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A Moment Inequality for Overall Decreasing Life Class of Life Distributions with Hypotheses Testing Applications

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ABSTRACT:*A moment inequality is derived for the system whose life distribution is in an overall decreasing life (ODL) class of life distributions. A new nonparametric test statistic for testing exponentiality against ODL is investigated based on this inequality. The asymptotic normality of the proposed statistic is presented. Pitman's asymptotic efficiency, power and critical values of this test are calculated to assess the performance of the test. Real examples are given to elucidate the use of the proposed test statistic in the reliability analysis. Wealso proposed a test for testing exponentiality versus ODL for right censored data and the power estimates of this test are also simulated for censored data for some commonly used distributions in reliability. Finally, real data are used as an example for practical problems.*

KEYWORDS:*life distributions, ODL, Moment inequalities,exponentiality U-statistic, asymptotic normality, efficiency, Monte Carlo method, power and censored data.*

I. INTRODUCTION AND MOTIVATION

The mathematical theory of reliability has developed in two main branches. The first branch is mostly constructed on the mathematical aspects of practical problems that face reliability and design engineers and reliability analysis specialists. The second branch concentrates on the mathematical and statistical aspects of reliability such as life testing and estimation.

Note that the assumption of exponentiality is the back bone in life testing, the theory of reliability, maintenance modeling, biometrics and biological science. It is very important because of its implications concerning the random mechanism operating in the experiment under consideration. In reliability theory, the exponential assumption may apply when one is dealing with failure times of items or equipment without any moving parts such as, for example, fuses, transistors, air monitors and bulbs. In these examples the failure is caused due to sudden shocks rather than wear and tear. The exponential assumption corresponds to assuming that this shock follows a Poisson process distribution. Testing for exponentiality of the failure time is, in effect, the same as testing the Poisson assumption about the process producing the shock that causes failure. An assumption of exponentially distributed life time's indicate that a used item is stochastically as good as new, so there is no reason to replace a functioning unit. This distribution is the most commonly used life distribution in applied probability analysis primarily. See Barlow and Proschan[10] and Zachks[33].

For more than four decades, investigates have led to declaring many families of life distributions to characterize aging. There are a several number of classes that have been presented in reliability, see for example Siddiqui and Bryson [31] Ahmad and abouammoh[7], Abu-Youssef [3], Mahmoud et al. [27] and Diab ([13],[14]). To categorize distributions based on their aging properties or their dual. Among the most practical aging classes of life distributions, the increasing failure rate (IFR). Many other have been extensively studied in the literature Barlow et al. [9],Proschan and Pyke[28]. For testing against increasing failure rate in expectation (IFRA), see Aly[8] and Ahmad [6]. For testing versus new better than used (NBU), see koul[24], and Kumazawa[25].

Ahmad [4], one of the authors presented moments inequalities for classes of life distributions IFR, NBU, NBUE and harmonic new better than used in expectation (HNBUE). For testing exponentiallity against an alternative among the classes NBUE, HNBUE, NBUFR, NBAFR, NBURFR, HNBUE, NRBU, DMRL, NBUC, NBUL and DVRL classes, we refer to Klefsjo ([22],[23]), Deshpaned et al. [11] Ahmad and Abuammoh[5], EL-Arishy et al. [16], Abdul Alim and Mahmoud [1], Mahmoud and Diab[26] and Diab et al. [12] respectively. Finally, for exponentiallity testing versus an alternative RNBU and HNRBUE, see El -Arishy, et al. [17] and Diab and Mahmoud [15].

Let*X* be a nonnegative continuous random variable with distribution function F, survival function $\bar{F} = 1 - F$, with finite mean, with survival of a device in operation at any time$t \geq 0$ is given by,

$$\bar{W}(t) = \frac{1}{\mu}\int_t^\infty \bar{F}(u)du \; t \geq 0 .$$

Sepehrifar et al. [30] defined the *ODL* .And investigated the probabilistic characteristics of this class of life distribution.

