

Properties Of Gsp-Separation Axioms In Topology

¹Govindappa Navalagi And ^{*2}R G Charantimath

¹Department of Mathematics KIT Tiptur 572202 ,Karnataka India

^{*2}Department of Mathematics KIT Tiptur 572202 ,Karnataka India

Corresponding Author: R G Charantimath

ABSTRACT: In this paper we define and study *gsp-separation axioms*, namely, $gsp-T_0$, $gsp-T_1$, $gsp-T_2$ $gsp-R_0$ and $gsp-R_1$ spaces using *gsp-open sets* due to J.Dontchev (1995). Also, we study the comparison of these *gsp-separation axioms* with the existing *gp-separation axioms* and *αg-separation axioms*. Further, we also introduce and study the notions of g^* -separations.

KEY WORDS: *semipreopen sets, gsp-closed sets, g^* -closed sets, preopen sets, gs-closed sets, gsp-irresoluteness, and strongly g^* -continuums functions*

DATE OF SUBMISSION: 09-07-2018

DATE OF ACCEPTANCE: 23-07-2018

I. INTRODUCTION

In 1982 A S Mashhour et al [8] have defined and studied the concept of pre-open sets and Spre-continuous functions of topology. In 1983 S.N.Deeb et al [5] have defined and studied the concepts of pre-closed sets, preclosure operator, p-regular spaces and pre-closed functions in topology. In 1998, T.Noiri et al [14] have defined the concepts of gp-closed sets and gp-closed functions in topology. In 2012, Navalagi et al. [12] have defined and studied the concepts of Generalized pre-separation axioms like, $gp-T_0$, $gp-T_1$, $gp-T_2$, $gp-R_0$ and $gp-R_1$ spaces using gp-open sets due to T.Noiri et al [14]. In 1986, D. Andrijivic [1] introduced and studied the notion of semipre open sets, semipreclosed sets, semipreinterior operator and semipre-closed operator in topological spaces. In 1965, Njstad [13] has defined the concept of α -open sets in topological spaces. In 1983, A.S.Mashhour et al [9] have defined and studied the concepts of α -closed sets, α -closure operator, α -continuity, α -openness and α -closedness in topology. For the first time, N.Levine [6] has introduced the notion g -closed sets and g -open sets in topology. In 1994, H.Maki et al [7] have defined and studied the concepts of αg -closed sets in topological spaces.

in topological spaces. Recently, in 2014 Thakur C.K.Raman et al [3 & 15] have defined and studied the concepts of αg -separation axioms in tology. In 1995, J.Dontchev [4] has defined and studied the concept of gsp -closed sets, gsp -open sets, gsp -continuous functions and gsp -irresoluteness in topology. In this paper, using gsp -open sets, we define and study the notions of $gsp-T_0$, $gsp-T_1$, $gsp-T_2$ $gsp-R_0$ and $gsp-R_1$ spaces.

II. PRELIMINARIES

Throughout this paper (X, τ) and (Y, σ) (or simply X and Y) denote topological spaces on which no separation axioms are assumed unless explicitly stated. If A be a subset of X , the closure of A and the interior of A is denoted by $Cl(A)$ and $Int(A)$, respectively.

We give the following define are useful in the sequel :

Definition 2.1: The subset of A of X is said to be.

- (i) A pre-open [8] set, if $A \subset Int(Cl(A))$
- (ii) A semi-pre open [1] set, if $A \subset Cl(Int(Cl(A)))$
- (iii) α -open [13] set, if $A \subset Int(Cl(Int(A)))$

The compliment of a pre-open (resp., semipre-open, α -open) set is called pre-closed [5] (resp., semipre-closed [1], α -closed [9]) set in space X . The family of all pre-open (resp. semipre-open, α -open) sets of a space X is denoted by $PO(X)$ (resp., $SPO(X)$, $\alpha O(X)$) and that of pre-closed (resp. semipre-closed, α -closed) sets of a space X is denoted by $PF(X)$, (resp. $SPF(X)$, $\alpha F(X)$).

