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Ratio and Product Type Estimators Using Stratified Ranked Set Sampling

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ABSTRACT: *In this paper, we propose a class of estimators for estimating the population mean of variable of interest using information on an auxiliary variable in stratified ranked set sampling. The bias and Mean Squared Error of proposed class of estimators are obtained to first degree of approximation. It has been shown that these methods are highly beneficial to the estimation based on Stratified Simple Random Sampling. Theoretically, it is shown that these suggested estimators are more efficient than the estimators in Stratified Random Sampling.*
Keywords*: Stratified ranked set sampling, Ratio and product type estimators, Ranked set sampling, Auxiliary variables, Mean squared error, Population mean, Efficiency.*

I. INTRODUCTION

Ranked set sampling is cost effective sampling procedure to the commonly used simple random sampling. RSS was first introduced by Mc Intyre in 1952. Samawi(1996) introduced the concept of Stratified Ranked Set Sampling to increase the efficiency of estimator of population mean. In this paper we propose estimators based on the modified ratio and product estimators.

In stratified ranked set sampling, for the h^{th} stratum of the population, first choose r_h independent samples each of size r_h , h = 1, 2, . . . , L. Rank each sample, and use RSS scheme to obtain L independent RSS samples of size r_h , one from each stratum. Let $\sum^{L}_{h=1}$= r. This complete one cycle of stratified ranked set sample. The cycle may be repeated m times until n = mr elements have been obtained. A modification of the above procedure is suggested here to be used for the estimation of the ratio using stratified ranked set sample. For the h th stratum, first choose r_h independent samples each of size r_h of independent bivariate elements from the h^{th} subpopulation (Stratum) , h = 1, 2, . . . , L. Rank each sample with respect to one of the variables say Y or X. Then use the RSS sampling scheme to obtain L independent RSS samples of size r_h one from each stratum. This complete one cycle of stratified ranked set sample. The cycle may be repeated m times until n = mr bivariate elements have been obtained. We will use the following notation for the stratified ranked set sample when the ranking is on the variable X. For the k th cycle and the h th stratum, the SRSS is denoted by $\{(Y_{h(1)k}, X_{h(1)k}),(Y_{h(2)k}, X_{h(2)k}), . . . ,(Y_{h(rh)k}, X_{h(rh)k})$: k = 1, 2, . . . , m; h = 1, 2, . . . , L} , where $Y_{h[i]}k$ is the i th Judgment ordering in the i th set for the study variable and $X_{h(i)}k$ is the i th order statistic in the i th set for the auxiliary variable.

II. SOME EXISTING ESTIMATORS AND NOTATIONS

Samawi and Siam (2003) gives the combined ratio estimator of the population mean Y using SRSS is as

$$\bar{y}_{R(str)} = \bar{y}_{(srss)} \left(\frac{\bar{x}_{(srss)}}{\bar{X}}\right) \quad 2.1$$

The combined product estimator of the population mean Y using SRSS can be also defined as

$$\bar{y}_{P(str)} = \bar{y}_{(srss)} \left(\frac{\bar{X}}{\bar{x}_{(srss)}}\right) \quad 2.2$$

Where $\bar{y}_{(srss)} = \sum_{h=1}^{L} W_h \ \bar{y}_{h(r_h)}$ and $\bar{x}_{(srss)} = \sum_{h=1}^{L} W_h \ \bar{x}_{h(r_h)}$