Definition 1.1: A life distribution *F*on $(0,\infty)$, with$F(0^{-})=0$ is called overall decreasing life (*ODL*) if,

$$\int_t^\infty \bar{W}(x)dx \le \mu\bar{W}(t), \qquad t \ge 0, \tag{1.1}$$

Where$\mu=\int_0^\infty \bar{F}(u)du$.

Remark 1.1: Life distribution *F* belongs to *ODL* if and onlyif$\frac{f(x)}{F(x)}<\frac{1}{\mu}$.

The rest of the article is structured as follows. The moment inequality developed in section 2, for *ODL* class of life distribution based on these inequalities test statistics for testing H_0 :is exponential against H_1 : is *ODL*and not exponential. In section 3, a new test statistic based on U-statistic is established and have exceptionally Pitman asymptotic efficiency of some of well-known alternatives, Monte Carlo null distribution critical points are simulated for sample sizes$n=5(1)30(5)50$and the power estimates of this test are also calculated at the significant level $\alpha=0.05$ for some common alternatives distribution followed by some numerical example. In section 4, we dealing with right-censored data and selected critical values are tabulated. Finally, the power estimates for censor data of this test are tabulated and numerical example are calculated.

II. MOMENT INEQUALITY

In this section we derive our main results. The following theorem gives the moment inequality for the *ODL*class of life distributions.

Theorem 2.1: If*F* is *ODL* ,then for all integer $r \ge 0$,

$$\frac{1}{(r+3)}\mu_{r+3} \le \mu\mu_{(r+2)}, \qquad r \ge 0. \tag{2.1}$$

Where

$$\mu_{(r)}=E(X^r)=r\int_0^\infty X^{r-1}\bar{F}(u)du.$$

Proof: Since *F*is said to be overall decreasing life *ODL* class, then from (1.1) we multiplying both sides by $t^r,\quad r\ge 0$ and integrating over $(0,\infty)$with respect to*t*,we get

$$\int_0^\infty t^r \int_t^\infty \bar{W}(x)dxdt \le \mu\int_0^\infty t^r\bar{W}(t)dt, \tag{2.2}$$

Now,

$$\begin{aligned} R.H.S &= \int_0^\infty t^r\bar{W}(t)dt,\\ &= \frac{1}{(r+1)}\int_0^\infty x^{r+1}\bar{F}(x)dx,\\ &= \frac{1}{(r+1)}E\left[\int_0^X x^{r+1}\,dx\right],\\ &= \frac{1}{(r+1)(r+2)}\mu_{(r+2)}. \end{aligned} \tag{2.3}$$

Also,

$$\begin{aligned} L.H.S &= \int_0^\infty t^r\int_t^\infty \bar{W}(x)dxdt,\\ &= \frac{1}{\mu(r+1)(r+2)}\int_0^\infty x^{r+2}\bar{F}(x)dx,\\ &= \frac{1}{\mu(r+1)(r+2)(r+3)}\mu_{r+3}. \end{aligned} \tag{2.4}$$

Using (2.3),(2.4) in (2.2) the result follows.

2.1 Testing against ODL class for non-censored data

Let $X_1,X_2,\dots,X_n$ be a random sample from a population with distribution function *F*. We test the null hypothesis $H_0:\bar{F}$ is exponential with mean μ against $H_1:\bar{F}$ is *ODL* and not exponential. Using Theorem (2.1), we can use the following quantity as a measure of departure from H_0 in favor of H_1

$$\delta_{ODL}(r) = \mu\mu_{(r+2)} - \frac{1}{(r+3)}\mu_{(r+3)} \tag{3.1}$$

Note that under $H_0 : \delta_{ODL}(r) = 0$,and it is positive under H_1.To make the test scale invariant under H_0 we use

$$\Delta_{ODL}(r) = \frac{\delta_{ODL}(r)}{\mu^{r+3}}$$

It could be estimated based on a random sample $X_1, X_2, \dots, X_n$ from F by

$$\hat{\Delta}_{ODL}(r) = \frac{\hat{\delta}_{ODL}(r)}{\bar{X}^{r+3}}$$

$$= \frac{1}{\bar{X}^{r+3}}\left[\frac{1}{n(n-1)}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(X_i X_j^{r+2} - \frac{1}{(r+3)}X_i^{r+3}\right)\right] \tag{3.2}$$