Definition 2.2[5] : The intersection of all pre-closed sets of X containing subset A is called the pre-closure of A and is denoted by $pCl(A)$.

Definition 2.3[1] : The intersection of all semipre-closed sets of X containing subset A is called the semipre-closure of A and is denoted by $spCl(A)$.

Definition 2.4[9] : The intersection of all α -closed sets of X containing subset A is called the α -closure of A and is denoted by $\alpha Cl(A)$.

Definition 2.5[5]: The union of all pre-open sets of X contained in A is called the pre-interior of A and is denoted by $pInt(A)$.

Definition 2.6[1]: The union of all semipre-open sets of X contained in A is called the semipre-interior of A and is denoted by $spInt(A)$.

Definition 2.7[9]: The union of all α -open sets of X contained in A is called the α -interior of A and is denoted by $\alpha Int(A)$.

Definition 2.8 : A sub set A of a space X is said to be :

- (i) a generalized closed (briefly, g- closed) [6] set if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in (X, τ)
- (ii) a α - generalized closed (briefly, αg - closed) [7] set if $\alpha Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in (X, τ)
- (iii) a generalized semi-preclosed (briefly, gsp- closed) [4] set if $spCl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ)
- (iv) a generalized pre -closed (briefly, gp- closed) [14] set if $pCl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ)

The complement of a g-closed (resp, αg -closed, gsp-closed, , gp-closed) set in X is called g-open (resp. αg -open, gsp- open, , gp- open) set in X . The family of all gsp-open sets of X is denoted by $GSPO(X)$.

Definition 2.9[12]: The intersection of all gp-closed sets of X containing subset A is called the gp-closure of A and is denoted by $gpCl(A)$.

Definition 2.10[3]: The intersection of all αg -closed sets of X containing subset A is called the αg -closure of A and is denoted by $\alpha gCl(A)$.

Definition 2.11 [12]: A space X is called generalized pre- T_1 (briefly written as $gp-T_1$) iff to each pair of distinct points x,y of X , there exists a pair of gp-open sets containing x but not y and the other containing y but not x .

Definition 2.12 [12] : A space X is said to be $gp-T_2$ space if for each pair of distinct points of X there exist disjoint gp-open sets containing them.

Definition 2.13 [3&15]: A space X is called α -generalized- T_0 (briefly written as $\alpha g-T_0$) iff to each pair of distinct points x,y of X , there exists a αg -open set containing one but not the other.

Definition 2.14 [3&15]: A space X is called α -generalized- T_1 (briefly written as $\alpha g-T_1$) iff to each pair of distinct points x,y of X , there exists a pair of αg -open sets containing x but not y and the other containing y but not x .

Definition 2.15 [10]: A space X is called semipre- T_0 (briefly written as semipre- T_0) iff to each pair of distinct points x,y of X , there exists a semipre-open set containing one but not the other.

Definition 2.16 [10]: A space X is called semipre- T_1 (briefly written as semipre- T_1) iff to each pair of distinct points x,y of X , there exists a pair of semipre-open sets containing x but not y and the other containing y but not x .

III. PROPERTIES OF GSP-SEPARATION AXIOMS

We, define the following

Definition 3.1: A space X is called $gsp-T_0$ iff to each pair of distinct points x,y of X , there exists a gsp -open set containing one but not the other .

Definition 3.2 : A space X is said to be $gp-T_0$ space if for each pair of distinct points of X there exists a gp -open set containing one but not the other.

