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Class of Estimators of Population Median Using New Parametric Relationship for Median

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ABSTRACT*: In this paper, we have defined a class of estimators of population median using the known information of population mean* $(\bar{X})$ *of the auxiliary variable making use of new parametric relationship for population median. We have derived the asymptotic expression for the MSE of any estimator of the proposed class and also its minimum value. As minimum MSE of all the estimators of defined class are same so to choose the optimum estimator of the class for the given population w.r.t.bias also, we have considered some important sub-classes of the generalized class. The optimum biases of the considered estimators are obtained (up to terms of order* n^{-1}*) and compared with each other. Theoretical results are supported by an empirical study based on twelve populations to show the superiority of the suggested estimator over others.*

KEYWORDS:*Auxiliary variable, SRSWOR, Bias, Mean square error, Median, Mode, Coefficient of skewness.*

I. INTRODUCTION

In many situations, population median is regarded as a more appropriate measure of central tedency than arithmetic mean such as when we are interested in the positional average as a measure of central tendency which is not affected much by extreme observations i.e. for skewed distributions or we are dealing with attributes or qualitative characters which can not be measured quantitatively but still can be arranged in ascending or decending order of magnitude. When it is unknown then in above situations, one is interested to estimate it. Initially, estimation of population median without auxiliary variable was materialized, after that some authors including Kuk and Mak (1989), Mak and Kuk(1993) , Garcia and Cebrian (2001), Singh et al. (2006), Al and Cingi (2010), Singh and Solanki (2013) used the known auxiliary information in estimation of population median.

Recently, Sharma et al (2016b) established the new parametric relationship for population median (M_d) as

$$M_d = \bar{Y} - \frac{k_1}{3}\frac{\mu_{30}}{S_y^2}$$

where for the Y- population $k_1 = \frac{\beta_y}{\lambda_y}$ is known constant. They proposed mean per unit estimator, the ratio-type and product-type estimators of population median M_d under the different situations as

$$\widehat{M}_{d_1}' = \bar{y} - \frac{\hat{k}_{1opt}}{3}\frac{m_{30}}{s_y^2},$$

$$\widehat{M}_{d_2}' = \bar{y}\frac{\bar{X}}{\bar{x}} - \frac{\hat{k}_{2opt}}{3}\frac{m_{30}}{s_y^2}\frac{\bar{X}}{\bar{x}},$$

and

$$\widehat{M}_{d_3}' = \bar{y}\frac{\bar{x}}{\bar{X}} - \frac{\hat{k}_{3opt}}{3}\frac{m_{30}}{s_y^2}\frac{\bar{x}}{\bar{X}}$$

where $\hat{k}_{1opt}\left(=\frac{\beta_y}{\lambda_y}\right)$, $\hat{k}_{2opt}\left(=\frac{\beta_y C_y+\beta_{1y}(C_x^2-C_{yx})+B}{C_y\left(\lambda_y+\beta_{1y}^2 C_x^2-2\beta_{1y}B\right)}\right)$ and $\hat{k}_{3opt}\left(=\frac{\beta_y C_y+\beta_{1y}(C_x^2+C_{yx})-B}{C_y\left(\lambda_y+\beta_{1y}^2 C_x^2+2\beta_{1y}B\right)}\right)$ are the conventional consistent estimators of the constants k_1, k_2 and k_3. Here the estimator $\widehat{M}_{d_1}'$uses no information on auxiliary variable x which is highly correlated with y, whereas $\widehat{M}_{d_2}'$ and $\widehat{M}_{d_3}'$ uses the known information of $\bar{X}$, which are of ratio-type and product-type estimators respectively.

In the present paper, we propose a class of estimators of population median using the new parametric relationship for population median when the population mean $(\bar{X})$ of the auxiliary variable is known. Asymptotic expressions for the Bias and MSE of any estimator of the proposed class and also its minimum value are obtained. We also consider some important members of the proposed class and up to the first degree of approximation the minimum MSE's of the considered estimators are same but biases are different. To have the rough idea about the optimum biases of the considered estimators and minmimum MSE of estimators of the class numerical illustration is given.

II. NOTATIONS AND EXPECTATIONS

Suppose a simple random sample of size n is drawn from a finite population of size N without replacement and observations on both study variables y and auxiliary variable x are taken. Let the values of variable y and x be denoted by Y_i and X_i respectively on the i^{th} unit of the population $i = 1,2 \dots N$ and the corresponding small letters y_i and x_i denote the sample values.