Where$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ is the sample mean, and μ is estimated by $\bar{X}$
Setting,

$$\phi(X_1, X_2) = X_1 X_2^{r+2} - \frac{1}{(r+3)}X_1^{r+3}$$

Again $\widehat{\Delta}_{ODL}(r)$ and $\frac{\hat{\delta}_{ODL}(r)}{\bar{X}^{r+3}}$ have the same limiting distribution. But since $\widehat{\Delta}_{ODL}(r)$ is the usual *U*-statistics theory,it is asymptotically normal and all we need to evaluate$Var\left(\frac{\hat{\delta}_{ODL}(r)}{\mu^{r+3}}\right)$. The following theorem summarized the large sample properties of $\widehat{\Delta}_{ODL}(r)$ or *U*-statistic.

Theorem 3.1: As $n \to \infty, \sqrt{n}\left(\widehat{\Delta}_{ODL}(r) - \delta_{ODL}(r)\right)$is asymptotically normal with mean zero and variance

$$\sigma_{(r)}^2 = Var\left[-\frac{1}{r+3}X^{r+3} + X^{r+2} + (r+2)!\,X - (r+2)!\right] \tag{3.3}$$

UnderH_0 this value reduced to

$$\sigma_0^2 = (2r+4)! - \left((r+2)!\right)^2 \tag{3.4}$$

Proof: Since $\widehat{\Delta}_{ODL}(r)$ and $\frac{\hat{\delta}_{ODL}(r)}{\bar{X}^{r+3}}$ have the same limiting distribution, weconcentrate on $\sqrt{n}\left(\widehat{\Delta}_{ODL}(r) -\right.$ *δODLr*. Now this is asymptotic normal with mean zero and variance*σ2=VarϕX1*, where

$$\phi(X_1) = E[\phi(X_1, X_2)|X_1] + E[\phi(X_2, X_1)|X_1] \tag{3.5}$$

But

$$\phi(X_1) = \mu_{(r+2)}X_1 - \frac{1}{(r+3)}X_1^{r+3} + X_1^{r+2} - \mu_{(r+2)} \tag{3.6}$$

Hence (3.3) follows.Under H_0

$$\phi(X_1) = (r+2)!\,X_1 - \frac{1}{(r+3)}X_1^{r+3} + X_1^{r+2} - (r+2)! \tag{3.7}$$

Thus it is easy to get σ_0^2as it is defined in (3.4). When $r = 0$,

$$\delta_{ODL} = \mu\mu_{(2)} - \frac{1}{3}\mu_{(3)} \tag{3.8}$$

In this case$\sigma_0 = 4.47214$ and the test statistic is

$$\hat{\delta}_{ODL} = \frac{1}{n(n-1)}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(X_i X_j^2 - \frac{1}{3}X_i^3\right). \tag{3.9}$$

And

$$\hat{\Delta}_{ODL} = \frac{\hat{\delta}_{ODL}}{\bar{X}^3}, \tag{3.10}$$

which is quite simple statistics. One can use the proposed test to calculate$\frac{\sqrt{n}\hat{\Delta}_{ODL}}{\sigma_0}$ and reject H_0 if $\frac{\sqrt{n}\hat{\Delta}_{ODL}}{\sigma_0} \geq Z_\alpha$,whereZ_α is the α −quintile of the standard normal distribution.