Clearly, every semipre- T_0 is $gsp-T_0$. Also, we have the following ,

Note 3.3 : In view of definitions of αg -closed, gp -closed sets and gsp -closed sets, the following is observed in [2] :

$$\alpha g\text{-closed set} \Rightarrow gp\text{-closed set} \Rightarrow gsp\text{-closed set}$$

Hence we have the following implication:

$$\alpha g\text{-}T_0\text{-space} \rightarrow gp\text{-}T_0\text{-space} \rightarrow gsp\text{-}T_0\text{-space}$$

We ,define the following

Definition 3.4 : A generalized semipre-closure of set A is denoted by $\text{gspCl}(A)$, is the intersection of all gsp-closed sets that contain A

We characterize gsp-T_0 -spaces in the following

Theorem 3.5: If in any topological space X , gsp-closures of distinct points are distinct, then X is gsp-T_0

Proof: Let $x, y \in X$, $x \neq y$ imply $\text{gspCl}(\{x\}) \neq \text{gspCl}(\{y\})$. Then there exists a point $z \in X$ such that z belongs one of two sets, say, $\text{gspCl}(\{y\})$ but not to $\text{gspCl}(\{x\})$. If we suppose that $z \in \text{gspCl}(\{x\})$, then $z \in \text{gspCl}(\{y\}) \subsetneq z \in \text{gspCl}(\{x\})$, which is contradiction. So, $y \in X - \text{gspCl}(\{x\})$, where $X - \text{gspCl}(\{x\})$ is gsp-open set which does not contain x . This shows that X is gsp-T_0 .

Next, we give the following

Theorem 3.6: A space X is gsp-T_0 iff $\text{gspCl}(\{x\}) \neq \text{gspCl}(\{y\})$ for every pair of distinct points x, y of X .

Proof follows from Th.3.5.

Theorem 3.7 : Every sub space of an gsp-T_0 space is gsp-T_0 space.

Proof: Let X be a space and (Y, τ^*) be a subspace of X where τ^* is the relative topology of τ on Y . Let x, y be two distinct points of Y . As $Y \subset X$, x and y are distinct points X . Since X is an gsp-T_0 space, there exists an gsp-open set G such that $x \in G$ but $y \notin G$. Then $G \cap Y$ is an gsp-open set in (Y, τ^*) which contains x but does not contain y . Hence (Y, τ^*) is an gsp-T_0 space.

We, define the following

Definition 3.8: A function $f: X \rightarrow Y$ is said to be point -gspclosure 1-1 iff $x, y \in X$ such that $\text{gspCl}(\{x\}) \neq \text{gspCl}(\{y\})$ then $f(\text{gspCl}(\{x\})) \neq f(\text{gspCl}(\{y\}))$

Theorem 3.9: If function $f: X \rightarrow Y$ is point -gspclosure 1-1 and X is gsp-T_0 then f is 1-1

Proof: Let $x, y \in X$ with $x \neq y$. Since X is gsp-T_0 , then $\text{gspCl}(\{x\}) \neq \text{gspCl}(\{y\})$ by Theorem 3.6. But f is point -gspclosure 1-1 implies that $f(\text{gspCl}(\{x\})) \neq f(\text{gspCl}(\{y\}))$. Hence $f(x) \neq f(y)$. Thus, f is 1-1.

Theorem 3.10: Let $f: X \rightarrow Y$ be a mapping from gsp-T_0 space X into gsp-T_0 space Y . Then f is point-gspclosure 1-1 iff f is 1-1

Prof follows from Theorem 3.5 above

Theorem 3.11: Let $f: X \rightarrow Y$ be an injective gsp-irresolute mapping. If Y is gsp-T_0 then X is gsp-T_0 .

Proof: Let $x, y \in X$ with $x \neq y$. Since f is injective and Y is gsp-T_0 , there exists a gspopen set V_x in Y such that $f(x) \in V_x$ and $f(y) \notin V_x$ or there exists a gspopen set V_y in Y such that $f(y) \in V_y$ and $f(x) \notin V_y$ with $f(x) \neq f(y)$. By $\text{gsp-irresoluteness}$ of f , $f^{-1}(V_x)$ is gspopen set in X such that $x \in f^{-1}(V_x)$ and $y \notin f^{-1}(V_x)$ or $f^{-1}(V_y)$ is gspopen set in X such that $y \in f^{-1}(V_y)$ and $x \notin f^{-1}(V_y)$. This shows that X is gsp-T_0 .

