not describe the activities going on inside the leather. The experimental result was based on the use of two experimental constantsused to fit data to the measured temperature variation inside the dryer. This papermodifiesthe mathematical model developed by Nordon and David [8] to determine the transient temperature and moisture concentration distribution of leather in the dryer. The distributions of these parameters are computed using the numerical method of finite-differences. The effects of operating parameters such as dryer temperature, humidity and initial moisture content of the leather samples were examined.

1. Mathematical model

Themathematical model derived in[8] is given by

$$D\frac{\partial C_A}{\partial x^2} = \frac{\partial C_F}{\partial t} + \frac{\partial C_A}{\partial t} (1)$$

and

$$\frac{\partial^2 T}{\partial x^2} = \rho C_P \frac{\partial T}{\partial t} - \frac{\partial C_F}{\partial t} \ (2)$$

The modified model of the differential equations are given by

$$D\frac{\partial C_A}{\partial x^2} = \frac{\partial C_F}{\partial t} + \frac{\partial C_A}{\partial t} \ (3)$$

and

$$k\frac{\partial^2 T}{\partial x^2} = \rho C_P \frac{\partial T}{\partial t} - \lambda\frac{\partial C_F}{\partial t} \ (4)$$

In which latent heat of evaporation (λ) and the thermal conductivity (k) are introduce to determine their effects on the leather with the boundary conditions for convective heatand mass transfer at the leather surface given as.

$$q = h_e(T_e - T) (5)$$

$$\overline{m} = h_m(C_e - C_A) \ (6)$$

The force that determines the rate of mass transfer inside the fabric is the difference between the relative humidity of the air in the pores and the leather. The rate of moisture exchange in this study is assumed to be proportional to the relative humidity difference. Hence, the rate equation for mass transfer is given as.

$$\frac{1}{\rho(1-\varepsilon)}\frac{\partial C_F}{\partial t} = K(y_A - y_F) \ \ (7)$$

With relative humidity of air and leather assumed to be

$$y_A = \frac{C_A RT}{P_s} \ (8)$$

$y_F = \frac{C_F}{\rho(1-\varepsilon)}$(9)

The rate constant in Equation (7) is an unknown empirical constant and the effect of this constant was examined.The value of the rate constant was varied from k = 1 to k =10. The resulting computed leather surface temperatures are compared in figure 1

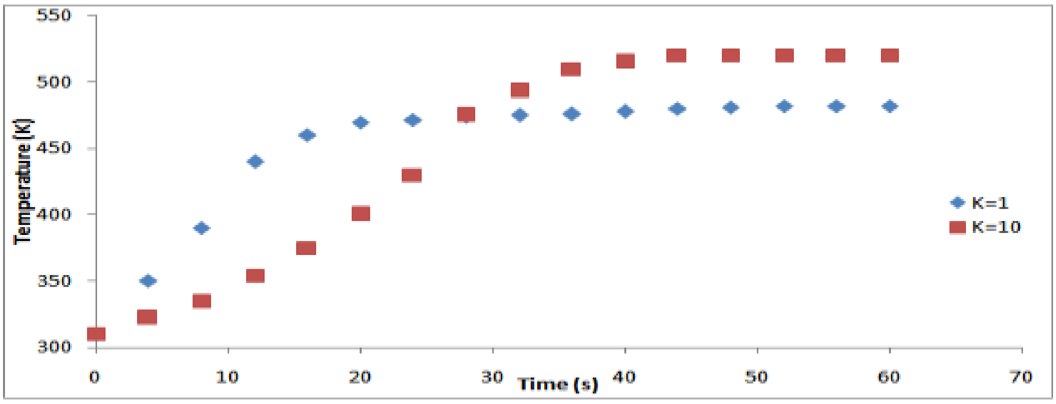

Figure 1: Rate constant effect on the surface temperature of leather

When the rate constant is small ($k < 1$) the evaporation rate is so small that the moisture content decreases slowly. Initially, the surface temperature increases rapidly, but later reduces. However when k is greater than 1.0 ($k > 1$), the effect of the rate constant on the surface temperature distribution is not significant. This implies that when the rate constant is greater than 1.0 ($k > 1$), the evaporation rate is high and the drying process is controlled by the moisture diffusion mechanism inside the fabric. The rate constant is assumedto be 1.0 ($k = 1$), in the computations below. The variables and the constants are as defined in the appendix.

2. Model prediction and discussion

The temperature and moisture content were computed using the modified model.The parameters used for the base condition are shown in Table 1.