To the first degree of approximation, the Biases and MSEs of $\bar{y}_{R(str)}$ and $\bar{y}_{P(str)}$ are respectively given by

$$B(\bar{y}_{R(str)}) = \bar{Y} \left[\sum_{h=1}^{L} \frac{w_h{}^2}{n_h} \left(\frac{s_{x_h}^2}{\bar{X}^2} - \frac{m}{n_h} \sum_{h=1}^{L} D_{x_h(i)}^2\right| - \bar{Y}\left[\sum_{h=1}^{L} \frac{w_h{}^2}{n_h} \left(\frac{s_{x_h y_h}}{\overline{\overline{XY}}} - \frac{m}{n_h} \sum_{h=1}^{L} D_{x_h(i) y_h(i)}\right)\right] \quad 2.3$$

$$B(\bar{y}_{R(str)}) = \bar{Y} \left[\left(\sum_{h=1}^{L} \frac{w_h{}^2}{n_h} \left(\frac{s_{x_h}^2}{\bar{X}^2} - \frac{s_{x_h y_h}}{\overline{XY}}\right) - \sum_{h=1}^{L} \frac{w_h{}^2}{n_h} \left(\frac{m}{n_h} \left(\sum_{h=1}^{L} D_{x_h(i)}^2 - \sum_{h=1}^{L} D_{x_h(i) y_h(i)}\right)\right]\right] \quad 2.4$$

$$B(\bar{y}_{P(str)}) = \left[\sum_{h=1}^{L} \frac{w_h{}^2}{n_h} \left(\frac{s_{x_h y_h}}{\overline{XY}} - \sum_{h=1}^{L} \frac{w_h{}^2}{n_h} \left(\frac{m}{n_h} \sum_{h=1}^{L} D_{x_h(i) y_h(i)}\right)\right]\right. \quad 2.5$$

And Mean Square Error is given by

$$MSE(\bar{y}_{R(str)}) = \sum_{h=1}^{L} \frac{w_h^2}{n_h}\left[\left(S_{y_h}^2 + R^2\ S_{x_h}^2 - 2R\,S_{x_h y_h}\right) - \bar{Y}^2\left(\frac{m}{n_h}\sum_{h=1}^{L}(D_{y_{h(i)}} - D_{x_{h(i)}})^2\right]\right. \qquad 2.6$$

And

$$MSE(\bar{y}_{P(str)}) = \sum_{h=1}^{L} \frac{w_h^2}{n_h}\left[\left(S_{y_h}^2 + R^2\ S_{x_h}^2 + 2R\,S_{x_h y_h}\right) - \bar{Y}^2\left(\frac{m}{n_h}\sum_{h=1}^{L}(D_{y_{h(i)}} + D_{x_{h(i)}})^2\right]\right. \qquad 2.7$$

Where $n_h = mr_h$, $D_{y_{h(i)}}^2 = \frac{\tau_{y_{h(i)}}^2}{\bar{Y}^2}$, $D_{x_{h(i)}}^2 = \frac{\tau_{x_{h(i)}}^2}{\bar{X}^2}$, $D_{x_{h(i)}y_{h(i)}} = \frac{\tau_{x_{h(i)}y_{h(i)}}}{\bar{X}\bar{Y}}$ and $\tau_{y_{h(i)}} = \mu_{y_{h(i)}} - \bar{Y}_h$,

$\tau_{x_{h(i)}} = \mu_{x_{h(i)}} - \bar{X}_h$, $\tau_{x_{h(i)}y_{h(i)}} = (\mu_{x_{h(i)}} - \bar{X}_h)(\mu_{y_{h(i)}} - \bar{Y}_h)$, where $\mu_{x_{h(i)}} = E(x_{h(i)})$

$\mu_{y_{h(i)}} = E(y_{h(i)})$, $\bar{Y}_h$ is the mean of h^{th} stratum for the variable Y and $\bar{X}_h$ is the mean of h^{th} stratum for the variable X.

III. PROPOSED ESTIMATORS

We propose ratio and product type estimators for population mean $\bar{Y}$ using SRSS as

$$\bar{y}_{str1} = \bar{y}_{(srss)} - \frac{\bar{x}_{(srss)}}{\bar{X}} + 1 \qquad 3.1$$

$$\bar{y}_{str2} = \bar{y}_{(srss)} - \frac{\bar{X}}{\bar{x}_{(srss)}} + 1 \qquad 3.2$$

Where $\bar{y}_{(srss)} = \sum_{h=1}^{L} W_h\ \bar{y}_{h(r_h)}$, and $\bar{x}_{(srss)} = \sum_{h=1}^{L} W_h\ \bar{x}_{h(r_h)}$ are the stratified ranked set sampling means for variable Y and X.

