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Higher-Order (F, α, β, ρ, d) –Convexity for Multiobjective Programming Problem

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ABSTRCT: *Higher-order (F, α, β, ρ, d)-convexity is considered. A multiobjective programming problem (MP) is considered. Mond-Weir and Wolfe type duals are considered for multiobjective programming problem. Duality results are established for multiobjective programming problem under higher-order (F, α, β, ρ, d)-convexity assumptions. The results are also applied for multiobjective fractional programming problem.*

Keywords*- Higher-order (F, α, β, ρ, d)-convexity; Sufficiency; Optimality conditionsMultiobjective Programming; Duality, Multiobjective fractional programming.*

I. INTRODUCTION

Convexity plays an important role in the optimization theory. In inequality constrained optimization the Kuhn-Tucker conditions are sufficient for optimality if the functions are convex. However, the application of the Kuhn-Tucker conditions as sufficient conditions for optimality is not restricted to convex problems as many mathematical models used in decision sciences, economics, management sciences, stochastics, applied mathematics and engineering involve non convex functions.

The concept of (F, ρ)-convexity was introduced by Preda[1] as an extension of F-convexity defined by Hanson and Mond [2] and ρ-convexitygeneralized convexity defined by Vial [3]. Ahmad [5] obtained a number of sufficiency theorems for efficient and properly efficient solutions under various generalized convexity assumptions for multiobjective programming problems.Liang et al. [8] introduced a unified formulation ofgeneralized convexity called (F,α, ρ,d)-convexity and obtained some optimality conditionsand duality results for nonlinearfractional programming problems.

Recently, Yuan et al. [12] introduced the concept of (C, α,ρ,d)-convexity which is the generalizationof (F,α, ρ,d)-convexity, and proved optimality conditions and duality theorems fornon-differentiable minimax fractional programming problems.

In this paper we have considered, higher-order (F,α, β, ρ,d)-convex functions. Under the generalized convexity, we obtain sufficient optimality conditions for multiobjective programming problem (MP). Mond-Weir and Wolfe type duals are considered for multiobjective programming problem. Duality results are established under for multiobjective programming problem under higher-order (F, α, β, ρ, d)-convexity assumptions. The results are also applied for multiobjective fractional programming problem. In the last we present Wolfeduality for (MP) and (MFP).

II. DEFINITIONS AND PRELIMINARIES

Definition 1. A functional $F:X \times X \times R^n \rightarrow R$ is said to be sublinear in the third variable, if forall $x, \bar{x} \in X$,

(i) $F(x, \bar{x}; a_1+a_2) \leqq F(x, \bar{x}; a_1) + F(x, \bar{x}; a_2)$, for all $a_1, a_2 \in R^n$; and

(ii) $F(x, \bar{x}; \alpha a) = \alpha F(x, \bar{x}; a)$ for all $\alpha \in R_+$, and $a \in R^n$.

From (ii), it is clear that $F(x, \bar{x}; 0) = 0$.

Gulati and Saini [13] introduced the class of higherorder(F, α, β, ρ, d)-convex functions as follows:
Let $X \subseteq R^n$be an open set. Let $\phi:X \rightarrow R$, $K:X \times R^n \rightarrow R$ be differentiable functions,
$F:X \times X \times R^n \rightarrow R$ be a sublinear functional in the third variable and $d:X \times X \rightarrow R$. Further,
let $\alpha, \beta: X \times X \rightarrow R_+ \setminus \{0\}$ and $\rho \in R$.

Definition 2. The function $\phi$is said to be higher-order (F, α, β, ρ, d)-convex at $\bar{x}$ with respect toK, if for all $x \in X$ and $p \in R^n$,

$$\phi(x) - \phi(\bar{x}) \geqq F(x, \bar{x}; \alpha(x, \bar{x})\{\nabla\phi(\bar{x}) + \nabla_p K(\bar{x}, p)\})$$
$$+ \beta(x, \bar{x})\{K(\bar{x}, p) - p^T\nabla_p K(\bar{x}, p)\} + \rho\, d^2(x, \bar{x}).$$

Remark 1. Let $K(\bar{x}, p) = 0$.
(i) Then the above definition becomes that of (F, α, ρ, d)-convex function introduced by Liang et al. [8].