Taking,

$$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i, \qquad \bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$$

$$S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^2, \qquad S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})^2$$

$$\mu_{rs} = \frac{1}{N}\sum_{i=1}^{N}(Y_i - \bar{Y})^r (X_i - \bar{X})^s, \qquad \lambda_{rs} = \frac{\mu_{rs}}{\mu_{20}^{r/2}\mu_{02}^{s/2}}$$

$$m_{30} = \frac{n}{(n-1)(n-2)}\sum_{i=1}^{n}(y_i - \bar{y})^3,$$

Obviously

$\lambda_{11} = \rho_{xy} = \rho$(Correlation between x and y)

$\lambda_{30} = \beta_{1y}$(Coefficient of skewness of y)

$\lambda_{40} = \beta_{2y}$(Coefficient of kurtosis of y)

Defining,

$$\delta_0 = \frac{\bar{y}}{\bar{Y}} - 1, \qquad \delta = \frac{s_y^2}{S_y^2} - 1$$

$$\epsilon = \frac{\bar{x}}{\bar{X}} - 1, \qquad \eta_1 = \frac{m_{30}}{\mu_{30}} - 1$$

For the sake of simplicity, assume that N is large enough as compares to n so that finite population correction (fpc) terms are ignored throughout.

For the given SRSWOR, we have the following expectations,

$$E(\delta_0) = E(\delta) = E(\epsilon) = 0, \qquad E(\delta_0^2) = \frac{1}{n}C_y^2$$

$$E(\epsilon^2) = \frac{1}{n}C_x^2, \qquad E(\delta_0\epsilon) = \frac{1}{n}C_{yx}$$

$$E(\epsilon\delta) = \frac{1}{n}\lambda_{21}C_x, \qquad E(\delta_0\delta) = \frac{1}{n}\lambda_{30}C_y = \frac{1}{n}\beta_{1y}C_y$$

and up to terms of order n^{-1}

$$E(\eta_1) = 0$$

$$E(\delta^2) = \frac{1}{n}(\lambda_{40} - 1) = \frac{1}{n}(\beta_{2y} - 1),$$

$$E(\eta_1^2) = \frac{1}{n}\frac{(\lambda_{60} - 6\lambda_{40} - \lambda_{30}^2 + 9)}{\lambda_{30}^2} = \frac{1}{n}\frac{(\lambda_{60} - 6\beta_{2y} - \beta_{1y}^2 + 9)}{\beta_{1y}^2},$$

$$E(\delta_0\eta_1) = \frac{1}{n}\frac{(\lambda_{40} - 3)}{\lambda_{30}}C_y = \frac{1}{n}\frac{(\beta_{2y} - 3)}{\beta_{1y}}C_y,$$

$$E(\delta\eta_1) = \frac{1}{n}\frac{(\lambda_{50} - 4\lambda_{30})}{\lambda_{30}} = \frac{1}{n}\frac{(\lambda_{50} - 4\beta_{1y})}{\beta_{1y}},$$

$$E(\epsilon\eta_1) = \frac{1}{n}\frac{(\lambda_{31} - 3\rho)}{\lambda_{30}}C_x = \frac{1}{n}\frac{(\lambda_{31} - 3\rho)}{\beta_{1y}}C_x.$$

III. PROPOSED CLASS OF ESTIMATORS

Sharma et al. (2016a) defined the class of estimators of population mode (M_o) as

$$\widetilde{M}_{og} = \widehat{M}_o t(u) \tag{3.1}$$

where the optimum values of two unknown constants k and $t(1)$ were determined by minimizing the MSE's up to terms of order n^{-1}, the minimum MSE was obtained as

$$MSE_{min} = \frac{1}{n}\bar{Y}^2 C_y^2\left[1 - \rho^2 - \frac{(B_y\rho + \beta_y)^2}{(\lambda_y - B_y^2)}\right] \tag{3.2}$$

where

$\lambda_y = \lambda_{60} - 6\beta_y + \beta_{0y}$

$\beta_y = \beta_{2y} - \beta_{1y}^2 - 3,$

$\beta_{0y} = \beta_{1y}^2\beta_{2y} - 2\beta_{1y}\lambda_{50} - 9,$

$B_y = \beta_{1y}\lambda_{21} - \lambda_{31} + 3\rho$

If we define the wider class of estimators of population mode (M_o) as

$$\widetilde{M}_{og} = t(\widehat{M}_o, u) \quad (3.3)$$

Two constants k and t_1 are involved in the class (3.3) and their optimum values determined by minimizing the MSE, up to terms of order n^{-1}, are

$$k = \frac{(B_y\rho + \beta_y)}{\{\lambda_y - B_y^2\}} \quad (3.4)$$

$$t_1 = -\frac{\bar{Y}C_y\{(B_y\beta_y + \rho\lambda_y)\}}{C_x\{\lambda_y - B_y^2\}}$$