To obtain the bias and MSE of $\bar{y}_{str1}$, we put

$\bar{y}_{(srss)} = \bar{Y}(1 + e_{0h})$ and $\bar{x}_{(srss)} = \bar{X}(1 + e_{1h})$, So that $E(e_{0h}) = E(e_{1h}) = 0$

$$V(e_{0h}) = E(e_{0h})^2 = V\left(\frac{\bar{y}_{(srss)}}{\bar{Y}^2}\right) = \left[\sum_{h=1}^{L}\frac{w_h^2}{n_h}\left(\frac{s_{y_h}^2}{\bar{Y}^2} - \frac{m}{n_h}\sum_{h=1}^{L} D_{y_{h(i)}}^2\right]\right.$$

$$V(e_{1h}) = E(e_{1h})^2 = V\left(\frac{\bar{x}_{(srss)}}{\bar{X}^2}\right) = \left[\sum_{h=1}^{L}\frac{w_h^2}{n_h}\left(\frac{s_{x_h}^2}{\bar{X}^2} - \frac{m}{n_h}\sum_{h=1}^{L} D_{x_{h(i)}}^2\right]\right.$$

$$E(e_{0h}, e_{1h}) = \sum_{h=1}^{L}\frac{w_h^2}{n_h}\left(\frac{s_{x_h y_h}^2}{\bar{Y}^2} - \frac{m}{n_h}\sum_{h=1}^{L} D_{x_{h(i)}y_{h(i)}}\right)$$

Expressing in terms of e's, we obtain

$$\bar{y}_{str1} = \bar{Y}(1 + e_{0h}) - \frac{\bar{X}(1+e_{1h})}{\bar{X}} + 1$$

$$\bar{y}_{str1} - \bar{Y} = \bar{Y}\,e_{0h} - e_{1h} \qquad 3.3$$

Taking expectation on both sides we get the bias of $\bar{y}_{str1}$ as

$$B(Y_{str1}) = 0 \qquad 3.4$$

Squaring both sides of ( ), neglecting terms e's having power greater than two and then taking expectation , we get the MSE of $\bar{y}_{str1}$ to the first degree of approximation as

$$MSE(\bar{y}_{str}) = \bar{Y}^2\left[\sum_{h=1}^{L}\frac{w_h^2}{n_h}\left(\frac{s_{y_h}^2}{\bar{Y}^2} - \frac{m}{n_h}\sum_{h=1}^{L} D_{y_{h(i)}}^2\right] - 2\bar{Y}\left[\sum_{h=1}^{L}\frac{w_h^2}{n_h}\left(\frac{s_{x_h y_h}}{\bar{X}\bar{Y}} - \frac{m}{n_h}\sum_{h=1}^{L} D_{x_{h(i)}y_{h(i)}}\right)\right]\right.\right.$$

$$+\bar{X}^2\left[\sum_{h=1}^{L}\frac{w_h^2}{n_h}\left(\frac{s_{x_h}^2}{\bar{X}^2} - \frac{m}{n_h}\sum_{h=1}^{L} D_{x_{h(i)}}^2\right]\right. \qquad 3.5$$

Expressing in terms of e's, we obtain

$$\bar{y}_{str2} = \bar{Y}(1 + e_{0h}) - \frac{\bar{X}}{\bar{X}(1+e_{1h})} + 1$$

$$\bar{y}_{str2} - \bar{Y} = \bar{Y}\,e_{0h} + e_{1h} - \frac{e_{1h}^2}{2} \qquad 3.7$$

