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b * -Open Sets in Bispaces

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ABSTRACT: *The notion of b-open sets in a topological space was introduced by D. Andrijevic [3] in 1996. Later Talal et al [16] introduced the idea of quasi b-open sets in a bitopological space. Here we have studied the ideas of b* -open sets and weakly b* -open sets in Alexandroff spaces. Also we have introduced the ideas of quasi b* -open sets , τ_i b* -open sets w.r.to τ_j and pairwise b* open sets in a more general structure of a bispace .*

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I. INTRODUCTION

The notion of a σ-space or simply a space or Alexandorff space was introduced by A. D. Alexandroff [1] in 1940 generalizing the idea of a topological space where only countable unions of open sets were taken to be open. The notion of a topological space was generalized to a bitopological space by J. C. Kelly [10] in 1963. In 2001 the idea of space was used by B. K. Lahiri and P. Das [12] to generalize this idea of bitopological space to a bispace.
N. Levine [14] introduced the concept of semi open sets in a topological space in 1963 and this was studied in a space by B. K. Lahiri and P. Das [13] and they critically took the matter of generalization in this setting. On the other hand the same was generalized by S. Bose [7] in the setting of a bitopological space (X, P, Q) using the idea of $P(Q)$ semi open sets with respect to $Q(P)$ etc. Later A. K. Banerjee and P. K. Saha studied the idea of pairwise semi open sets in a more general structure of a bispace.
Mashhour et al. [15] introduced the concept of preopen sets and precontinuity in a topological space. M. Jelic [8] generalized this idea of preopen sets and precontinuity in a bitopological space. Later Khedr et al. [9] and Kar et al. [11] studied further on precontinuity and semi precontinuity in a bitopological space. Recently and A. K. Banerjee and P. K. Saha studied the same in a bispace.
The idea of b-open sets in a topological space was given by D. Andrijevic [3] in 1996 where the class of b-open sets is larger than the union of the class of preopen sets and semi open sets.
In this paper we have studied a new class of sets called class of b^* -open sets in a Alexandroff space [1] which is a subclass of the sets of semi open sets and preopen sets. Indeed this class is not alike with the class of b-open sets as introduced in a topological space. However we have studied the idea of weakly b^*-open sets which is the same as b-open sets in a topological space.
Talal et al [16] introduced the idea of quasi b-open sets and quasi b-generalized open sets in a bitopological space. Here we have studied analogously quasi b^*-open sets and quasi b^*-generalized open sets in a bispace. Also we have introduced a new type of sets which are named as τ_1 b^*-open set w. r. to τ_2 and pairwise b^*-open sets and we have investigated some of its important properties.

II. PRELIMINARIES

Throughout our discussion ,(X, τ_1, τ_2) or simply X stands for a bispace , R stands for the set of real numbers , Q stands for the set of rational numbers , P for the set of irrational numbers , N for the set of natural numbers and sets are always subsets of X unless otherwise stated .

Definition 2.1[1]: A set X is called an Alexandroff space or simply a space if in it is chosen a system of subsets F satisfying the following axioms:
1) The intersection of a countable number of sets from F is a set in F.
2) The union of a finite number of sets from F is a set in F.
3) The void set φ is a set in F.

4) The whole set X is a set in F.

Sets of F are called closed sets. Their complementary sets are called open sets. It is clear that instead of closed sets in the definition of the space, one may put open sets with subject to the conditions of countable summability, finite intersectibility and the condition that X and void set φ should be open. The collection of all such open sets will sometimes be denoted by τ and the space by (X,τ). Note that, a topological space is a space but in general, τ is not a topology as can be easily seen by taking $X = R$ and τ as the collection of all F_σ sets in R.

Definition 2.2[1]: To every set M of a space (X,τ) we correlate its closure $\overline{M}$, the intersection of all closed sets containing M. The closure of a set M will be denoted by $\tau cl(M)$ or simply clM when there is no confusion about τ.

Generally the closure of a set in a space is not a closed set.

From the axioms, it easily follows that

1) $\overline{M \cup N} = \overline{M} \cup \overline{N}$; 2) $M \subset \overline{M}$; 3) $\overline{M} = \overline{\overline{M}}$; 4) $\overline{\varphi} = \varphi$. 5) $\overline{A} = A \cup A'$ where A' denotes the set of all limit point of A.

Definition 2.3[13]: The interior of a set M in a space, (X,τ) is defined as the union of all open sets contained in M and is denoted by τ-int(M) or int(M) when there is no confusion about τ.

Definition 2.4 [10]: A set X on which are defined two arbitrary topologies P, Q is called a bitopological space and is denoted by (X, P, Q).