Up to terms of order n^{-1}, the minimum MSE of the optimum estimator of class (3.3) is same as the minimum MSE of the optimum estimator of class (3.1) defined by Sharma et al. (2016a).
We, here propose a generalized class of estimators of population median (M_d) when $\bar{X}$ is known,

$$\widetilde{M}_{dg} = h(\widehat{M}_d, u) \quad (3.5)$$

where $\widehat{M}_d = \bar{y} - \frac{k}{3}\frac{m_{30}}{s_y^2}$ and k is constant whose value is given by (3.4). Whatever be the sample chosen, let $u = \frac{\bar{x}}{\bar{X}}$ assume values in a bounded closed convex subset R of the two-dimensional real space. Let $h(\widehat{M}_d, u)$ be a function of $\widehat{M}_d$ and u such that

$$h(M_d, 1) = M_d$$

and such that it satisfies the following conditions:

(i) The function $h(\widehat{M}_d, u)$ is continuous and bounded in R.

(ii) The first and second order partial derivatives of $h(\widehat{M}_d, u)$ exist and are continuous and bounded in R.

$$\Rightarrow h_1(M_d, 1) = 1$$

where $h_1(M_d, 1)$ is the first order partial derivative of function $h(\widehat{M}_d, u)$.

Note that the estimators of population median (M_d) defined by Sharma et al. (2016) are the members of the proposed class of estimators (3.5).

To find the biases and $MSE's$ of estimators of class $\widetilde{M}_{dg}$, we expand the function $h(\widehat{M}_d, u)$ about the value $(M_d, 1)$ in second-order Taylor's series, writing it in terms of $\delta_0, \delta, \epsilon, \eta_1$and then taking the expectations given in section 2, up to terms of ordern^{-1}, we get,

$$Bias(\widetilde{M}_{dg}) = O(n^{-1}) \quad (3.6)$$

$$MSE(\widetilde{M}_{dg}) = \frac{1}{n}\left[\bar{Y}^2C_y^2 + h_1^2C_x^2 + 2h_1\bar{Y}C_{yx} + \frac{k^2}{9}\bar{Y}^2C_y^2\lambda_y - \frac{2}{3}k\bar{Y}^2C_y^2\beta_y + \frac{2}{3}h_1k\bar{Y}C_yC_xB_y\right] \quad (3.7)$$

as k is known constant, whose value is given by (3.4) above, so the only unknown constant here to find out is $h_1 \{= \partial h\partial uu=1$ whose value is determined by minimizing *MSE(Mdg).*

To obtain the minimum value of $MSE(\widetilde{M}_{dg})$ we differentiate (3.7) w.r.t. h_1, then equating to zero, we get,

$$h_1C_x + \bar{Y}\rho C_y + \frac{k}{3}\bar{Y}C_yB_y = 0$$

Solving above equation by substitute the value of k_{opt} for h_1, we get

$$h_1 = -\frac{\bar{Y}C_y\{3\rho\lambda_y - 2\rho B_y^2 + \beta_yB_y\}}{3C_x\{\lambda_y - B_y^2\}}$$

Substituting the values of pair $(k_1, t(1))$ in (3.7), we get,

$$MSE_{min}(\widetilde{M}_{dg}) = \frac{1}{n}\bar{Y}^2C_y^2\{1 - \rho^2 - \frac{5(B_y\rho + \beta_y)^2}{9\{\lambda_y - B_y^2\}}\}$$

From Srivastava and Jhajj (1983) results, here we can also say that the unknown population parameters in optimum values of constants will not create any problem for practical use of the proposed class $\widetilde{M}_{dg}$. We can construct the large number of estimators belonging to the proposed class $\widetilde{M}_{dg}$. Here it should be noted that the use of estimators of the proposed class $\widetilde{M}_{dg}$ require the optimum values of constants k andh_1, which are further functions of unknown population parameter. However, if it is possible to guess accurately the values of such parameters either through past experience or through a pilot sample survey, then the values of optimum constants so obtained by using these guessed values of parameters are close enough to the optimum values of constants and the resulting estimators will be better than the convention estimators. Even if we replace the parameters in the constants k and h_1 by their conventional consistent estimators then up to terms of order n^{-1}, the minimum $MSE(\widetilde{M}_{dg})$ remains the same.