We define the following mapping analogous to always semi-pre-open mapping.

Definition 3.12 : A mapping $f: X \rightarrow Y$ is said to be always gsp-open , if the image of every gsp-open set of X is gsp-open in Y .

Lemma 3.13 : The property of a space being gsp-T_0 is preserved under one-one, onto and always gsp-open mapping.

Proof: Let X be a gsp-T_0 space and Y be any topological space. Let $f: X \rightarrow Y$ be a one-one, onto always gsp-open mapping from X to Y . Let $u, v \in Y$ with $u \neq v$. Since f is one-one, onto, there exist distinct points $x, y \in X$. Such that $f(x) = u$, $f(y) = v$. Since X is on gsp-T_0 space. There exists gsp-open set G in X such that $x \in G$ but $y \notin G$. Since f is always gsp-open , $f(G)$ is an gsp-open set containing $f(x) = u$ but not containing $f(y) = v$. Thus there exists an gsp-open set $f(G)$ in y such that $u \in f(G)$ but $v \notin f(G)$ and hence Y is an gsp-T_0 space.

Next, we define the following.

Definition 3.14 : A sub set A of a space X is called a gspD-set if there are two gsp-open subsets U and V such that $U \neq X$ and $A = U - X$.

Clearly, every gsp-open set is gspD-set .

We, define the following

Definition 3.15: A space X is called a gsp-D_0 if for any disjoint pair of points x and y of X there exists a gspD-set of X containing x but not y or a gspD-set of X containing y but not x .

Clearly, every gsp-T_0 space is gsp-D_0 space.

We prove the following

Theorem 3.16 : If $f: X \rightarrow Y$ is gsp-irresolute surjective function and A is a gspD-set in Y , then the inverse image of A is a gspD-set in X .

Proof: Let A be a gspD-set in Y . Then there are gsp-open sets U_1 and U_2 in Y such that $A = U_1 - U_2$ and $U_1 \neq Y$. By the gsp-irresoluteness of f , $f^{-1}(U_1)$ and $f^{-1}(U_2)$ are gsp-open set in X . Since $U_1 \neq Y$, we have $f^{-1}(U_1) \neq X$. Hence $f^{-1}(A) = f^{-1}(U_1) - f^{-1}(U_2)$ is a gspD-set.

We define the following .

Definition 3.17 : A space (X, τ) is gsp- T_1 if and only if for $x, y \in X$ such that $x \neq y$, there exists a gsp-open set containing x but not y and there is a gsp-open set containing y but not x .

It is easy to verify the following :

- (i) Every semipre- T_1 space is an gsp- T_1 space
- (ii) Every gsp- T_1 space is an gsp- T_0 space
- (iii) Every sg- T_1 space is an gsp- T_1 space . Also,

In view of above Note-3.3 , we have the following implication :

$$\text{ag-}T_1\text{-space} \rightarrow \text{gp-}T_1\text{-space} \rightarrow \text{gsp-}T_1\text{-space}$$

Theorem 3.18 : A space X is an gsp- T_1 space if and only if $\{x\}$ is gsp-closed in X for every $x \in X$.

Proof: Let x, y be two distinct points X such that $\{x\}$ and $\{y\}$ are gsp-closed. Then $X - \{x\}$ and $Y - \{y\}$ are gsp-open in X such that $y \in X - \{x\}$ but $x \notin X - \{x\}$ and $x \in X - \{y\}$ but $y \notin X - \{y\}$. Hence, X is an gsp- T_1 space.

