

Spacings Between and Units & Tens Place Digits in Primes till One Trillion in Arithmetical Progressions $9n + K$

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ABSTRACT : Prime numbers in six arithmetical progressions $9n + k$, for $1 \leq k \leq 9$, $\gcd(k, 9) = 1$, are considered. As assured by Dirichlet's Theorem, each of these six arithmetical progressions contains infinite number of primes in it. Selecting range of 1 to 1 trillion and 12 blocks of increasing sizes $1 - 10^n$, $1 \leq n \leq 12$, the spacings between primes of same form are determined in these blocks. The minimum and maximum spacings between them, the first & last prime starters with such spacings, and the frequencies of such spacings are obtained. Then, for primes of all forms $9n + k$, the digits in units place and tens place are analyzed.

KEYWORDS: Arithmetical progressions, block-wise spacings, prime, prime digits

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I. INTRODUCTION

Numbers is so fundamental a notion that human understanding, use, study and exploration of mathematics has begun with these numbers in ancient days. Amongst numbers, the fundamental ones are positive integers. The multiplicative building blocks of positive integers are prime numbers. A prime number is a positive integer $p > 1$ having no positive divisors other than 1 and p . From more than two millennia [1], we know that there are infinitely many primes.

Amongst sequences, the primary ones are arithmetical progressions. An arithmetical progression is a sequence of form $an + b$, where a and b are fixed positive integers and n is integer varying from 0 to ∞ . Of course, it contains infinitely many integers, all of which are those which when divided by a give remainder b . As there are a remainder 0, 1, 2, . . . , $a - 1$, when division by a is carried out, there are a such arithmetical progression for each of these values of b . Well-known theorem due to Dirichlet [2] states that if $\gcd(a, b) = 1$, then $an + b$ contains infinitely many primes in it otherwise not. Extensive study of primes in arithmetical progressions $2n + 1$, $3n + k$, $4n + k$, $5n + k$, $6n + k$, $7n + k$, $8n + k$ has been done recently [3] – [10].

We choose $a = 9$, so that there are 6 numbers that are relatively prime to 9, viz., 1, 2, 4, 5, 7, and 8. For each of these values as k , $9n + k$ contains infinitely many primes. The density distribution of primes in them is available [11].

II. MINIMUM SPACING BETWEEN PRIMES OF FORM $9n + k$ IN BLOCKS OF 10^n

We continue with the recent adopted approach of considering blocks of size 10^n for analyzing primes in them. It was possible to choose ranges as high as 10^{12} due to best choice of algorithms [12]-[18] to generate primes efficiently within least time using minimum resources. The ease of Java programming language [19] was harnessed to implement these algorithms on multiple computers simultaneously.

Neglecting specific blocks having no primes, the minimum spacing between primes of forms $9n + k$ in blocks of 10 powers are determined to be 18, except for $9n + 2$ for which only it is 9. For larger block sizes, the minimum spacing value generally doesn't increase; it remains same ahead for blocks of higher powers of 10.

The smallest considered block-size 10 is special here as the minimum block spacing happens to be 18 and it cannot occur within the blocks of 10 for forms other than $9n + 2$.

Since there is only one prime of form $9n + 3$, there is no question of computing its spacing with other prime of this form!

The other values are shown graphically.

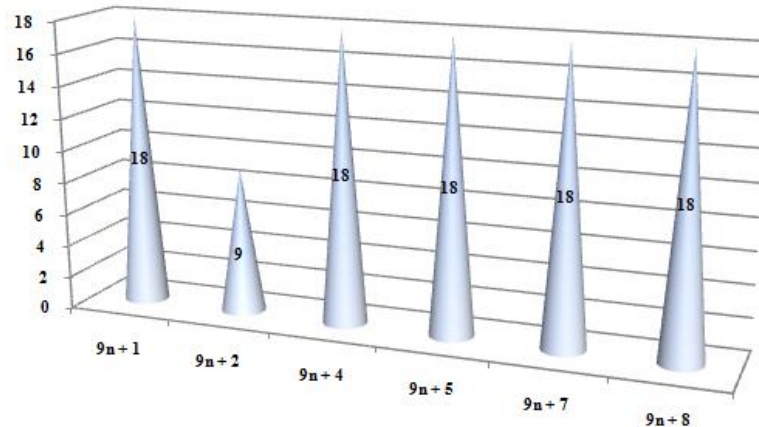


Figure 1: Minimum Block Spacing between Primes of form $9n+k$

Except the block-size of 10, the first and the last primes in the 10^n sized blocks with minimum spacing in between them are as follows:

Table 1 : First Starters of Minimum Block Spacings between Primes of form $9n+k$ in Blocks of 10^n

Sr. No.	Blocks of Size (of 10 Power)	First Prime with Respective Minimum Block Spacing					
		Form $9n+1$	Form $9n+2$	Form $9n+4$	Form $9n+5$	Form $9n+7$	Form $9n+8$
1.	10	Not Found	Not Found	Not Found	Not Found	Not Found	Not Found
2.	$100 \leq 10^n, 2 \leq n \leq 12$	19	2	13	5	43	53

Letting 10^{12} as our inspection limit, the last primes in the 10 power blocks with minimum block spacing also get settled.