Remarks:

(i.) Up to terms of order n^{-1},

$$MSE_{min}(\widetilde{M}_{dg}) < MSE(\hat{\bar{Y}}_{lr})$$

iff

$$\frac{5\left(B_y\rho+\beta_y\right)^2}{9\left\{\lambda_y-B_y^2\right\}}>0$$

(ii.) Special Case of Bivariate Normal Population

Let $(Y,X)\sim N(\mu_y,\mu_x,\sigma_y^2,\sigma_x^2,\rho)$, then we have $\lambda_{60}=15$, $\lambda_{40}=3$, $\lambda_{31}=3\rho$, $\lambda_{22}=1+2\rho^2$,$\lambda_{r,s}=0$ if $r+s$ is odd. Also, $X\sim N(\mu_x,\ \sigma_x^2)$ and $Y\sim N(\mu_y,\ \sigma_y^2)$.

Using these values, we get,

$$MSE_{min}\left(\widetilde{M}_{dg}\right)=MSE\left(\hat{\bar{Y}}_{lr}\right)=\frac{1}{n}S_y^2(1-\rho^2)$$

IV. SOME IMPORTANT MEMBERS OF THE PROPOSED CLASS

Any estimator, which satisfies the stated regularities conditions of the proposed class of estimators (3.5), is a member of the class. So we can construct a large number of estimators of M_d. All the estimators of the class though have the same minimum MSE (up to terms of order n^{-1}) but their biases are different. To choose the optimum estimator of the proposed class, we have to choose that estimator which has the minimum MSE as well as the minimum bias. Hence to choose the optimum estimator of the class, we take into consideration the following important sub-classes of the proposed generalized class (3.5) as

$$\widetilde{M}_{dg}^{(1)}=\widehat{M}_d+\alpha_1(u-1) \quad (4.1)$$

$$\widetilde{M}_{dg}^{(2)}=\widehat{M}_d\exp(\alpha_2 logu) \quad (4.2)$$

$$\widetilde{M}_{dg}^{(3)}=\widehat{M}_d\{1+\alpha_3(u-1)\} \quad (4.3)$$

and

$$\widetilde{M}_{dg}^{(4)}=\widehat{M}_d\exp\{\alpha_4(u-1)\} \quad (4.4)$$

Expanding above four estimators in a second order Taylor's series and using the expectations given in section II, we obtain,

$$Bias\left(\widetilde{M}_{dg}^{(1)}\right)=\frac{1}{n}\frac{k}{3}\bar{Y}C_y\{\lambda_{50}-\beta_{1y}(\beta_{2y}+3)\}$$

$$Bias\left(\widetilde{M}_{dg}^{(2)}\right)=\frac{1}{n}\left[\frac{k}{3}\bar{Y}C_y\{\lambda_{50}-\beta_{1y}(\beta_{2y}+3)\}+\alpha_2\bar{Y}C_{yx}+\frac{k}{3}\alpha_2\bar{Y}C_yC_xB_y-\frac{1}{2}\alpha_2M_dC_x^2+\frac{1}{2}\alpha_2^2M_dC_x^2\right]$$

$$Bias\left(\widetilde{M}_{dg}^{(3)}\right)=\frac{1}{n}\left[\frac{k}{3}\bar{Y}C_y\{\lambda_{50}-\beta_{1y}(\beta_{2y}+3)\}+\alpha_3\bar{Y}\rho C_x+\frac{k}{3}\alpha_3\bar{Y}C_xB_y\right]$$

$$Bias\left(\widetilde{M}_{dg}^{(4)}\right)=\frac{1}{n}\left[\frac{k}{3}\bar{Y}C_y\{\lambda_{50}-\beta_{1y}(\beta_{2y}+3)\}+\alpha_4\bar{Y}C_{yx}+\frac{k}{3}\alpha_4\bar{Y}C_yC_xB_y+\frac{1}{2}\alpha_4^2M_dC_x^2\right]$$

and

$$MSE\left(\widetilde{M}_{dg}^{(1)}\right)=\frac{1}{n}\left[\bar{Y}^2C_y^2+\alpha_1^2C_x^2+2\alpha_1\bar{Y}C_{yx}+\frac{k^2}{9}\bar{Y}^2C_y^2\lambda_y-\frac{2}{3}k\bar{Y}^2C_y^2\beta_y+\frac{2}{3}k\alpha_1\bar{Y}C_yC_xB_y\right]$$

$$MSE\left(\widetilde{M}_{dg}^{(i)}\right)=\frac{1}{n}\left[\bar{Y}^2C_y^2+\alpha_i^2M_d^2C_x^2+2\alpha_iM_d\bar{Y}C_{yx}+\frac{k^2}{9}\bar{Y}^2C_y^2\lambda_y-\frac{2}{3}k\bar{Y}^2C_y^2\beta_y+\frac{2}{3}k\alpha_iM_d\bar{Y}C_yC_xB_y\right];i=2,3,4.$$

where$k$ is known constant, whose value is given by (3.4) above and the only unknown constant here to find out is α_i, $i=1,2,3,4,$ whose value is determined by minimizing the respective $MSE(\widetilde{M}_{dg}^{(i)})$. Then the MSE of $\widetilde{M}_{dg}^{(i)},i=1,2,3,4$ are minimised for

$$\alpha_1=-\frac{\bar{Y}C_y\{3\rho\lambda_y-2\rho B_y^2+\beta_yB_y\}}{3C_x\{\lambda_y-B_y^2\}},$$