2.2 The Pitman asymptotic efficiency

This subsection includes the calculations of the asymptotic efficiencies of the *ODL* test statistic. We use the concept of Pitman's asymptotic efficiency (*PAE*) which is defined as $PAE(\Delta_r(\theta)) = \frac{\left[\frac{d}{d\theta}\Delta_r(\theta)\right]_{\theta\to\theta_0}}{\sigma_0}$. These calculations are done by using the following common alternative families:

(i) Linear failure rate family (LFR)

$$\bar{F}_1(x) = e^{-x-\frac{\theta}{2}x^2}, x \geq 0, \theta \geq 0,$$

(ii) Makeham family

$$\bar{F}_2(x) = e^{-x-\theta(x+e^{-x}-1)}, x \geq 0, \theta \geq 0,$$

(iii) Weibull family

$$\bar{F}_3(x) = e^{-x^{\theta}}, x > 0, \theta \geq 0,$$

(iv) Gamma family

$$\bar{F}_4(x) = \int_x^\infty e^{-u}\, u^{\theta-1} du/\Gamma(\theta), x > 0, \theta \geq 0.$$

Since,

$$\delta_{ODL}(r) = \mu(\theta)\mu_{(r+2)}(\theta) - \frac{1}{(r+3)}\mu_{(r+3)}(\theta).$$

Then we define

$$\grave{\delta}_{ODL}(r)\big|_{\theta\to\theta_0} = \mu(\theta_0)\grave{\mu}_{(r+2)}(\theta_0) + \grave{\mu}(\theta_0)\mu_{(r+2)}(\theta_0) - \frac{1}{(r+3)}\grave{\mu}_{(r+3)}(\theta_0).$$

$$\grave{\delta}_{ODL}\big|_{\theta\to\theta_0} = \left(\int_0^\infty \bar{F}_{\theta_0}(x)\,dx\right)\left(2\int_0^\infty x\bar{F}^{\backslash}_{\theta_0}(x)\,dx\right) + \left(2\int_0^\infty x\bar{F}_{\theta_0}(x)\,dx\right)\left(\int_0^\infty \bar{F}^{\backslash}_{\theta_0}(x)\,dx\right) - \int_0^\infty x^2\bar{F}^{\backslash}_{\theta_0}(x)\,dx$$

Note that H_0(the exponential distribution) is attained at $\theta_0 = 0$ in (i), (ii) and at $\theta_0 = 1$ in (iii), (iv) Direct calculations for the families in (i) and (ii) are given at $r = 0$.

(i) Linear failure rate

$$PAE(\Delta_r(\theta)) = \frac{1}{\sigma_0}|(r+2)(r+2)!|.$$

(ii) Makeham family

$$PAE(\Delta_r(\theta)) = \frac{1}{\sigma_0}\left|(r+2)!\left(\frac{1}{2} - 2^{-(3+r)}\right)\right|.$$

Using mathematica program to calculate *PAE* for Weibull family and Gamma family at $r = 0$.

(iii) Gamma family

$$PAE(\Delta_r(\theta)) = 0.298142.$$

(iv) Weibull family

$$PAE(\Delta_r(\theta)) = 0.67082.$$

Direct calculations of the asymptotic efficiencies of *ODL* test are given in Table 1. At$r = 0$.

Table 1. Pitman asymptotic efficiencies for *ODL* test

Efficiency	Linear failure rate	Makeham family	Weibull family	Gamma family
$PAE(\Delta_r(\theta))$	0.894427	0.167705	0.67082	0.298142

It is clear from Table 1,we can see that the new *ODL* test statistic is more efficiency.

III. MONTE CARLO NULL DISTRIBUTION CRITICAL POINTS

In practice, simulated percentiles for small samples are commonly used by applied statisticians and reliability analyst. We have simulated the upper percentile values for 90%, 95%, 98% and 99%. Table 2 presented these percentile values of the statistics $\hat{\Delta}_{ODL}$ in (3.10) and the calculations are based on 5000 simulated samples of sizes $n = 5(1)30(5)50$.