Table 2: Last Starters of Minimum Block Spacings between Primes of form $9n+k$ in Blocks of 10^n

Sr. No.	Blocks of Size (of 10 Power)	Last Prime with Respective Minimum Block Spacing					
		Form $9n+1$	Form $9n+2$	Form $9n+4$	Form $9n+5$	Form $9n+7$	Form $9n+8$
1.	10	Not Found	Not Found	Not Found	Not Found	Not Found	Not Found
2.	100	999,999,993,061	2	999,999,999,571	999,999,998,141	999,999,998,719	999,999,999,269
3.	$1,000 \leq 10^n, 3 \leq n \leq 12$	999,999,993,061	2	999,999,999,571	999,999,999,599	999,999,998,719	999,999,999,269

We compare them graphically.

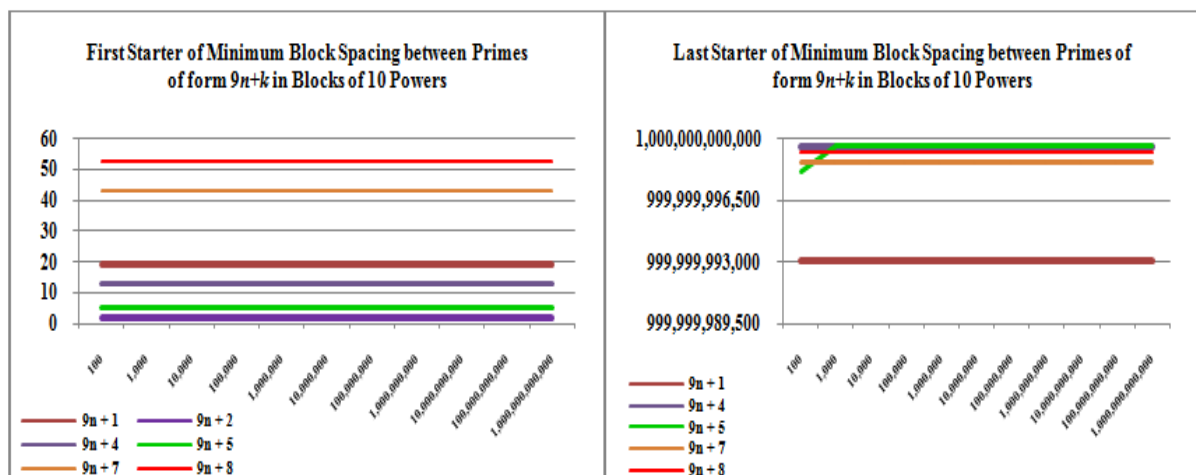


Figure 2: First & Last Starters of Minimum Block Spacing's between Primes of form $9n+k$ in Blocks of 10^n .

The next probe was the number of times these minimum block spacing's occur between primes of all forms $9n+1, 9n+2, 9n+4, 9n+5, 9n+7$ and $9n+8$.

Table 3: Frequency of Minimum Block Spacings between Primes of form $9n + k$

Sr. No.	Blocks of Size (of 10 Power)	Number of Times Minimum Block Spacing Occurring for Primes					
		Form $9n + 1$	Form $9n + 2$	Form $9n + 4$	Form $9n + 5$	Form $9n + 7$	Form $9n + 8$
1.	10	Not Found	Not Found	Not Found	Not Found	Not Found	Not Found
2.	100	519,579,826	1	519,577,729	519,598,836	519,567,951	519,628,161
3.	1,000	613,109,535	1	613,104,793	613,126,240	613,090,178	613,150,965
4.	10,000	622,462,678	1	622,457,690	622,477,714	622,444,034	622,499,624
5.	100,000	623,399,572	1	623,392,352	623,413,434	623,379,136	623,435,657
6.	1,000,000	623,492,878	1	623,485,910	623,507,109	623,473,116	623,528,991
7.	10,000,000	623,502,264	1	623,495,209	623,516,545	623,482,436	623,538,327
8.	100,000,000	623,503,242	1	623,496,152	623,517,442	623,483,341	623,539,300
9.	1,000,000,000	623,503,328	1	623,496,251	623,517,527	623,483,439	623,539,388
10.	10,000,000,000	623,503,336	1	623,496,258	623,517,539	623,483,445	623,539,396
11.	100,000,000,000	623,503,337	1	623,496,260	623,517,540	623,483,446	623,539,396
12.	1,000,000,000,000	623,503,337	1	623,496,260	623,517,540	623,483,446	623,539,396

There is increase in the number of times the minimum spacing occurs for primes of all forms except for primes of form $9n + 2$. The following analysis has omitted from $9n + 2$.