Conversely, let X be an gsp- T_1 space and x be any arbitrary point of X . If $y \in X - \{x\}$, then $y \neq x$. Now the space being gsp- T_1 and y is a point different from x , there exists an gsp-open set G_y such that $y \in G_y$ but $x \notin G_y$. Thus for each $y \in X - \{x\}$, there exists an gsp-open set G_y such that $y \in G_y \subset X - \{x\}$. Therefore $\bigcup \{G_y \mid y \neq x\} \subset X - \{x\}$ which implies that

$$X - \{x\} \subset \bigcup \{G_y \mid y \neq x\} \subset X - \{x\}.$$

Therefore , $X - \{x\} = \bigcup \{G_y \mid y \neq x\}$. Since G_y gsp-open in X and the union of gsp-open sets in X is gsp-open in X , $X - \{x\}$ is gsp-open in X and so $\{x\}$ is gsp-closed.

Recall the following.

Definition 3.19 [11] : A topological space (X, τ) is ags-symmetric if for any x and y in X , $x \in \text{agsCl}(\{y\})$ implies $y \in \text{agsCl}(\{x\})$.

We , define the following .

Definition 3.20 : A topological space (X, τ) is semipre symmetric if for x and y in x , $x \in \text{semipreCl}(\{y\})$ implies $y \in \text{semipreCl}(\{x\})$.

Definition 3.21 : A topological space (X, τ) is gsp-symmetric if for any x and y in X , $x \in \text{gspCl}(\{y\})$ implies $y \in \text{gspCl}(\{x\})$.

Clearly , every semipre-symmetric space is gsp-symmetric space.

Theorem 3.22 : If $\{x\}$ is gsp-closed for each x in X then a space X is semipre-symmetric.

Proof: Suppose $x \in \text{gspCl}(\{y\})$ and $y \notin \text{gspCl}(\{x\})$. Since $\{y\} \subset X - \text{gspCl}(\{x\})$ and $\{y\}$ is gsp-closed, $\text{gspCl}(\{y\}) \subset X - \text{gspCl}(\{x\})$. Thus $x \in X - \text{gspCl}(\{y\})$, a contradiction.

Theorem 3.23 : If a space X is extremely disconnected (i.e., closure of every open set is open) and semipre-symmetric, then $\{x\}$ is gsp-closed, for each x in X .

Proof: Suppose $\{x\} \subset U$ where U is semipre-open and $\text{gspCl}(\{x\}) \not\subset U$.

Then $\text{gspCl}(\{x\}) \cap (X - U) \neq \emptyset$ Let $y \in \text{gspCl}(\{x\}) \cap (X - U)$. We have $x \in \text{gspCl}(\{x\}) \subset (X - U)$ and $x \notin U$, a contradiction. Hence $\{x\}$ is gsp-closed in X .

Corollary 3.24 : If X is extremely disconnected, then X is gsp- T_1 if and only if X is semipre-symmetric.

Proof: Obvious

Next, we have the following invariant properties.

Theorem 3.25 : Let $f: X \rightarrow Y$ be an gsp-irresolutes injective map. If Y is gsp- T_1 , then X is gsp- T_1 .

Proof: Assume that Y is gsp- T_1 . Let $x, y \in Y$ be such that $x \neq y$. Then there exists a pair of gsp-open sets u, v in Y such that $f(x) \in u$, $f(y) \in v$ and $f(x) \notin v$, $f(y) \notin u$. Then $x \in f^{-1}(u)$, $y \in f^{-1}(v)$, $x \notin f^{-1}(v)$ and $y \notin f^{-1}(u)$. Since f is gsp-irresolute, X is gsp- T_1 .

Corollary 3.26: A topological space (X, τ) is gsp-T_1 if and only if every finite subset of X is gsp-closed .

We, define the following

Definition 3.27 : A space X is called gsp-D_1 if for any distinct pair of points x and y of X there exists a gsp-D set of X containing x but y and a gsp-D set of X containing y but not x .

Clearly, every gsp-T_1 space is gsp-D_1 space.

Theorem 3.28: If Y is a gsp-D_1 and $f: X \rightarrow Y$ is gsp-irresolute and bijective, then X is gsp-D_1 .