Taking expectation on both sides we get the bias of $\bar{y}_{str2}$ as

$$B(Y_{str2}) = -\frac{1}{2}\left\{\sum_{h=1}^{L}\frac{w_h^2}{n_h}\left(\frac{s_{x_h}^2}{\bar{X}^2} - \frac{m}{n_h}\sum_{h=1}^{L} D_{x_{h(i)}}^2\right\}\right. \qquad 3.8$$

and MSE is

$$MSE(\bar{y}_{str\ 2})=\bar{Y}^2\left[\sum_{h=1}^{L}\frac{w_h^2}{n_h}\left(\frac{s_{y_h}^2}{\bar{Y}^2}-\frac{m}{n_h}\sum_{h=1}^{L}D_{y_{h(i)}}^2\right]+2\bar{Y}\left[\sum_{h=1}^{L}\frac{w_h^2}{n_h}\left(\frac{s_{x_hy_h}}{\bar{X}\bar{Y}}-\frac{m}{n_h}\sum_{h=1}^{L}D_{x_{h(i)}y_{h(i)}}\right)\right]$$
$$+\bar{X}^2\left[\sum_{h=1}^{L}\frac{w_h^2}{n_h}\left(\frac{s_{x_h}^2}{\bar{X}^2}-\frac{m}{n_h}\sum_{h=1}^{L}D_{x_{h(i)}}^2\right]\qquad 3.9$$

Now we propose Exponential type ratio and product estimators for population mean $\bar{Y}$ using SRSS as

$$\bar{y}_{str3}=\sum_{h=1}^{L}W_h\,y_h\,e^{\left(\frac{\bar{x}_h-\bar{X}_h}{\bar{x}_h+\bar{X}_h}\right)}\qquad 3.10$$

$$\bar{y}_{str4}=\sum_{h=1}^{L}W_h\,y_h\,e^{\left(\frac{x_h-\bar{x}_h}{\bar{X}_h+\bar{x}_h}\right)}\qquad 3.11$$

Expressing $\bar{y}_{str3}$ in terms of e's, we obtain

$$\bar{y}_{str3}=\sum_{h=1}^{L}W_h\,y_h\,e^{\left(\frac{\bar{X}_h(1+e_{1h})-\bar{X}_h}{\bar{X}_h(1+e_{1h})+\bar{X}_h}\right)}$$

$$\bar{y}_{str3}-\bar{Y}=\bar{Y}\left[\frac{e_{1h}}{2}-\frac{e_{1h}{}^2}{8}+e_{0h}+\frac{e_{0h}e_{1h}}{2}\right]\qquad 3.12$$

The bias and MSE of $\bar{y}_{str3}$ is given by

$$B(Y_{str3})=\bar{Y}\left[\frac{1}{2}\sum_{h=1}^{L}\frac{w_h^2}{n_h}\left(\frac{s_{x_hy_h}}{\bar{X}\bar{Y}}-\frac{m}{n_h}\sum_{h=1}^{L}D_{x_{h(i)}y_{h(i)}}\right)-\frac{1}{8}\sum_{h=1}^{L}\frac{w_h^2}{n_h}\left(\frac{s_{x_h}^2}{\bar{X}^2}-\frac{m}{n_h}\sum_{h=1}^{L}D_{x_{h(i)}}^2\right]\qquad 3.13$$

And

$$MSE\quad(\bar{y}_{str\ 3})=\bar{Y}^2\left[\sum_{h=1}^{L}\frac{w_h^2}{n_h}\left(\frac{s_{y_h}^2}{\bar{Y}^2}-\frac{m}{n_h}\sum_{h=1}^{L}D_{y_{h(i)}}^2+\sum_{h=1}^{L}\frac{w_h^2}{n_h}\left(\frac{s_{x_hy_h}}{\bar{X}\bar{Y}}-\frac{m}{n_h}\sum_{h=1}^{L}D_{x_{h(i)}y_{h(i)}}\right)\right.$$
$$\left.+\frac{1}{4}\sum_{h=1}^{L}\frac{w_h^2}{n_h}\left(\frac{s_{x_h}^2}{\bar{X}^2}-\frac{m}{n_h}\sum_{h=1}^{L}D_{x_{h(i)}}^2\right)\right]\qquad 3.14$$