Definition 2.5[12]: Let X be a nonempty set. If τ_1 and τ_2 be two collections of subsets of X such that (X, τ_1) and (X, τ_2) are two spaces, then X is called a bispace and is denoted by (X, τ_1, τ_2).

Definition 2.6 [15] : Let (X, τ) be a topological space . A subset A of X is said to be preopen if $A \subset int\left(cl\left(A\right)\right)$.

Definition 2.7 [11] : A subset A of bitopological space (X, τ_1, τ_2) is said to be (τ_i, τ_j) preopen (or briefly (i, j) preopen) if there exists $U \in \tau_i$ such that $A \subset U \subset \tau_j cl(A)$ or equivalently $A \subset \tau_i int\left(\tau_j cl(A)\right)$.

A subset A is said to be pairwise preopen if it is (i, j) preopen for $i, j = 1,2; i \neq j$.

III. WEAKLY SEMI OPEN AND WEAKLY PREOPEN SETS IN SPACES

Definition 3.1 (cf. [14]) : A subset A in an Alexandroff space (i.e., space) (X, τ) is called weakly semi open if $A \subset cl\left(int\, A\right)$.

Definition 3.2 ([13]) : A set A in a space (X, τ) is said to be semi open if there exists an open set G such that $G \subset A \subset clG$. .

Definition 3.3 (cf. [15]) : A subset A in a space (X, τ) is called weakly preopen if $A \subset int\left(cl\, A\right)$.

Definition 3 .4 ([6]) : A subset A in a space (X, τ) is said to be preopen if there exists an open set U such that $A \subset U \subset cl(A)$.

Clearly countable union of semi open sets is semi open and countable union of preopen sets is preopen.

Note 3.1 : Note that a subset A of a space X is semi-open if and only if there exists an open set U such that $U \subset A$ and $cl\, U = cl\, A$. Also a subset A of a space X is pre-open if and only if there exists an open set V such that $A \subset V$ and $cl\, A = cl\, V$.

Note 3.2 : Note that in a topological space the idea of weakly semi-open sets and semi-open sets are same but a weakly semi-open set may not be semi-open in a space as shown in the example 3.4

[13] . Similarly the idea of weakly pre-open sets and pre-open set are same in a topological space but in a space a weakly pre-open set may not be pre-open set as shown in the following example:

Example 3.1 : Let $X = [1,2]$ and $\tau = \{X, \varphi, F_i\}$ where F_i's are the countable subsets of irrational in $[1,2]$. Consider $B = ([1,2] - Q) - \{\sqrt{2}\}$. Then $cl(B) = X - \{\sqrt{2}\}$ and $int(cl(B)) = B$. Therefore, $B \subset int(cl(B))$. Again, there does not exists any τ-open set U such that $B \subset U \subset cl(B)$. So B is weakly preopen but not preopen.

IV. b *- OPEN SETS IN SPACES

Definition 4.1 (cf. [3]) : A set A in a space (X, τ) is called weakly b^*-open if $A \subset int\,(\,cl\,A\,) \cup\ cl\,(int\,A\,)$.

Clearly every weakly semi open set is weakly b^*-open and every weakly preopen set is weakly b^*-open.

Definition 4.2 : A set A in a space (X, τ) is called b^*-open if there exist open sets U and V such that $U \subset A \subset V$ and $cl\,U = cl\,V$.

Theorem 4.1 : Let $A \subset X$, then A is b^*-open if and only if A is both semi-open and preopen.

Proof : If A is b^*-open then there exist open sets U and V such that $U \subset A \subset V$ and $cl\,U = cl\,V$. So $U \subset A \subset V \subset cl\,V = cl\,U$ which implies that A is semi-open. Again $A \subset V \subset cl\,V = cl\,U = cl\,A$ implies A is pre-open.
Conversely , if A is both semi-open and pre-open then there exist open sets U and V such that $U \subset A \subset cl\,U$ and $A \subset V \subset cl\,A$ which gives $cl\,U \subset cl\,A \subset cl\,(cl\,U) = cl\,U$. So $cl\,U = cl\,A$ and $cl\,A \subset cl\,V \subset cl\,(cl\,A) = cl\,A$. So $cl\,V = cl\,A$. Thus $U \subset A \subset V$ and $cl\,U = cl\,A = cl\,V$. So A is b^*-open.

Note 4..1 : It can be easily checked that every open set is b^*-open.

Theorem 4.2: Every b^*-open set is weakly b^*-open .