(ii)If $\alpha(x, \bar{x}) = 1$, we obtain the definition of (F, ρ)-convex function given by Preda [1].

(iii) If $\alpha(x, \bar{x}) = 1$, ρ= 0 and $F(x, \bar{x};\nabla\phi(\bar{x})) = \eta^T(x, \bar{x})\nabla\phi(\bar{x})$ for a certain map $\eta: X \times X \to R^n$, then (F, α, β, ρ, d)-convexity reduces to the invexity in Hanson [7].
(iv) If F is convex with respect to the third argument, then we obtain the definition of (F, α, ρ, d)-convex function introduced by Yuan et al. [12].

Remark 2. Let $\beta(x, \bar{x}) = 1$.

(i)If $K(\bar{x}, p) = \frac{1}{2} p^T \nabla^2 \phi\left(\bar{x}\right) p$, then the above inequality reduces to the definition of second order (F, α, ρ, d)-convex function given by Ahmad and Husain [6].

(ii) If $\alpha(x, \bar{x}) = 1$, ρ= 0, $K(\bar{x}, p) = \frac{1}{2} p^T \nabla^2 \phi\left(\bar{x}\right) p$, and $F(x, \bar{x}; a) = \eta^T(x, \bar{x})a$, where $\eta: X \times X \to R^n$, the above definition becomes that of η -bonvexity introduced by Pandey [10].

We consider the following multiobjective programming problem:

(MP) Minimize $\phi_i(x) \square (\phi_1(x), \phi_2(x), \ldots \phi_k(x))$,

Subject to $h_j(x) \leqq 0$; $x \in X$,

where X is an open subset of $R^n$and the functions $\phi_i : \{\phi_1, \phi_2, \ldots, \phi_k\} : X \to R^n$ and $h_j : \{h_1, h_2, \ldots, h_m\} : X \to R^m$ are differentiable on X. Let $S = \{x \in X: h_j(x) \leqq 0\}$ denote the set of all feasible solutions for(MP).

Proposition 1 (Kuhn-Tucker Necessary Optimality Conditions [9])**.** Let $\bar{x} \in S$ be an optimal solution of (MP) and let h_j satisfy a constraint qualification [Theorem 7.3.7 in 5]. Then thereexists a $\bar{v} \in R^m$ such that

$$\sum_{i=1}^{k} \bar{\mu}_i \nabla \phi_i(\bar{x}) + \sum_{j=1}^{m} \bar{v}_j \nabla h_j(\bar{x}) = 0, \quad (2.1)$$

$$\sum_{j=1}^{m} \bar{v}_j^T h_j(\bar{x}) = 0, \quad (2.2)$$

$$\bar{\mu}_i \geq 0, \bar{v}_j \geq 0, \ h_j(\bar{x}) \leq 0, \quad (2.3)$$

where $\nabla h_j(\bar{x})$ denotes the n× m matrix $\left[\nabla h_1(\bar{x}), \nabla h_2(\bar{x}), \ldots, \nabla h_m(\bar{x})\right]$.

III. SUFFICIENT OPTIMALITY CONDITIONS

In this section, we have established Kuhn-Tucker sufficient optimality conditions for (MP) under (F, α, β, ρ, d)-convexity assumptions.

Theorem 1. Let $\bar{x} \in S$ and $\bar{v} \in R^m$ satisfy (2.1)-(2.3). If

(i) $\phi_i$is higher-order (F, α, β, ρ_1, d)-convex at $\bar{x}$ with respect to K,

(ii) $\bar{v}^T h$ is higher-order (F, α, β, ρ_2, d)-convex at $\bar{x}$ with respect to −K, and
(iii) $\rho_1 + \rho_2 \geqq 0$,

then $\bar{x}$ is an optimal solution of the problem (MP).