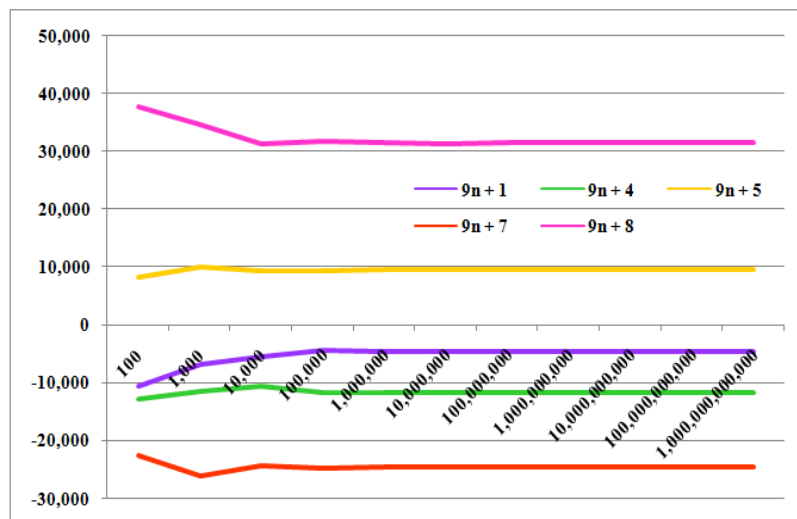


Figure 3: Deviation in Occurrences of Minimum Block Spacing between Primes of form $9n+k$ in Blocks of 10 Powers.

It is found that the occurrences of minimum block spacings for primes of form $9n + 5$ and $9n + 8$ is always more than those of forms $9n + 1$, $9n + 4$ and $9n + 7$.

III. MAXIMUM SPACING BETWEEN PRIMES OF FORM $9N + K$ IN BLOCKS OF 10^N

The maximum spacing between primes for form $9n + k$ in blocks of 10 powers initially increases with increase in the block size.

Table 4: Maximum Block Spacing between Primes of form $9n + k$.

Sr. No.	Blocks of Size (of 10 Power)	Maximum Spacing Between Primes					
		Form $9n + 1$	Form $9n + 2$	Form $9n + 4$	Form $9n + 5$	Form $9n + 7$	Form $9n + 8$
1.	10	Not Found	Not Found	Not Found	Not Found	Not Found	Not Found
2.	100	90	90	90	90	90	90
3.	1,000	990	990	990	990	990	990
4.	10,000	2,988	3,330	3,060	3,114	2,970	3,114
5.	100,000	2,988	3,330	3,060	3,114	3,042	3,114
6.	1,000,000	2,988	3,330	3,060	3,114	3,060	3,114
7.	10,000,000	2,988	3,330	3,060	3,114	3,060	3,114
8.	100,000,000	2,988	3,330	3,060	3,114	3,060	3,114
9.	1,000,000,000	2,988	3,330	3,060	3,114	3,060	3,114
10.	10,000,000,000	2,988	3,330	3,060	3,114	3,060	3,114
11.	100,000,000,000	2,988	3,330	3,060	3,114	3,060	3,114
12.	1,000,000,000,000	2,988	3,330	3,060	3,114	3,060	3,114

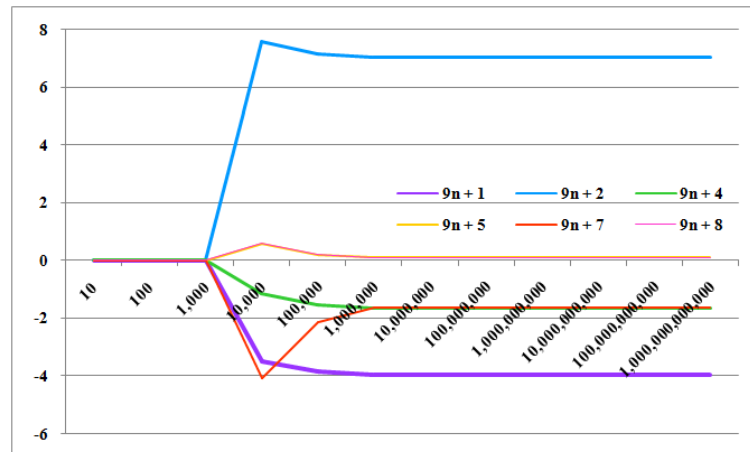


Figure 4: % Deviation of Maximum Block Spacing in 10^n Blocks between Primes of forms $9n+k$ from Average.