Parameter	**Value with Units**
Dryer Temperature	450.00K
Heat Transfer coefficient	70.00W/m^2 K
Mass Transfer coefficient	0.08m/s
Leather Thickness	1.20mm
Porosity	0.90
Initial Moisture	50.00%RH
Drying Air Moisture	0.02Kg/m^3

Table 1: Base condition parameters

The transient temperatures of the surface and centre of the leather were computed, using the data in Table 1. From Figure 2, observed that the surface and centre temperature increase rapidly at the initial stage up tosaturation temperature, at that point the moisture in the leather starts evaporating.

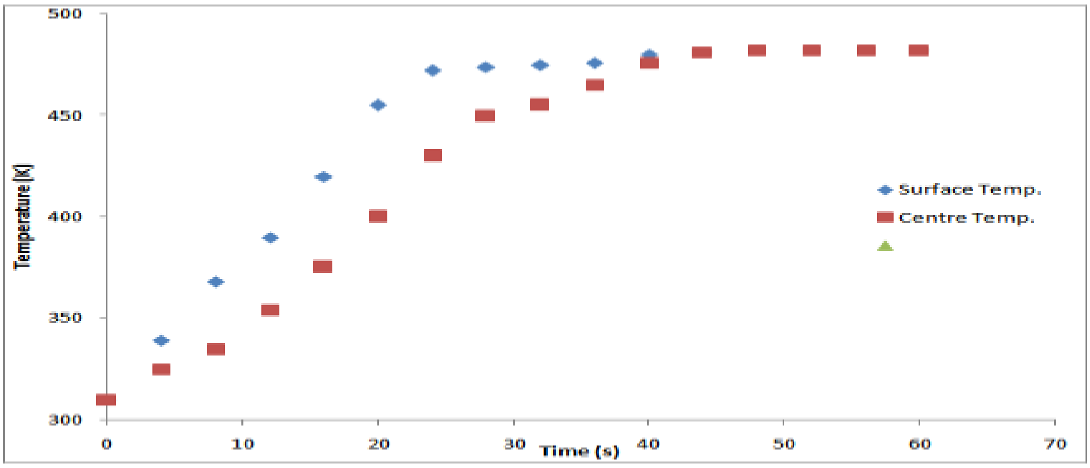

Figure 2: Temperature variation atthe surface and centre of leather.

The difference between the surface temperature and the centre temperature increases due to the different moisture contents of the surface and the centre.

At this stage, the leather starts to dry from the surface, and the moisture in the interior is transferred to the leather surface which decreases the moisture contentof the leather. Therefore, the surface and the centre temperatures converge to reach the external air temperature.

The moisture variations of the surface and the centre of the leather were also computed and are shown in figure 3. Initially, the surface moisture content decreases rapidly, but later this rate reduces due to moisture transferred to the external air from the leather surface. The centre moisture content remains constant for a short time, and then decreases rapidly because the moisture content difference between the surface and the interior of the leather becomes high.

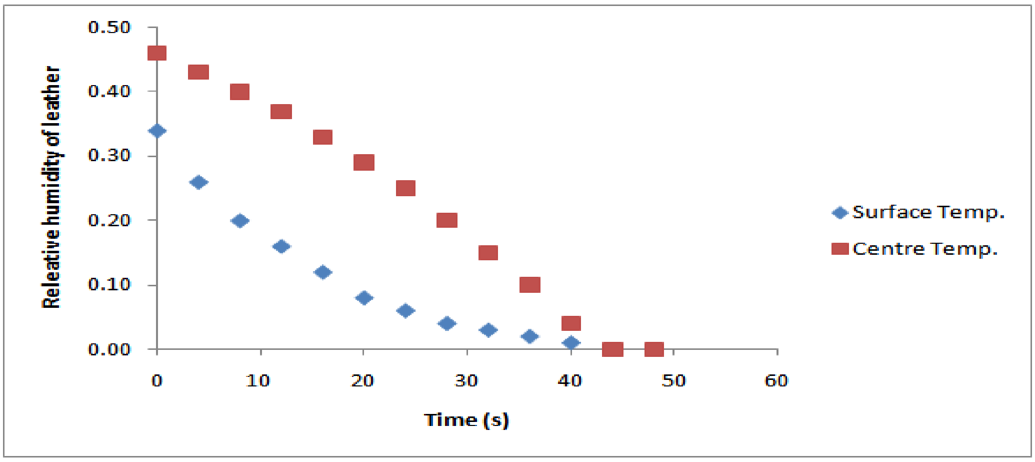

Figure 3: Moisture content of leather at the surface and centre

After drying out, both centre and the surface moisture content converge to reach the external air moisture content, and dryer air temperature.