And

$$\alpha_i=-\frac{\bar{Y}C_y\{3\rho\lambda_y-2\rho B_y^2+\beta_yB_y\}}{3M_dC_x\{\lambda_y-B_y^2\}};i=2,3,4.$$

and the optimum biases and minimum MSE are given as,

$$Bias_{opt}\,(\widetilde{M}_{dg}^{(1)})=\frac{1}{n}\frac{(B_y\rho+\beta_y)}{3(\lambda_y-B_y^2)}\bar{Y}C_y\{\lambda_{50}-\beta_{1y}(\beta_{2y}+3)\}$$

$$Bias_{opt}\,(\widetilde{M}_{dg}^{(2)})=\frac{1}{n}\frac{\bar{Y}C_y}{3(\lambda_y-B_y^2)}\left[(B_y\rho+\beta_y)\{\lambda_{50}-\beta_{1y}(\beta_{2y}+3)\}-\frac{\bar{Y}C_y\left(3\rho\lambda_y-2\rho B_y^2+\beta_yB_y\right)^2}{6M_d(\lambda_y-B_y^2)}+\frac{C_x}{2}\left(3\rho\lambda_y-2\rho B_y^2+\beta_yB_y\right)\right]$$

$$Bias_{opt}(\widetilde{M}_{dg}^{(3)}) = \frac{1}{n}\frac{\bar{Y}C_y}{3(\lambda_y - B_y^2)}\left[(B_y\rho + \beta_y)\{\lambda_{50} - \beta_{1y}(\beta_{2y}+3)\} - \frac{\bar{Y}C_y(3\rho\lambda_y - 2\rho B_y^2 + \beta_y B_y)^2}{3M_d(\lambda_y - B_y^2)}\right]$$

$$Bias_{opt}(\widetilde{M}_{dg}^{(4)}) = \frac{1}{n}\frac{\bar{Y}C_y}{3(\lambda_y - B_y^2)}\left[(B_y\rho + \beta_y)\{\lambda_{50} - \beta_{1y}(\beta_{2y}+3)\} - \frac{\bar{Y}C_y(3\rho\lambda_y - 2\rho B_y^2 + \beta_y B_y)^2}{6M_d(\lambda_y - B_y^2)}\right]$$

and

$$MSE_{min}\left(\widetilde{M}_{dg}^{(i)}\right) = \frac{1}{n}\bar{Y}^2C_y^2\left\{1 - \rho^2 - \frac{5(B_y\rho + \beta_y)^2}{9\{\lambda_y - B_y^2\}}\right\}; i = 1,2,3,4.$$

V. COMPARISION W.R.T. BIASES

Theorem 1. Up to terms of order n^{-1},

$$\left|Bias_{opt}\left(\widetilde{M}_{dg}^{(1)}\right)\right| < \left|Bias_{opt}\left(\widetilde{M}_{dg}^{(2)}\right)\right|$$

iff

$$\left[Bias_{opt}\left(\widetilde{M}_{dg}^{(1)}\right)\right]^2 < \left[Bias_{opt}\left(\widetilde{M}_{dg}^{(2)}\right)\right]^2$$

when

$$G > \frac{6C_x(\lambda_y - B_y^2)}{L_1^2}\left[2L_2 + \frac{C_xL_1}{2}\right] \; or \; G < \frac{3C_x(\lambda_y - B_y^2)}{L_1}$$

where $L_1 = 3\rho\lambda_y - 2\rho B_y^2 + \beta_y B_y$, $L_2 = (B_y\rho + \beta_y)\{\lambda_{50} - \beta_{1y}(\beta_{2y}+3)\}$ and $G = \frac{\bar{Y}C_y}{M_d}$.