The first prime starters of forms $9n + k$ having maximum spacings with immediate next prime within various 10^n blocks are as follows.

Table 5: First Starters of Maximum Block Spacings between Primes of form $9n + k$ in Blocks of 10^n .

Sr. No.	Blocks of Size (of 10 Power)	First Prime with Respective Maximum Block Spacing					
		Form $9n + 1$	Form $9n + 2$	Form $9n + 4$	Form $9n + 5$	Form $9n + 7$	Form $9n + 8$
1.	10	Not Found	Not Found	Not Found	Not Found	Not Found	Not Found
2.	100	8,101	5,303	4,909	1,103	907	10,601
3.	1,000	1,022,268,007	1,350,587,009	635,502,001	475,519,001	1,624,552,009	290,932,001
4.	10,000	773,859,500,839	826,489,712,747	654,189,572,767	895,515,273,923	877,245,900,541	682,345,396,169
5.	100,000	773,859,500,839	826,489,712,747	654,189,572,767	895,515,273,923	661,912,829,449	682,345,396,169
6.	1,000,000	773,859,500,839	826,489,712,747	654,189,572,767	895,515,273,923	524,288,597,137	682,345,396,169
7.	10,000,000	773,859,500,839	826,489,712,747	654,189,572,767	895,515,273,923	524,288,597,137	682,345,396,169
8.	100,000,000	773,859,500,839	826,489,712,747	654,189,572,767	895,515,273,923	524,288,597,137	682,345,396,169
9.	1,000,000,000	773,859,500,839	826,489,712,747	654,189,572,767	895,515,273,923	524,288,597,137	682,345,396,169
10.	10,000,000,000	773,859,500,839	826,489,712,747	654,189,572,767	895,515,273,923	524,288,597,137	682,345,396,169
11.	100,000,000,000	773,859,500,839	826,489,712,747	654,189,572,767	895,515,273,923	524,288,597,137	682,345,396,169
12.	1,000,000,000,000	773,859,500,839	826,489,712,747	654,189,572,767	895,515,273,923	524,288,597,137	682,345,396,169

And the last prime starters of these forms with maximum spacings with immediate next prime within 10^n blocks are also determined.

Table 6: Last Starters of Maximum Block Spacings between Primes of form $9n + k$ in Blocks of 10^n .

Sr. No.	Blocks of Size (of 10 Power)	Last Prime with Respective Maximum Block Spacing					
		Form $9n + 1$	Form $9n + 2$	Form $9n + 4$	Form $9n + 5$	Form $9n + 7$	Form $9n + 8$
1.	10	Not Found	Not Found	Not Found	Not Found	Not Found	Not Found
2.	100	999,999,989,803	999,999,984,701	999,999,999,301	999,999,979,403	999,999,983,401	999,999,936,701
3.	1,000	999,991,116,001	999,971,605,001	999,953,184,001	999,846,698,009	999,899,386,009	999,999,428,003
4.	10,000	773,859,500,839	826,489,712,747	960,700,146,187	895,515,273,923	877,245,900,541	682,345,396,169
5.	100,000	773,859,500,839	826,489,712,747	960,700,146,187	895,515,273,923	661,912,829,449	682,345,396,169
6.	1,000,000	773,859,500,839	826,489,712,747	960,700,146,187	895,515,273,923	524,288,597,137	682,345,396,169
7.	10,000,000	773,859,500,839	826,489,712,747	960,700,146,187	895,515,273,923	524,288,597,137	682,345,396,169
8.	100,000,000	773,859,500,839	826,489,712,747	960,700,146,187	895,515,273,923	524,288,597,137	682,345,396,169
9.	1,000,000,000	773,859,500,839	826,489,712,747	960,700,146,187	895,515,273,923	524,288,597,137	682,345,396,169
10.	10,000,000,000	773,859,500,839	826,489,712,747	960,700,146,187	895,515,273,923	524,288,597,137	682,345,396,169
11.	100,000,000,000	773,859,500,839	826,489,712,747	960,700,146,187	895,515,273,923	524,288,597,137	682,345,396,169
12.	1,000,000,000,000	773,859,500,839	826,489,712,747	960,700,146,187	895,515,273,923	524,288,597,137	682,345,396,169

Their comparative trend is depicted in the graphical representation.