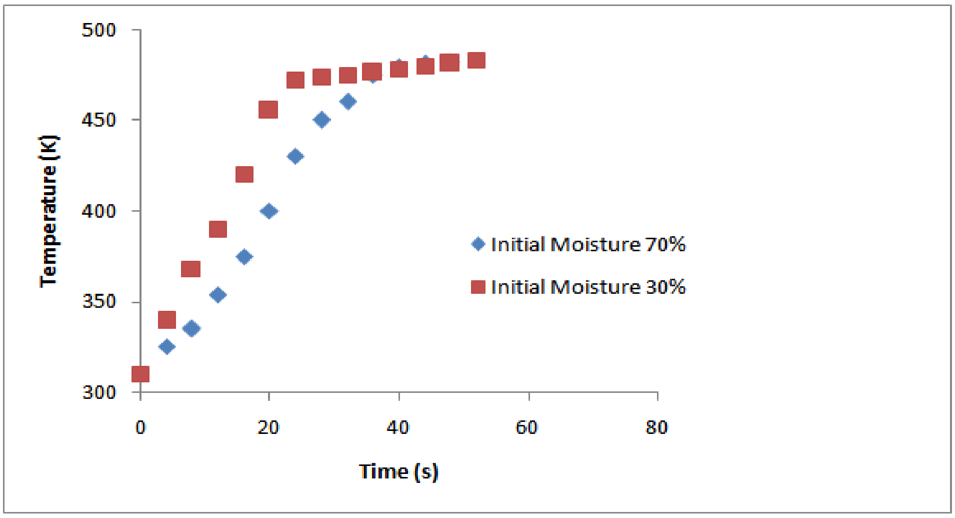

Figure 4: Initial moisture content of fabrics effect

This model is used to predict the effect of certain parameters on the temperature variation of the leather.

These parameters include the operation conditions of the dryer, such as the initial moisture content of the leather, heat and mass transfer coefficient, drying air moisture content, and dryer's air temperature.

Figure 4 shows the computed result of the effect of the initial moisture content of the leather. When the initial moisture content is high, the temperature rise is relatively small and drying takes a long time.This may be because the higher moisture content needs much more heat for evaporation from the leather. Also, the saturation temperature for higher moisture content is lower, and the temperature rise in the initial stage is comparatively small.

The leather temperature was computed to investigate the effect of heat and mass transfer coefficient.

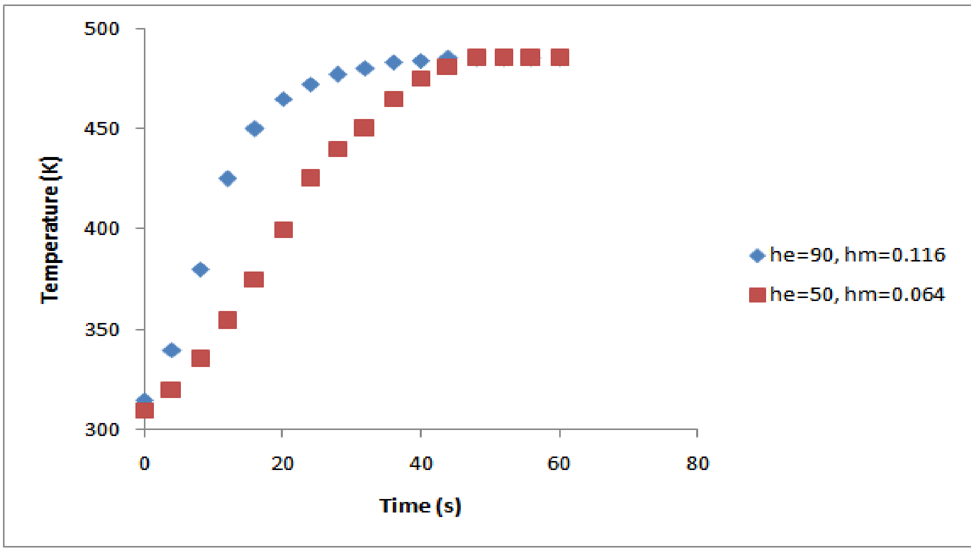

Figure 5: Heat and mass transfer coefficients effect

A similarity was assumed between heat and mass transfer coefficient and were determined using this assumption.The computed results are compared in Figure 5.When the heat and mass transfer coefficients are high, the leather temperature rise is higher and the time required for drying is relatively small

The drying air moisture content and the computed values of the model are shown in figure 6.When the moisture content is lowtemperature increase is relatively small, and hence the time required for complete drying is smallcomparatively long.