Theorem 2. Up to terms of order n^{-1},

$$\left|Bias_{opt}\left(\widetilde{M}_{dg}^{(1)}\right)\right| < \left|Bias_{opt}\left(\widetilde{M}_{dg}^{(3)}\right)\right|$$

iff

$$\left[Bias_{opt}\left(\widetilde{M}_{dg}^{(1)}\right)\right]^2 < \left[Bias_{opt}\left(\widetilde{M}_{dg}^{(3)}\right)\right]^2$$

when

$$G > \frac{6L_2(\lambda_y - B_y^2)}{L_1^2}.$$

Theorem 3. Up to terms of order n^{-1},

$$\left|Bias_{opt}\left(\widetilde{M}_{dg}^{(1)}\right)\right| < \left|Bias_{opt}\left(\widetilde{M}_{dg}^{(4)}\right)\right|$$

iff

$$\left[Bias_{opt}\left(\widetilde{M}_{dg}^{(1)}\right)\right]^2 < \left[Bias_{opt}\left(\widetilde{M}_{dg}^{(4)}\right)\right]^2$$

when

$$G > \frac{12L_2(\lambda_y - B_y^2)}{L_1^2}.$$

Theorem 4. Up to terms of order n^{-1},

$$\left|Bias_{opt}\left(\widetilde{M}_{dg}^{(2)}\right)\right| < \left|Bias_{opt}\left(\widetilde{M}_{dg}^{(3)}\right)\right|$$

iff

$$\left[Bias_{opt}\left(\widetilde{M}_{dg}^{(2)}\right)\right]^2 < \left[Bias_{opt}\left(\widetilde{M}_{dg}^{(3)}\right)\right]^2$$

when

$$G > \frac{2(\lambda_y - B_y^2)}{L_1^2}\left[\left(L_2 - \frac{C_xL_1}{2}\right) + \sqrt{L_2^2 - \frac{C_x^2L_1^2}{2} - 4C_xL_1L_2}\right]$$

$$or \; G < \frac{2(\lambda_y - B_y^2)}{L_1^2}\left[\left(L_2 - \frac{C_xL_1}{2}\right) - \sqrt{L_2^2 - \frac{C_x^2L_1^2}{2} - 4C_xL_1L_2}\right].$$

Theorem 5. Up to terms of order n^{-1},

$$\left|Bias_{opt}\left(\widetilde{M}_{dg}^{(2)}\right)\right| < \left|Bias_{opt}\left(\widetilde{M}_{dg}^{(4)}\right)\right|$$

iff

$$\left[Bias_{opt}\left(\widetilde{M}_{dg}^{(2)}\right)\right]^2 < \left[Bias_{opt}\left(\widetilde{M}_{dg}^{(4)}\right)\right]^2$$

when

$$G > \frac{6(\lambda_y - B_y^2)}{L_1^2}\left[\frac{C_x L_1}{4} + L_2\right].$$

Theorem 6. Up to terms of order n^{-1},

$$\left|Bias_{opt}\left(\widetilde{M}_{dg}^{(3)}\right)\right| < \left|Bias_{opt}\left(\widetilde{M}_{dg}^{(4)}\right)\right|$$

iff

$$\left[Bias_{opt}\left(\widetilde{M}_{dg}^{(3)}\right)\right]^2 < \left[Bias_{opt}\left(\widetilde{M}_{dg}^{(4)}\right)\right]^2$$

when

$$G < \frac{4L_2(\lambda_y - B_y^2)}{L_1^2}.$$

VI. NUMERICAL ILLUSTRATIONS

To illustrate the result numerically, we have made computations for 12 populations taken from literature by using Microsoft Excel 2010.

The source of the populations, the nature of the variables, the values of $\bar{Y}$, k_1,μ_{20}, β_{1y} and ρ are listed in Table 1.

The efficiencies of proposed estimators are given in Table 2.

The absolute optimum biases of considered four important sub-classes of the proposed generalized class are given in Table 3. In Table 4, we compare optimum estimator of proposed class with all 22 existing estimators of different technique that are listed by Singh and Solanki (2013), 3 existing estimators defined by Sharma et al. (2016b) and the linear regression estimator of meadian M_d.