Proof. Let $\bar{x} \in S$. Since $\phi_i$is higher-order (F, α, β, ρ_1, d)-convex at $\bar{x}$ with respect to K, for all $x \in S$, we have

$\phi_i(x) - \phi_i(\bar{x}) \quad \geqq F(x, \bar{x}; \alpha(x, \bar{x})\{\bar{\mu}_i \nabla\phi_i(\bar{x}) + \nabla_p K(\bar{x}, p)\})$

$+ \beta(x, \bar{x})\{K(\bar{x}, p) - p^T\nabla_p K(\bar{x}, p)\} + \rho_1 d^2(x, \bar{x})$. (3.1)
Using (2.1), we get

$\phi_i(x) - \phi_i(\bar{x}) \geqq F(x, \bar{x}; \alpha(x, \bar{x})\{-\bar{v}_j \nabla h_j(\bar{x}) + \nabla_p K(\bar{x}, p)\})$

$+ \beta(x, \bar{x})\{K(\bar{x}, p) - p^T\nabla_p K(\bar{x}, p)\} + \rho_1 d^2(x, \bar{x})$. (3.2)

Also, $\bar{v}^T h$ is higher-order (F, α, β, ρ_2, d)-convex at $\bar{x}$ with respect to −K. Therefore

$\bar{v}^T h(x) - \bar{v}^T h(\bar{x}) \geqq F(x, \bar{x}; \alpha(x, \bar{x})\{\bar{v}^T \nabla h(\bar{x}) - \nabla_p K(\bar{x}, p)\})$

$- \beta(x, \bar{x})\{K(\bar{x}, p) - p^T\nabla_p K(\bar{x}, p)\} + \rho_2 d^2(x, \bar{x})$. (3.3)

Since $\bar{v}^T h(\bar{x}) = 0$, $\bar{v} \geq 0$ and $h(x) \leq 0$, we get

$$0 \geqq F(x, \bar{x}; \alpha(x, \bar{x})\{\bar{v}^T \nabla h(\bar{x}) - \nabla_p K(\bar{x}, p)\})$$
$$- \beta(x, \bar{x})\{K(\bar{x}, p) - p^T\nabla_p K(\bar{x}, p)\} + \rho_2 d^2(x, \bar{x}). \quad (3.4)$$

Adding the inequalities (3.2) and (3.4), we obtain

$\phi_i(x) - \phi_i(\bar{x}) \geqq (\rho_1 + \rho_2)\, d^2(x, \bar{x})$.

which by hypothesis (iii) implies,

$$\phi_i(x) \geqq \phi_i(\bar{x}).$$

Hence $\bar{x}$ is an optimal solution of the problem (MP).

IV. MOND WEIR DUALITY

In this section, we have established weak and strong duality theorems for the following MondWeir dual (MD) for (MP):

(MD) Maximize $\phi_i(u)$,

Subject to $\sum_{i=1}^{k} \mu_i \nabla \phi_i(u) + \sum_{j=1}^{m} v_j \nabla h_j(u) = 0$, (4.1)

$\bar{v}_j^T h_j(u) \geqq 0$, (4.2)

$u \in X, \mu_i \geqq 0, v_j \geqq 0, v \in R^m$. (4.3)

Theorem 2 (Weak Duality)**:**Let x and (u, v) be feasible solutions of (MP) and (MD) respectively.
Let
(i) $\phi_i$be higher-order (F, α, β, ρ_1, d)-convex at u with respect to K,
(ii) $\bar{v}^T h$ be higher-order (F, α, β, ρ_2, d)-convex at u with respect to −K, and
(iii) $\rho_1 + \rho_2 \geqq 0$.
Then

$$\phi_i(x) \geqq \phi_i(u).$$

Proof: By hypothesis (i), we have

$\phi_i(x) - \phi_i(u) \geqq F(x, u; \alpha(x, u)\{\bar{\mu} \nabla\phi_i(u) + \nabla_p K(u, p)\})$

$+ \beta(x, u)\{K(u, p) - p^T\nabla_p K(u, p)\} + \rho_1 d^2(x, u)$. (4.4)

Also hypothesis (ii) yields

$\bar{v}^T h(x) - \bar{v}^T h(u) \geqq F(x, u; \alpha(x, u)\{\bar{v}^T \nabla h(u) - \nabla_p K(u, p)\})$

$-\beta(x, u)\{K(u, p) - p^T\nabla_p K(u, p)\} + \rho_2 d^2(x, u)$.