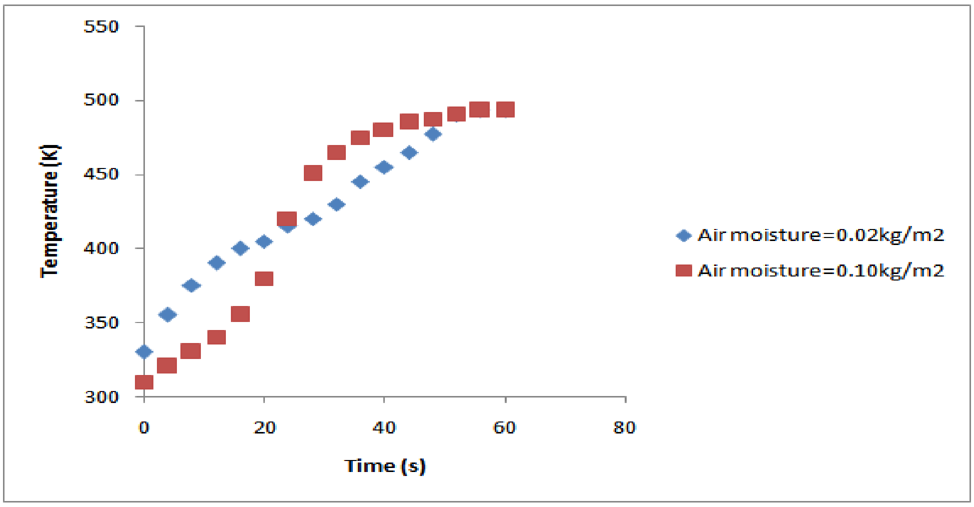

Figure 6: Drying air moisture content effect

When the moisture content is high, the initial temperature rise of the leather also becomes high.This is because the saturation temperature at the initial stage depends on the drying air moisture content. After the initial temperature rise, however, the temperature increase is relatively small and hence the time required for complete drying is long comparatively.The dryer air temperaturewas also investigated, and the computed results are shown in Figure 7. Consequently, when the dryer air temperature is high the temperature rise of the leather is excellent.

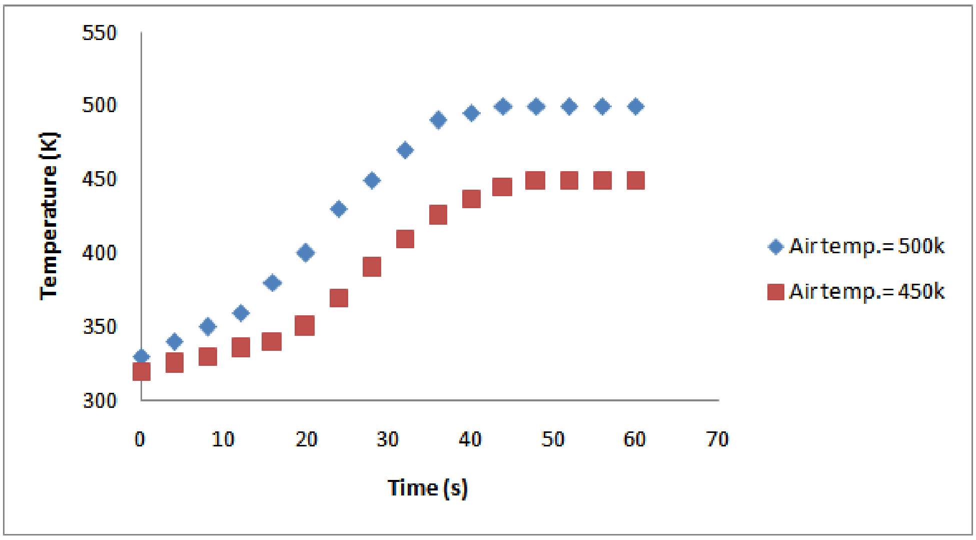

Figure 7: Drying's air temperature effect

II. CONCLUSION

The model developed in this work can be used to predict transient variations in temperature and moisture content distribution of leather in the dryer with reasonable accuracy.The effect of the temperature and humidity of the dryer, the initial moisture content of leather and the heat and mass transfer coefficient can be predicted. With this model, energy consumption can be potentially reduced by optimizing the drying conditions of the dryer.

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APPENDIX,

C_A	Moisture content of air in leather pore [kg/m^3]
C_e	Moisture content of external air [kg/m^3]
C_F	Moisture content in leather [kg/m^3]
C_P	Specific heat [kJ/kg K]
D	Diffusion coefficient [m^2/s]
G	Mass flow rate [kg/m^{2}s]
h_e	Heat transfer coefficient [W/m^2 K]
h_m	Mass transfer coefficient [m/s]
K	Rate constant [1/s]
k	Thermal conductivity [W/m K]
$\overline{m}$	Mass transfer rate [kg/m^2 s]
P_s	Saturation pressures [Pa]
q	Convection heat transfer rate [W/m^2]
R	Gas constant [kJ/K]
T	Temperature [K]
T_e	External air temperature [K]
t	Time [s]

$\hat{y}_A$	Relative humidity of air in pores of leather
$\hat{y}_F$	Relative humidity of leather
ε	Porosity
λ	Latent heat of evaporation [KJ/kg]
ρ	Density [kg/m^3]

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