Table 2. Critical values of statistic $\hat{\Delta}_{ODL}$.

n	90%	95%	98%	99%
5	0.6803	0.6930	0.7044	0.7100
6	0.6726	0.6844	0.6957	0.7032
7	0.6675	0.6799	0.6902	0.6961
8	0.6611	0.6741	0.6860	0.6928
9	0.6540	0.6691	0.6820	0.6873
10	0.6468	0.6645	0.6773	0.6852
11	0.6405	0.6597	0.6752	0.6828
12	0.6381	0.6563	0.6708	0.6791
13	0.6275	0.6524	0.6709	0.6780
14	0.6180	0.6413	0.6624	0.6719
15	0.6141	0.6406	0.6601	0.6706
16	0.6066	0.6354	0.6547	0.6660
17	0.6050	0.6319	0.6547	0.6646
18	0.5948	0.6232	0.6465	0.6630
19	0.5921	0.6214	0.6438	0.6579
20	0.5885	0.6198	0.6424	0.6517
21	0.5834	0.6128	0.6387	0.6519
22	0.5760	0.6083	0.6334	0.6449
23	0.5766	0.6066	0.6334	0.6443
24	0.5640	0.5982	0.6257	0.6411
25	0.5603	0.5944	0.6265	0.6423
26	0.5572	0.5915	0.6234	0.6369
27	0.5527	0.5888	0.6170	0.6322
28	0.5501	0.5855	0.6127	0.6301
29	0.5437	0.5776	0.6092	0.6256
30	0.5385	0.5770	0.6071	0.6250
35	0.5259	0.5650	0.5974	0.6109
40	0.5055	0.5488	0.5843	0.6016
45	0.4825	0.5289	0.5675	0.5888
50	0.4778	0.5182	0.5583	0.5764

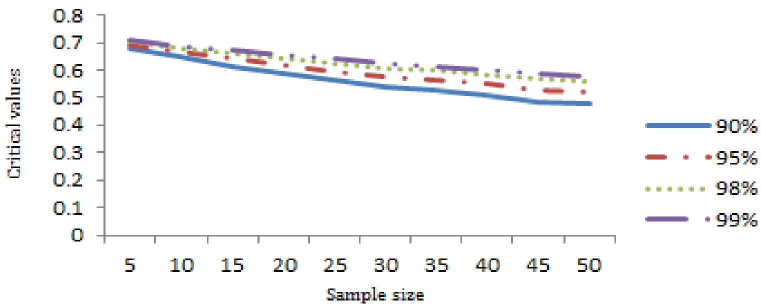

Fig. 1. Relation between critical values, sample size and confidence levels.

In view of Table 2, and Fig. 1, it is noticed that the critical values are increasing as the confidence level increasing and is almost decreasing as the sample size increasing.

3.1 The Power Estimates

Now, we present an estimation of the power estimate of the test statistic $\hat{\Delta}_{ODL}$ at the significance level $\alpha = 0.05$ using LFR, Gamma and Weibull distribution. The estimates are based on 5000 simulated samples for sizes $n = 10, 20$ and 30 with parameter $\theta = 2,3$ and 4.

Table 3. Power estimates using $\alpha = 0.05$.

Distribution	Parameter θ	Sample Size *n=10*	*n=20*	*n=30*
LFR	2	0.9988	0.9998	1.0000
	3	0.9998	1.0000	1.0000
	4	0.9992	1.0000	1.0000
Weibull	2	1.0000	1.0000	1.0000
	3	1.0000	1.0000	1.0000
	4	1.0000	1.0000	1.0000
Gamma	2	0.9978	0.9986	0.9988
	3	1.0000	1.0000	1.0000
	4	1.0000	1.0000	1.0000

It is clear from the Table 3 that our test has good powers for all alternatives and the power increases as the sample size increases. The power is getting as smaller as the *ODL* approaches the exponential distribution.