Proof : If A is b^*-open then there exist open sets U and V such that $U \subset A \subset V$ and $cl\,U = cl\,V$. Now $A \subset V$ implies that $int\,A \subset int\,V = V$ and so $cl\,(\,int\,A\,) \subset cl\,V$. Again, $U \subset A$ implies that $U = int\,U \subset int\,A$ and so $cl\,U \subset cl\,(int\,A\,)$. Therefore, $cl\,A = cl\,U \subset cl\,(\,int\,A\,) \subset cl\,V = cl\,A$, therefore, $cl\,A = cl\,(\,int\,A\,)$ i.e., $A \subset cl\,A = cl\,(\,int\,A\,)$. So A is weakly semi open.
Also $V \subset cl\,V$ implies that $V = int\,V \subset int\,(\,cl\,V\,)$. Again, $cl\,V = cl\,A$ i.e., $int\,(\,cl\,V\,) = int\,(\,cl\,A\,)$. Therefore, $A \subset V \subset int\,(\,cl\,V\,) = int\,(\,cl\,A\,)$. So A is weakly preopen set and hence A weakly b^*-open set.

Example 4.1 : Example of a b^*-open set which is not open.
Let $X = [0,2]$ and G_i be the countable subsets of irrational of $[\,0,2\,]$. Let $\tau = \{\,\varphi, X\,, G_i \cup \{\sqrt{3}\}\}$ then for each open set $G_i \cup \{\sqrt{3}\}$, $cl\,(G_i \cup \{\sqrt{3}\}) = X$. Let P = the set of all irrational in $[0,2]$. Clearly P is not open. But P is semi-open since for every open set $G_i \cup \{\sqrt{3}\}$, $G_i \cup \{\sqrt{3}\} \subset P \subset X = cl\,(G_i \cup \{\sqrt{3}\})$. Also P is pre -open . Since $P \subset X \subset cl\,P = X$. So P is b^*-open.

Theorem 4.3 : Countable union of b^*-open sets is b^*-open.

Proof : Let $\{A_i : i = 1,2,3, \ldots.\}$ be a countable collection of b^*-open sets. Then A_i is pre-open and semi-open for each $i = 1,2,3 \ldots \ldots$. So $\bigcup_{i=1}^{\infty} A_i$ is both pre –open and semi-open. So $\bigcup_{i=1}^{\infty} A_i$ is b^*-open.

Theorem 4.4 : Intersection of an open set and b^*-open set is b^*-open set.

Proof : Let A is τ-open set and B is b*-open set. So B is both semi open and preopen set. Since B is semi open there exists an τ-open set U such that $U \subset B \subset \tau cl(U)$, this implies $U \cap A \subset B \cap A \subset \tau cl(U) \cap A \subset \tau cl(U \cap A)$. Therefore, $B \cap A$ is semi open.
Again, since B is preopen open there exists an τ-open set V such that $B \subset V \subset \tau cl(B)$, this implies $B \cap A \subset V \cap A \subset \tau cl(B) \cap A \subset \tau cl(U \cap A)$. Therefore, $B \cap A$ is preopen and hence $B \cap A$ is b*-open.

Note 4.2 : Arbitrary union of b*-open sets may not be b*-open as shown in the following example :

Example 4.2 : Let $X = R$, $\tau = \{X, \varphi, G_i\}$ where G_i's are countable subsets of irrational numbers. Let P be the set of all irrational numbers . Then for any open set G_i, $(G_i \neq \varphi, X)$, $cl\, G_i = G_i \cup Q$ where Q is the set of all rational number. So there does not exist any open set G such that $G \subset P \subset cl\, G$ holds. So P is not semi-open and hence not b*-open. But each G_i is open and so b*-open and $\bigcup_i G_i = P$ which is not b*-open.

Note that a pre-open set may not be a b*-open set. We give below an example of preopen set which is not b*-open set.

Example 4.3 : Let $X = [1,2]$, $\tau = \{X, \varphi, G_i\}$ where G_i's are countable subset of irrational in $[1,2]$. Obviously, (X, τ) is a space but not a topological space. Consider $A = [1,2] - Q$. If G is any open set such that $G \subset A$ then $cl\, G = G \cup ([1,2] \cap Q)$ which does not contain A. So A is not semi open but A is preopen set because $A \subset X \subset cl(A) = X$. So A is preopen but not b*-open.
Clearly a semi open set may not be b*-open set as shown in the following example .

Example 4.4 : Take X, τ as example 4.3. Let $B = ([1,2] \cap Q) \cup \{\sqrt{2}\}$. Here $\{\sqrt{2}\} \subset B \subset cl(\{\sqrt{2}\}) = B$. So B is semi open set. Again, there does not exist any open set U, other than X which contains B and also $B \subset X \not\subset cl(B) = B$. So B is not preopen. Therefore, B is not b*-open set.