Expressing $\bar{y}_{str3}$ in terms of e's, we obtain

$$\bar{y}_{str4}=\sum_{h=1}^{L}W_h\,y_h\,e^{\left(\frac{\bar{x}_h-\bar{x}_h(1+e_{1h})}{\bar{X}_h+\bar{X}_h(1+e_{1h})}\right)}$$

$$\bar{y}_{str4}-\bar{Y}=\bar{Y}\left[-\frac{e_{1h}}{2}+e_{0h}+\frac{3}{8}e_{1h}{}^2-\frac{e_{0h}e_{1h}}{2}\right]\qquad 3.15$$

The bias and MSE of $\bar{y}_{str4}$ is given by

$$B(Y_{str4})=\bar{Y}\left[\frac{3}{8}\sum_{h=1}^{L}\frac{w_h^2}{n_h}\left(\frac{s_{x_h}^2}{\bar{X}^2}-\frac{m}{n_h}\sum_{h=1}^{L}D_{x_{h(i)}}^2-\frac{1}{2}\sum_{h=1}^{L}\frac{w_h^2}{n_h}\left(\frac{s_{x_hy_h}}{\bar{X}\bar{Y}}-\frac{m}{n_h}\sum_{h=1}^{L}D_{x_{h(i)}y_{h(i)}}\right)\right]\qquad 3.16$$

$$MSE\quad(\bar{y}_{str\ 4})=\bar{Y}^2\left[\sum_{h=1}^{L}\frac{w_h^2}{n_h}\left(\frac{s_{y_h}^2}{\bar{Y}^2}-\frac{m}{n_h}\sum_{h=1}^{L}D_{y_{h(i)}}^2-\sum_{h=1}^{L}\frac{w_h^2}{n_h}\left(\frac{s_{x_hy_h}}{\bar{X}\bar{Y}}-\frac{m}{n_h}\sum_{h=1}^{L}D_{x_{h(i)}y_{h(i)}}\right)\right.$$
$$\left.+\frac{1}{4}\sum_{h=1}^{L}\frac{w_h^2}{n_h}\left(\frac{s_{x_h}^2}{\bar{X}^2}-\frac{m}{n_h}\sum_{h=1}^{L}D_{x_{h(i)}}^2\right)\right]\qquad 3.16$$

IV. EFFICIENCY COMPARISONS

In this section ,we have compared the proposed ratio and product estimators to the SRSS

1. $MSE(\bar{y}_{R(str)})-MSE(\bar{y}_{str1})\geq 0$

i.e. $MSE\overline{(y}_{R(str)})\geq MSE(\bar{y}_{str1})$

2 .$MSE(\bar{y}_{P(str)})-MSE(\bar{y}_{str2})\geq 0$

i.e. $MSE\overline{(y}_{P(str)})\geq MSE(\bar{y}_{str2})$

3 .$MSE(\bar{y}_{R(str)})-MSE(\bar{y}_{str3})\geq 0$

i.e. $MSE\overline{(y}_{R(str)})\geq MSE(\bar{y}_{str3})$

4. $MSE(\bar{y}_{R(str)})-MSE(\bar{y}_{str4})\geq 0$

i.e. $MSE\overline{(y}_{R(str)})\geq MSE(\bar{y}_{str4})$

The present paper deals with ratio and product type estimators of finite population mean in stratified ranked set sampling . The bias and MSE of proposed estimators under large approximation are derived. The proposed estimators for the population mean using SRSS are asymptotically more efficient than the usual estimators in SRSS .

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