3.2 Applications Using Complete (Uncensored) Data

Here, we present some of good real examples to illustrate the use of our test statistics $\hat{\Delta}_{\boldsymbol{ODL}}$ in the case of non-censored data at 95% confidence level.

Example 1: The data set of 40 patients suffering from blood cancer (Leukemia) from one of ministry of health hospitals in Saudi Arabia sees Abouammoh et al. [2]. The ordered life times (in years)

0.315	0.496	0.616	1.145	1.208	1.263	1.414	2.025	2.036	2.162
2.211	2.370	2.532	2.693	2.805	2.910	2.912	3.192	3.263	3.348
3.348	3.427	3.499	3.534	3.767	3.751	3.858	3.986	4.049	4.244
4.323	4.381	4.392	4.397	4.647	4.753	4.929	4.973	5.074	4.381

It was found that $\hat{\Delta}_{ODL} = 0.678917$ and this value exceeds the tabulated critical value in Table 2. It is evident that at the significant level %95 this data set has *ODL* property.

Example 2: The following data in keating et al.[21] set on the time, in operating days, between successive failures of air conditioning equipment in an aircraft. These data are recorded

3.750	0.417	2.500	7.750	2.542	2.042	0.583	1.000	2.333	0.833
3.292	3.500	1.833	2.458	1.208	4.917	1.042	6.500	12.917	3.167
1.083	1.833	0.958	2.583	5.417	8.667	2.917	4.208	8.667	

by using (3.3) it is found that $\hat{\Delta}_{ODL} = 0.373289$ which is less than the critical value of Table 2, then we accept the null hypothesis. This means that the data set has the exponential property.

Example 3: Using the data set given in Grubbs [19], this data set gives the times between arrivals of 25 customers at a facility

1.80	2.89	2.93	3.03	3.15	3.43	3.48	3.57	3.85	3.92
3.98	4.06	4.11	4.13	4.16	4.23	4.34	4.73	4.53	4.62
4.65	4.84	4.91	4.99	5.17					

It is easily to show that $\hat{\Delta}_{ODL} = 0.668617$ which is greater than the critical value ofTable 2. Thenwe accept H_1 which states that the data set have *ODL* property and not exponential.

Example 4: Consider the well-known Darwin data (Fisher [18]) that represent the differences in heights between cross- and self-fertilized plants of the same pair grown together in one pot

4.25	8.708	0.583	2.375	2.25	1.333	2.792	2.458
5.583	6.333	1.125	0.583	9.583	2.75	2.542	1.417

We can see that the value of test statistic for the data set by formula (3.10) is given by $\hat{\Delta}_{ODL}= 2.41461$ and this value greater than the tabulated critical value in Table 2. This means that the set of data have *ODL*property and not exponential.

IV. TESTING AGAINST ODL CLASS FOR CENSORED DATA

In this section, a test statistic is proposed to test H_0versus $H_1$with randomly right-censored data. Such a censored data is usually the only information available in a life-testing model or in a clinical study where patients may be lost (censored) before the completion of a study. This experimental situation can formally be modeled as follows. Suppose *n* objects are put on test, and $X_1, X_2, \dots, X_n$ denote their true life time. We assume that $X_1, X_2, \dots, X_n$ be independent, identically distributed (i.i.d.) according to a continuous life distribution *F*. Let $Y_1, Y_2, \dots, Y_n$ be (i.i.d.) according to a continuous life distribution *G*. Also we assume that $X's$ and $Y's$ are independent.

In the randomly right-censored model, we observe the pairs $(Z_j, \delta_j), j = 1, \dots, n$ where$Z_j = min(X_j, Y_j)$ and

$$\delta_j = \begin{cases} 1 \ \ if Z_j = X_j (j - thobservationisuncensored). \\ 0 \ \ if Z_j = Y_j (j - thobservationiscensored). \end{cases}$$

Let $Z(0) = 0 < Z(1) < Z(2) < \cdots < Z(n)$ denote the ordered $Z's$ and $\delta_{(j)}$ is the δ_j corresponding to $Z_{(j)}$ respectively.