Table 1: Description of populations

Sr. No.	Source	y	x	$\bar{Y}$	k_1	μ_{20}	β_{1y}	ρ
1	Murthy (1967), p.91 (1-35)	Cultivated area (acres)	Holding size (acres)	2.3650	-0.2217	1.5818	0.9119	0.3685
2	Murthy (1967), p.398	No. of absentees	No. of workers	9.6512	0.0442	42.1341	1.5575	0.6608
3	Murthy (1967), p.399	Area under wheat in 1964	Cultivated area in 1961	199.4412	-0.0220	21900.893 6	1.1295	0.9043
4	Chakravarty et al.(1967), p-183	Length(cm) measured by 1st person	Length(cm) measured by 2nd person	4.9737	-0.0437	0.1346	-0.0546	0.9317
5	Chakravarty et al.(1967), p-207	Weight (kg) of male	Height (cm) of male	29.2625	-0.0240	6.5836	0.3670	0.7709
6	Chakravarty et al.(1967), p-207	Weight (kg) of female	Height (cm) of female	28.5313	-0.3896	1.8109	0.1099	0.2306
7	Chakravarty et al.(1967), p-185 (1-35)	Weight (lb) of Kayastha males	Stature (cm) of Kayastha males	82.2000	-0.2012	191.7029	0.0439	0.8578
8	Chakravarty et al.(1967), p-185 (1-76)	Weight (lb) of Kayastha males	Stature (cm) of Kayastha males	89.4211	0.0516	278.4806	0.6068	0.4361
9	Chochran (1999), p-325	Total number of persons	Average persons per room	101.1000	-0.3015	214.6900	0.3248	0.6515
10	Maddala&Lahiri (1992), p-316	Consumption per capital of Lamb	Deflated prices of Lamb	4.5188	-0.0281	0.2103	-0.6578	-0.7517
11	Guajrati (2004), p-27,(1-50)	Price per dozen(cent) in 1990	Egg production in 1991 (million)	78.2880	0.0111	445.3787	0.9959	-0.3096
12	http://content.hccfl.edu	Highway fuel efficiency of vehicles (in miles)	Weightof vehicles (in 1000 lbs.)	30.6154	-0.2045	15.6213	0.0549	-0.8978

Table 2: $n^{-1} \times MSE's$ of $\hat{M}_{d_1}$, $\hat{M}_{d_2}$, $\hat{M}_{d_3}$, $\hat{\hat{Y}}_R$, $\hat{\hat{Y}}_P$, $\tilde{M}_{dg}$ and $\tilde{M}_{lr}$ up to terms of order n^{-1}

$n^{-1} * MSE's$ of							
Pop. No.	$\hat{M}_{d_1}$	$\hat{M}_{d_2}$	$\hat{M}_{d_3}$	$\hat{\hat{Y}}_R$	$\hat{\hat{Y}}_P$	$\tilde{M}_{dg}$	$\tilde{M}_{lr}$
1	1.4201	7.2895	14.9915	-	-	**1.2843**	1.3670
2	40.3890	22.9990	90.9751	23.7459	-	**22.9937**	23.7380
3	20935.5069	4172.6821	66661.3002	4286.4483	-	**3971.8559**	3992.7274
4	0.1201	0.0201	0.4947	0.0201	-	**0.0175**	0.0178
5	6.5145	3.9238	10.5213	3.9590	-	**2.6658**	2.6713
6	1.5012	1.8675	2.6333	-	-	**1.4462**	1.7146
7	142.0275	79.4387	237.1650	105.5227	-	**45.2976**	50.6533
8	270.2905	228.2034	539.9383	237.2253	-	**216.9131**	225.5076
9	176.6954	125.6235	554.3948	135.1725	-	**106.0736**	123.5609
10	0.2005	0.6691	0.1023	-	0.1023	**0.0912**	0.0915
11	445.3506	10052.1851	7317.9041	-	-	**402.4026**	402.7018
12	10.6760	59.4203	6.4059	-	6.7647	**2.7241**	3.0308

Table 3: $n^{-1} \times \left|Bias_{opt}\right|$ of $\tilde{M}_{dg}^{(i)}$, $i = 1, 2, 3, 4$, up to terms of order n^{-1}

Pop. No.	$\left\|Bias_{opt}(\tilde{M}_{1d})\right\|$	$\left\|Bias_{opt}(\tilde{M}_{2d})\right\|$	$\left\|Bias_{opt}(\tilde{M}_{3d})\right\|$	$\left\|Bias_{opt}(\tilde{M}_{4d})\right\|$
1	0.0355	0.2832	0.0485	**0.0065**
2	0.7019	0.5424	1.5368	**0.4174**
3	**1.0521**	24.3762	125.6609	63.3565
4	**0.0008**	0.0025	0.0223	0.0107
5	**0.0042**	0.0425	0.1376	0.0709
6	0.0383	**0.0355**	0.0406	0.0395
7	**0.0131**	0.4398	1.4981	0.7425
8	1.8005	1.9346	**1.1957**	1.4981
9	0.4794	**0.3064**	1.2792	0.8793
10	**0.0044**	0.0253	0.0210	0.0083
11	0.2729	3.9549	0.2967	**0.0119**
12	**0.0172**	0.4966	0.3830	0.2001