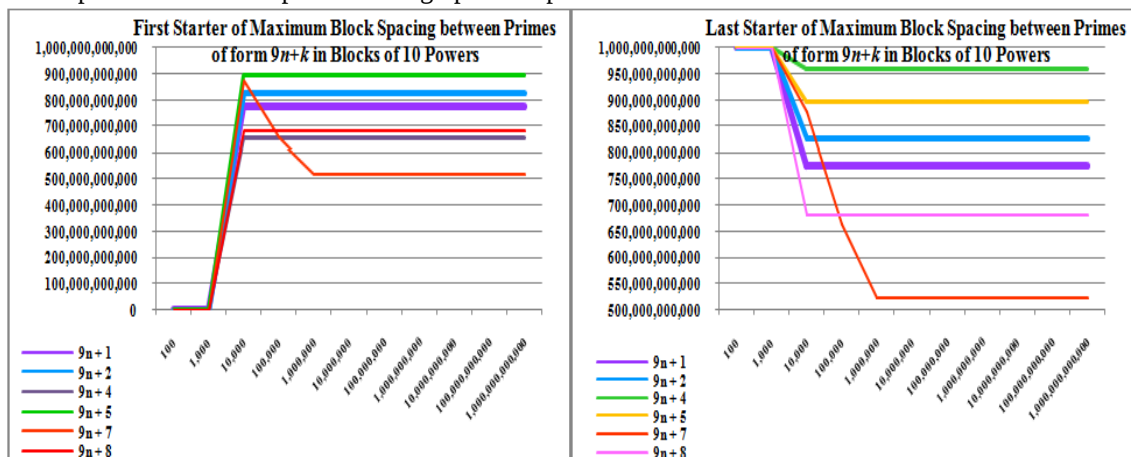


Figure 5: First & Last Starters of Maximum Block Spacings between Primes of form $9n + k$ in Blocks of 10^n .

The frequency of occurrence of maximum block spacings primes of these forms is determined.

Table 7: Frequency of Maximum Block Spacings between Primes of form $9n + k$.

Sr. No.	Blocks of Size (of 10 Power)	Number of Times Maximum Block Spacing Occurring for Primes					
		Form $9n + 1$	Form $9n + 2$	Form $9n + 4$	Form $9n + 5$	Form $9n + 7$	Form $9n + 8$
1.	10	Not Found	Not Found	Not Found	Not Found	Not Found	Not Found
2.	100	55,824,479	55,816,858	55,825,661	55,826,295	55,832,785	55,823,342
3.	1,000	10,357	10,269	10,246	10,418	10,184	10,562
4.	10,000 & $10^4, 4 \leq n \leq 12$	1	1	2	1	1	1

For all rest 10 power blocks for all prime forms under consideration, except $9n + 4$, the maximum block spacing occurs only once, while for $9n + 4$ it continues to occur twice, yielding following pattern for their deviation from respective averages.

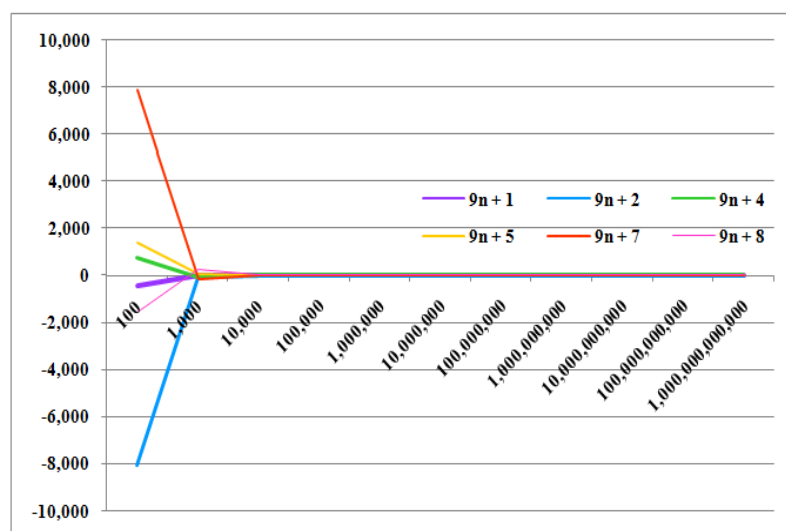


Figure 6: Deviation of Number of Occurrences of Maximum Block Spacings from Average.

IV. UNITS & TENS PLACE DIGITS IN PRIME PAIR STARTERS OF FORM $9N + K$

There are only 6 possible digits in units place for primes. The number of primes of form $9n + k$ till 1 trillion with these different digits in units place are determined.

Table 8: Number of Primes of form $9n + k$ with Different Units Place Digits till One Trillion.

Sr. No.	Digit in Units Place	Number of Primes of form					
		$9n+1$	$9n+2$	$9n+4$	$9n+5$	$9n+7$	$9n+8$
1.	1	1,567,004,624	1,566,989,672	1,566,984,806	1,566,997,630	1,566,979,403	1,567,004,845
2.	2	0	1	0	0	0	0
3.	3	1,566,993,251	1,566,998,508	1,566,995,086	1,566,993,706	1,566,996,592	1,567,002,760
4.	5	0	0	0	1	0	0
5.	7	1,566,991,208	1,566,989,775	1,567,010,785	1,567,011,481	1,567,000,688	1,566,993,063
6.	9	1,566,982,953	1,567,003,059	1,566,998,808	1,567,010,747	1,566,995,316	1,566,983,249

Here form $9n + 3$ is not seen but it contains single prime with 3 in units place. Their graphical comparison is done in which digits 2 and 5 are dropped as they come only once and form $9n + 3$ also.