Proof: Suppose that Y is a gsp-D_1 space. Let x and y be any pair of distinct points in X . Since f is injective and Y is gsp-D_1 , there exist gsp-D sets G_x and G_y of Y containing $f(x)$ and $f(y)$ respectively, such that $f(y) \notin G_x$ and $f(x) \notin G_y$. By Theorem 3.16, $f^{-1}(G_x)$ and $f^{-1}(G_y)$ are gsp-D sets in X containing x and y respectively. This implies that X is a gsp-D_1 space.

We, define and study the concept of gsp-R_0 spaces in the following :

Definition 3.29: Let X be a topological space and $A \subset X$. Then the generalized pre-kernel of A denoted by $\text{gsp-ker}(A)$, is defined to be the set $\text{gsp-ker}(A) = \{G \in \text{GSPO}(X) | A \subset G\}$

Lemma 3.30 : Let X be a topological space and $x \in X$. Then $y \in \text{gsp-ker}(\{x\})$ if and only if $x \in \text{gspCl}(\{y\})$

Proof: Suppose that $y \in \text{gsp-ker}(\{x\})$. Then there exists a gsp-open set V containing x such that $y \notin V$. Therefore, we have $x \in \text{gspCl}(\{y\})$.

Conversely, Suppose that $x \in \text{gsp-ker}(\{y\})$. Then there exists a gsp-open set V containing y such that $x \notin V$. Therefore, we have $y \in \text{gspCl}(\{x\})$.

Lemma.3.31: Let X be a topological space and A be a subset of X . Then $\text{gsp-ker}(A) = \{x \in X | \text{gspCl}(\{x\}) \cap A \neq \emptyset\}$

Proof: Let $x \in \text{gsp-ker}(A)$ and suppose $\text{gspCl}(\{x\}) \cap A = \emptyset$. Hence $x \in X \setminus \text{gspCl}(\{x\})$ which is a gsp-open set containing A . This is absurd. Since $x \in \text{gsp-ker}(A)$, consequently, $\text{gspCl}(\{x\}) \cap A \neq \emptyset$. Next, let $\text{gspCl}(\{x\}) \cap A \neq \emptyset$ and suppose that $x \notin \text{gsp-ker}(A)$. Then there exists gsp-open set U containing A and $x \notin U$. Let $y \in \text{gspCl}(\{x\}) \cap A$, hence, U is a gsp-neighbourhood of y where $x \notin U$. But this is a contradiction, Therefore $x \in \text{gsp-ker}(A)$ and the claim.

Now, we define the following

Definition 3.32: A space X is said to be gsp-R_0 space if every gsp-open set contains the gsp-closure of each of its singletons.

Clearly, every gsp-R_0 space is gsp-T_1 space.

We recall the following :

Definition 3.33 [12]: A topological space (X, τ) is said to be gp-R_0 space if every gp-open set contains the gp-closure of each of its singletons.

Definition 3.34 [3]: A topological space X is said to be ag-R_0 space if $\text{agCl}(\{x\}) \subset U$ Whenever U is ag-open and $x \in U$.

Hence, we have the following w.r.t. Note-3.3:

$$\text{ag-R}_0\text{-space} \rightarrow \text{gp-R}_0\text{-space} \rightarrow \text{gsp-R}_0\text{-space}$$

Now, we characterize the gsp-R_0 spaces in the following.

Theorem 3.35: For any topological space X the following properties are equivalent:

- (i) X is gsp-R_0 space;
- (ii) For any $F \in \text{GSPC}(X, \mathbb{T})$ $x \notin F \Rightarrow F \subset U$ and $x \notin U$ for some $U \in \text{GSPC}(X, \mathbb{T})$;
- (iii) For any $F \in \text{GSPC}(X, \mathbb{T})$ $x \notin F \Rightarrow F \cap \text{gspCl}(\{x\}) = \emptyset$;
- (iv) For any distinct points x and y either $\text{gspCl}(\{x\}) = \text{gspCl}(\{y\})$ or $\text{gspCl}(\{x\}) \cap \text{gspCl}(\{y\}) = \emptyset$.