Table 4: MSE and Relative Efficiencies of Population Median Class

	MSE		Relative	Efficiency
Estimators	**Pop.I**	**Pop.II**	**Pop.I**	**Pop.II**
$V(\hat{M}_y)$	565443.57	565443.57	100.00	100.00
$MSE(\hat{M}_r)$	988372.76	536149.50	57.21	105.46
$MSE_{min}\ (\hat{M}_d)$				
$MSE_{min}\ (\hat{M}_y^{(G)})$	552636.13	508766.02	102.32	111.14
$MSE_{min}\ (\hat{M}_i)$				
$MSE_{min}\ (t_4)$	630993.68	478781.74	89.61	118.10
$MSE_{min}\ (t_5)$	499412.60	499412.60	113.22	113.22
$MSE_{min}\ (t_6)$	630979.49	478784.18	89.61	118.10
$MSE_{min}\ (t_7)$	630367.71	478806.00	89.70	118.09
$MSE_{min}\ (t_8)$	522345.11	488388.99	108.25	115.78
$MSE_{min}\ (t_9)$	630993.63	478781.75	89.61	118.10
$MSE_{min}\ (t_{10})$	489754.69	493940.28	115.45	114.48
$MSE_{min}\ (t_{11})$	630993.67	478781.74	89.61	118.10
$MSE_{min}\ \{\hat{M}_d^{(1)}\}$	489569.06	495484.97	115.50	114.12
$MSE_{min}\ \{\hat{M}_d^{(2)}\}$	489395.24	454675.78	115.54	124.36
$MSE_{min}\ \{\hat{M}_d^{(3)}\}$	3220.01	51355.17	17560.30	1101.05
$MSE_{min}\ \{\hat{M}_{d1}^{(4)}\}$	480458.29	454616.16	117.69	124.38
$MSE_{min}\ \{\hat{M}_{d2}^{(4)}\}$	489395.24	454675.78	115.54	124.36
$MSE_{min}\ \{\hat{M}_{d3}^{(4)}\}$	480459.82	454616.17	117.69	124.38
$MSE_{min}\ \{\hat{M}_{d4}^{(4)}\}$	480525.30	454616.32	117.67	124.38
$MSE_{min}\ \{\hat{M}_{d5}^{(4)}\}$	487375.11	454660.89	116.02	124.37

$MSE_{min}\{\hat{M}_{d6}^{(4)}\}$	480458.30	454616.16	117.69	124.38
$MSE_{min}\{\hat{M}_{d7}^{(4)}\}$	489260.97	454672.34	115.57	124.36
$MSE_{min}\{\hat{M}_{d8}^{(4)}\}$	480458.29	454616.16	117.69	124.38
$MSE_{min}(\hat{M}_{d_1})$	2155601.93	2155601.93	26.23	26.23
$MSE_{min}(\hat{M}_{d_2})$	187364.86	241764.01	301.79	233.88
$MSE_{min}(\hat{M}_{d_3})$	6887379.49	7187700.83	8.21	7.87
$MSE_{min}(\hat{Y}_{lr})$	168489.40	183861.68	335.60	307.54
$\boldsymbol{MSE_{min}(\tilde{M}_{dg})}$	**164833.35**	**178024.51**	**343.04**	**317.62**

From table 2, in which we compared the estimators of similar type, we observe that, upto the terms of order n^{-1},$MSE_{min}(\tilde{M}_{dg})$ is less than $MSE_{min}(\hat{M}_{d_1})$, $MSE_{min}(\hat{M}_{d_2})$, $MSE_{min}(\hat{M}_{d_3})$, $MSE(\hat{\bar{Y}}_R)$, $MSE(\hat{\bar{Y}}_P)$and even smaller than $MSE_{min}(\hat{Y}_{lr})$, which are very interesting results.
From table 3, it is clearly seen that among all the four important types of estimators, the bias of first sub-class of estimators $\left(\tilde{M}_{dg}^{(1)}\right)$, which is of regression type, is less in most of the populations.
From table 4, we can see that the efficiency of the proposed optimum estimator of class $\tilde{M}_{dg}$ is very much high as compare to estimators of different technique.

VII. CONCLUTION

In this study, when $\bar{X}$ is known then we have proposed the generalized class of estimators of population median which includes the estimators defined by Sharma et al. (2016). The lower bound for MSE for the class of estimators has been obtained. To choose optimum estimators w.r.t. MSE and bias, important types of sub-classes of proposed generalized class are considered. Their optimum biases have been obtained and compared with each other.

Empirically we have shown that the sub-class of regression-type estimators $\tilde{M}_{dg}^{(1)} = \hat{M}_d + \alpha_1(u-1)$ are optimum estimators of population median w.r.t. bias and MSE, as well as very simple as compared to the exisiting ones.

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