V. QUASI b* -OPEN SETS IN BISPACES

Definition 5.1(cf. [16]): Let (X, τ_1, τ_2) be a bispace. A subset $A \subset X$ is said to be quasi b*-open in (X, τ_1) and if $A = U \cup V$ for some $U \in$ B*O (X, τ_1) and $V \in$ B*O (X, τ_2) where B*O (X, τ_1) and B*O (X, τ_2) are respectively the class of b*-open sets in (X, τ_1). and (X, τ_2). The complement of a quasi b* open set is called quasi b* closed.
By $QB^*O(X, \tau_1, \tau_2)$ (resp. $QB^*C(X, \tau_1, \tau_2)$) we denote the class of all quasi b*-open (resp. quasi b*-closed) subsets of bispace (X, τ_1, τ_2).

Theorem 5.1 : Countable union of quasi b*-open sets is quasi b*-open.

Proof: Let $A_i = U_i \cup V_i$ be quasi b*-open sets for $i = 1,2,3, \dots \dots \infty$ where $U_i \in B^*O(X, \tau_1)$ and $V_i \in B^*O(X, \tau_2)$ i.e., U_i is both preopen and semi open in (X, τ_1) and V_i is both preopen and semi open in (X, τ_2).
Now $A_1 \cup A_2 \cup \dots \dots . . = (U_1 \cup V_1) \cup (U_2 \cup V_2) \cup \dots.$

$$= \left(\bigcup_{i=1}^{\infty} U_i \right) \cup \left(\bigcup_{i=1}^{\infty} V_i \right)$$

Where $\bigcup_{i=1}^{\infty} U_i \in B^*O(X, \tau_1)$ and $\bigcup_{i=1}^{\infty} V_i \in B^*O(X, \tau_2)$. Hence the result.

Theorem 5.2 : In a bispace (X, τ_1, τ_2), $B^*O(X, \tau_i) \subset QB^*O(X, \tau_1, \tau_2)$ holds for $i = 1, 2$.

Proof : For any set $A \in B^*O(X, \tau_i)$, $A = A \cup \emptyset$ where $\emptyset \in B^*O(X, \tau_j)$ for $i \neq j; i, j = 1,2$. So $A \in QB^*O(X, \tau_1, \tau_2)$.

Theorem 5.3 : If a subset $A \in \tau_1 \cap \tau_2$ in (X, τ_1, τ_2) and a subset $B \in QB^*O(X, \tau_1, \tau_2)$ then $A \cap B \in QB^*O(X, \tau_1, \tau_2)$.

Proof : Since $B \in QB^*O(X, \tau_1, \tau_2)$, so B can be written as $B = B_1 \cup B_2$ where $B_1 \in B^*O(X, \tau_1)$ and $B_2 \in B^*O(X, \tau_2)$. So $A \cap B_1 \in B^*O(X, \tau_1)$ and $A \cap B_2 \in B^*O(X, \tau_2)$, by theorem 4.4. So $A \cap B = A \cap (B_1 \cup B_2) = (A \cap B_1) \cup (A \cap B_2) \in QB^*O(X, \tau_1, \tau_2)$.

Definition 5.2 (cf.[16]) : The quasi b^*-closure of a subset A in a bispace (X, τ_1, τ_2) is defined to be the set $qb^*cl(A) = \cap \{F : F \in QB^*C(X, \tau_1, \tau_2),\ A \subset F\}$.

Definition 5.3 (cf. [16]) : A subset A in a bispace (X, τ_1, τ_2) is called quasi b^*-generalized closed (simply, quasi-b^*g-closed) if $qb^*cl(A) \subset U$ whenever $A \subset U$ and $U \in QB^*C(X, \tau_1, \tau_2)$ The complement of a quasi b-generalized closed set is called quasi b^*-generalized open (simply quasi-b^*g-open).

Theorem 5.4 : $x \in qb^*cl(A)$ if, and only if, for each $V \in QB^*O(X, \tau_1, \tau_2)$ containing x, $A \cap V = \emptyset$.

Proof : Let there exists a set $V \in QB^*O(X, \tau_1, \tau_2)$ containing x such that $A \cap V \neq \emptyset$. This implies that $A \subset X - V$, where $(X - V) \in QB^*C(X, \tau_1, \tau_2)$. So $qb^*cl(A) \subset X - V$. Since $x \notin X - V$, $x \notin qb^*cl(A)$. Therefore, contrapositively, if $x \in qb^*cl(A)$ then every set $V \in QB^*O(X, \tau_1, \tau_2)$ containing x intersects A.

Conversely, let every $V \in QB^*O(X, \tau_1, \tau_2)$ containing x intersects A and let $x \notin qb^*cl(A)$. Then there exists a set $F \in QB^*C(X, \tau_1, \tau_2)$ containing A such that $x \notin F$. So $X - F$ is a set in $QB^*O(X, \tau_1, \tau_2)$ containing x such that $(X - F) \cap A = \emptyset$ reW* nMCID 90>B7.52( )-2(th)7(at)-3calleduasraposs31 504.55 Tm[( )] TJETBT1 0

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