Proof: (i) \Rightarrow (ii): Suppose $F \in \text{GSPC}(X, \mathbb{T})$ and $x \notin F$. Then by (i) $\text{gspCl}(\{x\}) \subset X \setminus F$. Set $U = X \setminus \text{gspCl}(\{x\})$ then $U \in \text{GSPC}(X, \mathbb{T})$, $F \subset U$ and $x \notin U$

(ii) \Rightarrow (iii): Let $F \in \text{GSPC}(X, \mathbb{T})$, $x \notin F$. Therefore, there exists $U \in \text{GSPC}(X, \mathbb{T})$ such that $F \subset U$ and $x \notin U$. Since $U \in \text{GSPC}(X, \mathbb{T})$, $U \cap \text{gspCl}(\{x\}) = \emptyset$. and $F \cap \text{gspCl}(\{x\}) = \emptyset$.

(iii) \Rightarrow (iv): Suppose that $\text{gspCl}(\{x\}) \neq \text{gspCl}(\{y\})$ for distinct points $x, y \in X$. There exist $z \in \text{gspCl}(\{x\})$ such that $z \notin \text{gspCl}(\{y\})$. One can also assume that $z \in \text{gspCl}(\{y\})$ such that $z \notin \text{gspCl}(\{x\})$. There exists

$V \in \text{GSPPO}(X, \mathcal{T})$ such that $y \notin V$ and $z \in V$. Hence $x \in V$. Therefore we obtain $x \notin \text{gspCl}(\{y\})$. By (iii) we obtain $\text{gspCl}(\{x\}) \cap \text{gspCl}(\{y\}) = \emptyset$. The proof of otherwise is similar.

(iv) \Rightarrow (i): Let $V \in \text{GSPPO}(X, \mathcal{T})$ and $x \in V$. For each $y \notin V$, $x \neq y$ and $x \notin \text{gspCl}(\{y\})$. This show that $\text{gspCl}(\{x\}) \neq \text{gspCl}(\{y\})$. By (iv) $\text{gspCl}(\{x\}) \cap \text{gspCl}(\{y\}) = \emptyset$ for each $y \in X|V$. Hence $\text{gspCl}(\{x\}) \cap (\cup\{\text{gspCl}(\{y\}) \mid y \in X|V\}) = \emptyset$. On the other hand, since $V \in \text{GSPPO}(X, \mathcal{T})$ and $y \notin X|V$. We have $\text{gspCl}(\{y\}) \subset X|V$. Therefore $X|V = \cup\{\text{gspCl}(\{y\}) \mid y \in X|V\}$. Therefore we obtain $(X|V) \cap \text{gspCl}(\{x\}) = \emptyset$ and $\text{gspCl}(\{x\}) \subset V$. Hence (X, \mathcal{T}) is gsp-R_0 space.

Finally, we define and study the following.

Definition 3.36: A space X is said to be a gsp-R_1 if for x, y in X with $\text{gspCl}(\{x\}) \neq \text{gspCl}(\{y\})$, there exists disjoint gsp -open sets U and V such that $\text{gspCl}(\{x\}) \subset U$ and $\text{gspCl}(\{y\}) \subset V$.

We recall the following

Definition 3.37[12] : A topological space X is said to be gp-R_1 space if for x, y in X with $\text{gpCl}(\{x\}) \neq \text{gpCl}(\{y\})$, there exist disjoint gp -open sets U and V such that $\text{gpCl}(\{x\})$ is a subset of U and $\text{gpCl}(\{y\})$ is a subset of V .

Definition 3.38[3] : A topological space X is said to be $\alpha\text{g-R}_1$ space if for x, y in X with $\alpha\text{gCl}(\{x\}) \neq \alpha\text{gCl}(\{y\})$, there exist disjoint αg -open sets U and V such that $\alpha\text{gCl}(\{x\})$ is a subset of U and $\alpha\text{gCl}(\{y\})$ is a subset of V

In view of Note-3.3 , we have the following :

$\alpha\text{g-R}_1\text{-space} \rightarrow \text{gp-R}_1\text{-space} \rightarrow \text{gsp-R}_1\text{-space}$

We, prove the following

Theorem 3.39: If X is gsp-R_1 , then X is gsp-R_0 -space.