Using the censored data $(Z_j, \delta_j), j = 1, \dots, n$ Kaplan and Meier [20] proposed the product limit estimator.

$$\bar{F}_n(X) = 1 - F_n(X) = \prod_{[j: Z_{(j)} \le X]} \left\{\frac{n-j}{n-j+1}\right\}^{\delta_{(j)}}, X \in [0, Z_n]$$

Now, for testing $H_0 : \Delta_{ODL}= 0$, against $H_1 : \Delta_{ODL}> 0$,using the randomly right censored data, we propose the following test statistic:

$$\hat{\Delta}^c_{ODL}= \frac{1}{\mu^3}\left\{\mu\mu_2 - \frac{1}{3}\mu_3\right\}.$$

For computational purposes, $\hat{\Delta}^c_{ODL}$may be rewritten as

$$\hat{\Delta}^c_{ODL}= \frac{1}{\psi^3}\left\{\psi\Omega - \frac{1}{3}\Phi\right\}. \qquad (4.1)$$

Where

$$\psi = \sum_{i=1}^{n}\prod_{m=1}^{i-1} C_m^{\delta(m)} \left(Z_{(i)} - Z_{(i-1)}\right), \qquad \Omega = 2\sum_{j=1}^{n} Z_{(j)} \prod_{p=1}^{j-1} C_p^{\delta(p)} \left(Z_{(j)} - Z_{(j-1)}\right),$$

and,

$$\Phi = 3\sum_{i=1}^{n}\left(Z_{(i)}\right)^2 \prod_{q=1}^{i-1} C_q^{\delta(q)} \left(Z_{(i)} - Z_{(i-1)}\right).$$

Where $C_k = \frac{n-k}{n-k+1}$.

Table 4.gives the critical values percentiles of $\hat{\Delta}^c_{ODL}$test for sample sizes $n = 5(5)30(10)70,81,86$.based on 5000 replications.

Table 4. Critical values for percentiles of $\hat{\Delta}^{c}_{ODL}$ test

n	90%	95%	98%	99%
5	0.09651	0.13859	0.20003	0.25226
10	0.08166	0.12182	0.17443	0.21400
15	0.06461	0.09050	0.13029	0.15792
20	0.05159	0.07348	0.10401	0.12619
25	0.04419	0.06126	0.08593	0.10513
30	0.03738	0.05214	0.07011	0.08734
40	0.03067	0.04256	0.05678	0.06852
50	0.02538	0.03482	0.04918	0.05767
60	0.02244	0.03059	0.04002	0.04884
70	0.01968	0.02658	0.03639	0.04561
81	0.01735	0.02344	0.03169	0.03797
86	0.01646	0.02233	0.02917	0.03653

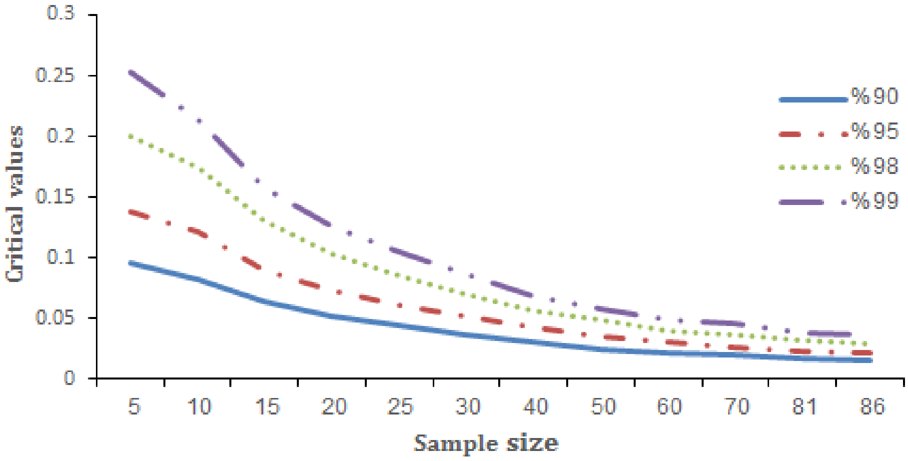

Fig. 2. Relation between critical values, sample size and confidence levels.