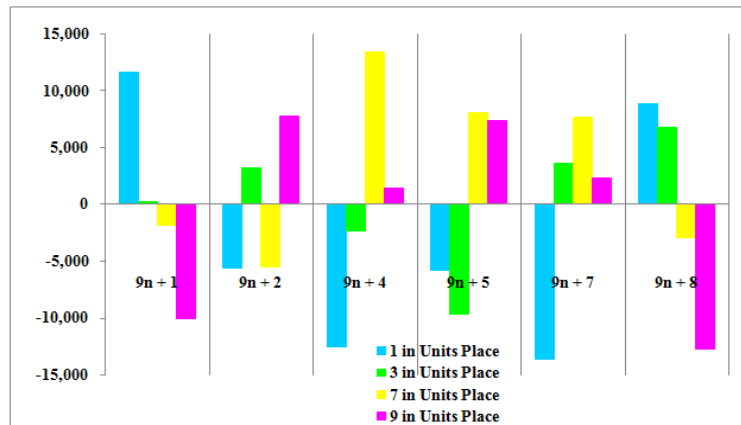


Figure 7: Deviation of Units Place Digits of Primes of form $9n + k$ from Average.

Now we consider two digits together; tens and units place.

Table 9: Number of Primes of form $9n + k$ with Different Tens and Units Places Digits till One Trillion.

Sr. No.	Digit in Tens and Units Places	Number of Primes of form					
		$9n+1$	$9n+2$	$9n+4$	$9n+5$	$9n+7$	$9n+8$
1.	01	156,699,616	156,704,835	156,692,274	156,707,529	156,699,443	156,697,527
2.	02	0	1	0	0	0	0
3.	03	156,699,856	156,697,194	156,698,921	156,704,254	156,695,993	156,702,823
4.	05	0	0	0	1	0	0
5.	07	156,698,906	156,697,564	156,701,881	156,702,837	156,696,461	156,703,835
6.	09	156,692,800	156,699,670	156,705,925	156,707,543	156,695,888	156,696,211
7.	11	156,703,791	156,696,225	156,702,341	156,698,749	156,696,265	156,694,260
8.	13	156,696,320	156,698,627	156,705,423	156,695,762	156,699,046	156,705,526
9.	17	156,696,128	156,699,839	156,697,015	156,700,885	156,698,305	156,697,133
10.	19	156,700,382	156,708,183	156,700,301	156,702,687	156,705,367	156,707,647
11.	21	156,704,752	156,704,138	156,697,105	156,700,649	156,697,131	156,703,676
12.	23	156,699,975	156,698,446	156,702,444	156,702,764	156,700,401	156,701,083
13.	27	156,706,824	156,701,250	156,701,232	156,702,358	156,695,587	156,700,121
14.	29	156,702,100	156,693,347	156,698,268	156,703,782	156,702,414	156,697,518
15.	31	156,705,822	156,700,119	156,697,998	156,696,147	156,701,570	156,701,640
16.	33	156,698,250	156,708,214	156,697,860	156,698,239	156,705,642	156,689,429
17.	37	156,709,740	156,693,406	156,706,692	156,707,093	156,687,710	156,694,195
18.	39	156,696,077	156,702,360	156,700,990	156,694,885	156,704,560	156,696,491
19.	41	156,699,303	156,693,561	156,697,377	156,702,536	156,697,267	156,699,962
20.	43	156,698,872	156,701,600	156,703,009	156,703,345	156,693,642	156,697,125
21.	47	156,695,130	156,703,635	156,695,627	156,696,822	156,704,460	156,702,058
22.	49	156,707,632	156,693,857	156,694,029	156,702,067	156,703,759	156,699,432
23.	51	156,708,753	156,698,873	156,698,318	156,698,481	156,700,397	156,700,058
24.	53	156,698,325	156,704,914	156,692,542	156,696,019	156,703,474	156,700,313
25.	57	156,696,962	156,693,554	156,703,468	156,698,845	156,699,173	156,700,993
26.	59	156,700,643	156,696,175	156,702,206	156,699,770	156,700,441	156,700,287
27.	61	156,693,979	156,698,836	156,699,616	156,700,171	156,699,466	156,704,042
28.	63	156,701,237	156,699,115	156,698,923	156,693,658	156,702,579	156,699,854
29.	67	156,700,984	156,702,053	156,703,449	156,698,549	156,699,640	156,698,682
30.	69	156,695,077	156,704,309	156,699,433	156,689,864	156,691,193	156,692,548
31.	71	156,700,206	156,698,415	156,701,089	156,701,107	156,698,875	156,696,797
32.	73	156,701,992	156,700,806	156,697,957	156,699,040	156,694,501	156,702,651
33.	77	156,690,236	156,702,422	156,698,850	156,695,838	156,708,703	156,700,574
34.	79	156,698,112	156,700,343	156,697,995	156,698,062	156,695,529	156,698,785
35.	81	156,695,321	156,694,210	156,697,794	156,699,392	156,694,191	156,699,095
36.	83	156,699,100	156,695,763	156,697,535	156,702,909	156,694,278	156,702,315
37.	87	156,700,172	156,693,711	156,700,305	156,700,528	156,704,698	156,699,640
38.	89	156,696,932	156,706,999	156,697,599	156,703,360	156,698,884	156,698,234
39.	91	156,695,081	156,700,460	156,700,894	156,692,869	156,694,798	156,707,788
40.	93	156,699,324	156,693,829	156,700,472	156,697,716	156,707,036	156,701,641
41.	97	156,696,126	156,702,341	156,702,266	156,707,686	156,705,951	156,695,812
42.	99	156,693,198	156,697,816	156,702,042	156,708,727	156,697,281	156,696,096