Proof: Let U be a gsp -open and $x \in U$. If $y \notin U$ then since $x \notin \text{gspCl}(\{y\})$, $\text{gspCl}(\{x\}) \cap \text{gspCl}(\{y\}) = \emptyset$. Hence there exists a gsp -open V such that $\text{gspCl}(\{x\}) \subset V$ and $x \notin V$, which implies $y \notin \text{gspCl}(\{x\})$. Thus $\text{gspCl}(\{y\}) \subset U$. Therefore (X, \mathcal{T}) is gsp-R_0 space.

REFERENCE

- [1]. D.Andrijevic, Semipreopen sets, Math.Vensik 38(1),(1986), 24-32.
- [2]. I. Arokiarani, K.Balachandran and J.Dontchev "Some characterizations of gp -irresolute and gp -contiuous maps between Topological spaces" Men.Fac.Sci.Kochi Univ(Math) 20(1999), 93-104
- [3]. S.Balasubramanian and Ch.Chaitanya" On αg -Separation Axioms" International Journal Of Mathematics Archive-3(3), 2012,Page: 877-888
- [4]. J.Dontchev ,On generalizingsemi-pre open sets,Mem.Fac.Sci. Kochi.Univ. .Ser. A.Math,6(1995), 35-48.
- [5]. S.N.El-Deeb, I.A. Hasanein, A.S.Mashhour and T. Noiri, On p -regular spaces,Bull Math. Soc. Sci. Math. R.S.Roumanie (N.S), 27(75), (1983), 311-315.
- [6]. N.Levine, Generalized closed sets in Topology, Rend.Cric.Math.Palermo,19(2)(1970), 89-96.
- [7]. H.Maki, R.Devi and K.Balachandran, Associated topologies of generalized α -closed sets & α -generalized closed sets, Mem.Rac.Sci.Kochi.Univ.Ser.A.Math, 15(1994), 51-63.
- [8]. A.S. Mashhoor, M.E. Abd El-Monsef and S.N .El-Deeb, On Pre continuous and Weak Precontinuous Mappings, Proc. Math. Phys. Soc. Egypt, 53(1982), pp.47-53.
- [9]. A. S. Mashhour, I. A. Hasanein and S. N. El-Deeb, On α - continuous and α -open mapping, Acta. Math. Hungar. 41 (1983), 213-218.
- [10]. G B Navalagi " Definition Bank in General Topology" Topology Atlast Preprint# 449
- [11]. G.Navalagi, M.Rajmani and K.Viswanathan αg s-Separation Axioms In Topology Inernational Joueal of General Topology Vol. 1,No. 1 (June 2007),p. 43-53
- [12]. Govidapp Navalagi and Girish Sajjanshetter " generalized Pre- R_0 spaces". IJECSM. Vol.3,No.1, January-June 2012, pp 7-12.
- [13]. O.Njstad , On some classes of nearly open sets, Pacific Jour.of Mathematics, 15(1965),pp. 961-970.
- [14]. T. Noiri, H. Maki and J. Umehara, Generalized preclosed functions, Mem.Fac.Sci.Kochi. Univ. Ser.A, Math, 19(1998)13-24
- [15]. Thakur C.K.Raman, Vidyottama kumari, M.K.Sharma " α -generalized & α^+ -Separation Axioms for Topological Space" ISOR Journal of Mathematics (IOSR-JM) Volume10. Issue 3 Ver.VI (May-June 2014),pp 32-36.

R G Charantimath"Properties Of Gsp-Separation Axioms In Topology". International Journal of Mathematics and Statistics Invention (IJMSI), vol. 06, no. 04, 2018, pp. 04-09.