We noticed that from Table 4 and Fig. 2, the critical values are increasing as the confidence level increasing and is almost decreasing as the sample size increasing.

4.1 The power estimates for $\hat{\Delta}^{c}_{ODL}$

Her, we present an estimation of the power for testing exponentiality Versus *ODL*. Using significance level $\alpha = 0.05$with suitable parameter values of θat $n = 10,20\ and\ 30$, and for commonly used distributions in reliability such as LFR family, Gamma family and Weibull family alternatives which include in Table 5.

Table 5. Power estimates

Distribution	Parameter θ	Sample Size *n=10*	*n=20*	*n=30*
LFR	2	0.9974	0.9992	0.9982
	3	0.9996	1.0000	1.0000
	4	0.9998	1.0000	1.0000
Weibull	2	0.9968	0.9990	0.9994
	3	0.9990	1.0000	1.0000
	4	0.9954	1.0000	1.0000
Gamma	2	0.9294	0.9480	0.9548
	3	0.9334	0.9682	0.9662
	4	0.9310	0.9768	0.9784

We notice from Table 5. We can show that our test has a good power, and the power increases as the sample size increases.

4.2 Applications Using Incomplete (Censored) Data

Example 1.

Consider the data of Susarla and Vanryzin [32], which represent 81 survival times (in weeks) of patients melanoma. Out of these 46 represents non-censored data and the ordered values are:

13	14	19	19	20	21	23	23	25	26
26	27	27	31	32	34	34	37	38	38
40	46	50	53	54	57	58	59	60	65
65	66	70	85	90	98	102	103	110	118
124	130	136	138	141	234				

The ordered censored observations are:

16	21	44	50	55	67	73	76	80	81
86	93	100	108	114	120	124	125	129	130
132	134	140	147	148	151	152	152	158	181
190	193	194	213	215					

Now, taking into account the whole set of survival data (both censored and uncensored). It was found that the value of test statistic for the data set by formula (4.1) is given by $\widehat{\Delta}^{c}_{ODL} = 0.0492696$ and this value greater than the tabulated critical value in Table 5. This means that the data set has *ODL* property.

Example 2.

On the basis of right censored data for lung cancer patients from Pena [29]. These data consists of 86 survival times (in month) with 22 right censored:

The whole life times (non-censored data):

0.99	1.28	1.77	1.97	2.17	2.63	2.66	2.76	2.79	2.86
2.99	3.06	3.15	3.45	3.71	3.75	3.81	4.11	4.27	4.34
4.4	4.63	4.73	4.93	4.93	5.03	5.16	5.17	5.49	5.68
5.72	5.85	5.98	8.15	8.62	8.48	8.61	9.46	9.53	10.05
10.15	10.94	10.94	11.24	11.63	12.26	12.65	12.78	13.18	13.47
13.96	14.88	15.05	15.31	16.13	16.46	17.45	17.61	18.2	18.37
19.06	20.7	22.54	23.36						

The ordered censored observations are:

11.04	13.53	14.23	14.65	14.91	15.47	16.49	17.05	17.28	17.88
17.97	18.83	19.55	19.58	19.75	19.78	19.95	20.04	20.24	20.73
21.55	21.98								

We now account the whole set of survival data (both censored and uncensored), and computing the test statistic given by formula (4.1). It was found that $\widehat{\Delta}^{c}_{ODL} = 0.433052$ which is exceeds the tabulated value in Table 5. It is evident that at the significant level 0.95 .Then this data set has *ODL* property.

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