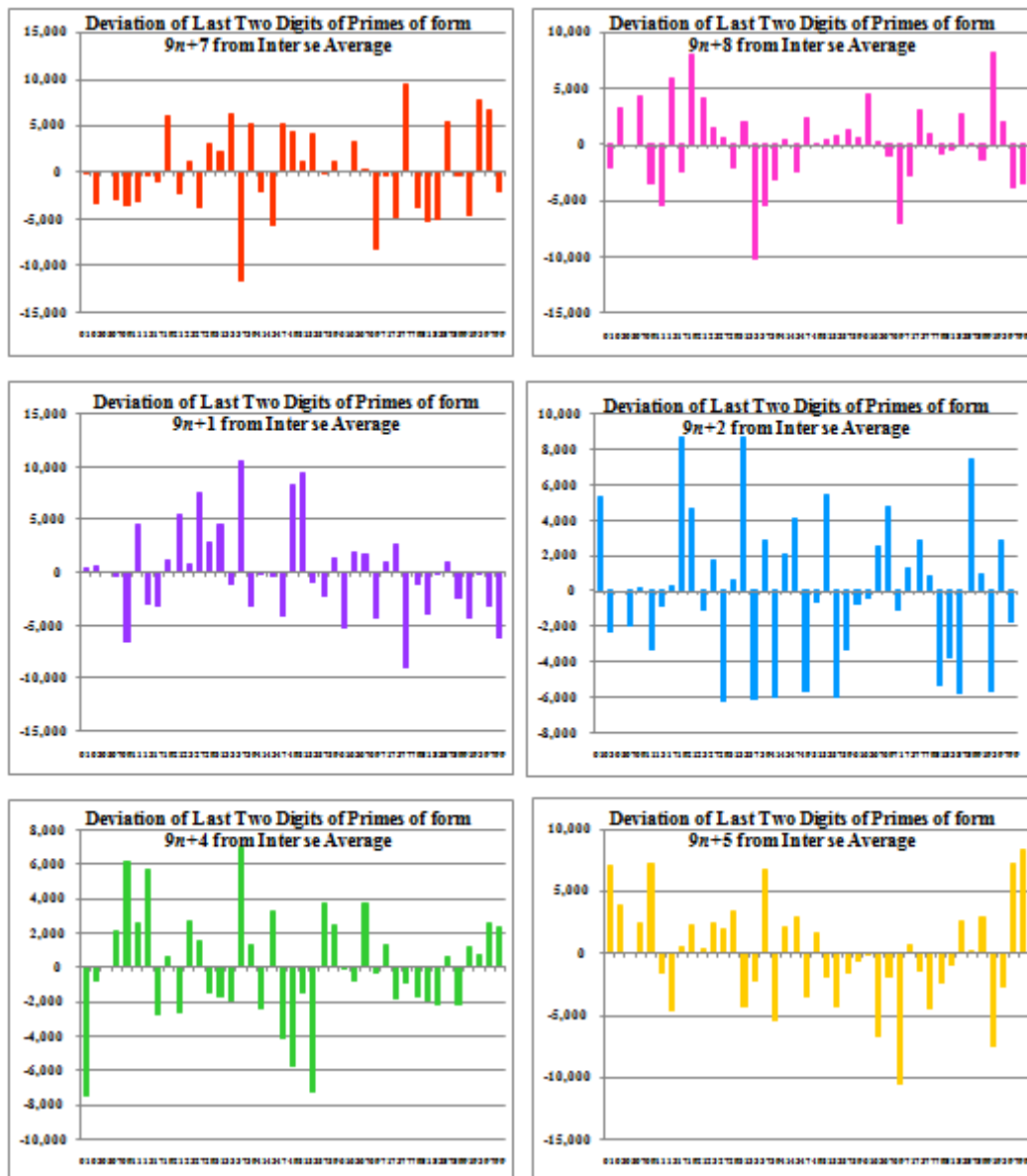


Figure 8: Deviation of Last 2 Digits of Primes of form $9n + k$ from Inter se Average.

Here also form $9n + 3$ is omitted. Quite random behavior is seen, as expected!

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REFERENCES

- [1]. Euclid (of Alexandria), Elements, Book IX, (300 BC).
- [2]. P.G.L. Dirichlet, Beweis des Satzes, dass jede unbegrenzte arithmetische Progression, deren erstes Glied und Differenz ganze Zahlen ohne gemeinschaftlichen Factor sind, unendlich viele Primzahlen enthält, (Abhand. Ak. Wiss. Berlin, 1837).
- [3]. Neeraj Anant Pande, Analysis of Primes Less than a Trillion, International Journal of Computer Science & Engineering Technology (ISSN: 2229-3345), Vol. 6, No. 06, 2015, 332 – 341.
- [4]. Neeraj Anant Pande, Analysis of Primes in Arithmetical Progressions $3n + k$ up to a Trillion, IOSR Journal of Mathematics, Volume 11, Issue 3 Ver. IV, 2015, 72-85.
- [5]. Neeraj Anant Pande, Analysis of Primes in Arithmetical Progressions $4n + k$ up to a Trillion, International Journal of Mathematics and Computer Applications Research, Vol. 5, Issue 4, 2015, 1-18.

- [6]. Neeraj Anant Pande, Analysis of Primes in Arithmetical Progressions $5n + k$ up to a Trillion, Journal of Research in Applied Mathematics, Volume 2, Issue 5, 2015, 14-29.
- [7]. Neeraj Anant Pande, Analysis of Primes in Arithmetical Progressions $6n + k$ up to a Trillion, International Journal of Mathematics and Computer Research, Volume 3, Issue 6, 2015, 1037-1053.
- [8]. Neeraj Anant Pande, Analysis of Primes in Arithmetical Progressions $7n + k$ up to a Trillion, International Journal of Mathematics and Its Applications, Accepted, 2016.
- [9]. Neeraj Anant Pande, Block-wise Distribution of Primes less than a Trillion in Arithmetical Progressions $8n + k$, IOSR Journal of Mathematics, 2016, Communicated.
- [10]. Neeraj Anant Pande, Spacings Between and Units & Tens Place Digits in Primes till One Trillion in Arithmetical Progressions $8n + k$, American International Journal of Research in Science, Technology, Engineering and Mathematics, 2016, Communicated.
- [11]. Neeraj Anant Pande, Block-wise Density Distribution of Primes less than a Trillion in Arithmetical Progressions $9n + k$, International Journal of Advances in Mathematics and Statistics, 2016, Communicated.
- [12]. Neeraj Anant Pande, Evolution of Algorithms: A Case Study of Three Prime Generating Sieves, Journal of Science and Arts, Year 13, No.3(24), 2013, 267-276.
- [13]. Neeraj Anant Pande, Algorithms of Three Prime Generating Sieves Improvised Through Nonprimality of Even Numbers (Except 2), International Journal of Emerging Technologies in Computational and Applied Sciences, Issue 6, Volume 4, 2013, 274-279.
- [14]. Neeraj Anant Pande, Algorithms of Three Prime Generating Sieves Improvised by Skipping Even Divisors (Except 2), American International Journal of Research in Formal, Applied & Natural Sciences, Issue 4, Volume 1, 2013, 22-27.
- [15]. Neeraj Anant Pande, Prime Generating Algorithms through Nonprimality of Even Numbers (Except 2) and by Skipping Even Divisors (Except 2), Journal of Natural Sciences, Vol. 2, No.1, 2014, 107-116.
- [16]. Neeraj Anant Pande, Prime Generating Algorithms by Skipping Composite Divisors, International Journal of Computer Science & Engineering Technology, Vol. 5, No. 09, 2014, 935-940.
- [17]. Neeraj Anant Pande, Improved Prime Generating Algorithms by Skipping Composite Divisors and Even Numbers (Other Than 2), Journal of Science and Arts, Year 15, No.2(31), 2015, 135-142.
- [18]. Neeraj Anant Pande, Refinement of Prime Generating Algorithms, International Journal of Innovative Science, Engineering & Technology, Vol. 2 Issue 6, 2015, 21-24.
- [19]. Herbert Schildt, Java : The Complete Reference, 7th Edition, Tata McGraw Hill, 2006.