By (4.2), (4.3) and $h_j(x) \leqq 0$, it follows that

$0 \geqq F(x, u; \alpha(x, u)\{\bar{v}^T \nabla h(u) - \nabla_p K(u, p)\})$

$-\beta(x, u)\{K(u, p) - p^T\nabla_p K(u, p)\} + \rho_2 d^2(x, u)$. (4.5)

Adding the inequalities (4.4), (4.5) and applying the properties of sublinear functional, we Obtain

$\phi_i(x) - \phi_i(u) \geqq F(x, u; \alpha(x, u)[\bar{\mu} \nabla\phi_i(u) + \bar{v}^T \nabla h_j(u)]) + \rho_1 d^2(x, u) + \rho_2 d^2(x, u)$.

which in view of (4.1) implies

$$\phi_i(x) - \phi_i(u) \geqq (\rho_1 + \rho_2)\, d^2(x, \bar{x}).$$

Using hypothesis (iii) in the above inequality, we get

$$\phi_i(x) \geqq \phi_i(u).$$

Remark 3. A constraint qualification is not required to establish weak duality. It has been erroneously assumed in Theorem 3.4 in [4].

Theorem 3(Strong Duality)**:**Let $\bar{x}$ be an optimal solution of the problem (MP) and let h satisfya constraint qualification. Further, let Theorem 2 hold for the feasible solution $\bar{x}$ of (MP) andall feasible solutions (u, v) of (MD). Then there exists a $\bar{v} \in R^m_+$such that $(\bar{x}, \bar{v})$ is an optimalsolution of (MD).

Proof. Since $\bar{x}$ is an optimal solution for the problem (MP) and h satisfies a constraint qualification, by Proposition 1 there exists a $\bar{v} \in R^m_+$ such that the Kuhn-Tucker conditions, (2.1)- (2.3) hold. Hence $(\bar{x}, \bar{v})$ is feasible for (MD).

Now let (u, v) be any feasible solution of (MD). Then by weak duality (Theorem 2), wehave

$$\phi_i(\bar{x}) \geqq \phi_i(u).$$

Therefore $(\bar{x}, \bar{v})$ in an optimal solution of (MD).

V. APPLICATION IN MULTIOBJECTIVE FRACTIONAL PROGRAMMING

If$\phi_I : X \to R$ is defined by

$$\phi_i(x) = \frac{f_i(x)}{g_i(x)}$$

where f , g : X → R, $f_i(x) \geqq 0$ and $g_i(x) > 0$ on X, then the multiobjective programming problem (MP) becomes the following multiobjective fractional programming problem (MFP):

(MFP) Minimize $\phi_i(x) = \dfrac{f_i(x)}{g_i(x)}$

Subject to$h_j(x) \leqq 0$, $x \in X$.

We now prove the following result, which gives higher-order $(F, \bar{\alpha}, \bar{\beta}, \rho, \bar{d})$-convexity of the ratio function $f_i(x)/g_i(x)$.

Theorem 4. Let $f_i(x)$ and $-g_i(x)$ be higher-order (F, α, β, ρ, d)-convex at $\bar{x}$ with respect to thesame function K.Then the multiobjective fractional function $\dfrac{f_i(x)}{g_i(x)}$ is higher-order $(F, \bar{\alpha}, \bar{\beta}, \rho, \bar{d})$-convex at $\bar{x}$ withrespect to $\bar{K}$, where

$$\bar{\alpha}(x, \bar{x}) = \alpha(x, \bar{x})\frac{g_i(\bar{x})}{g_i(x)},$$

$$\bar{\beta}(x, \bar{x}) = \beta(x, \bar{x})\frac{g_i(\bar{x})}{g_i(x)},$$

$$\bar{K}(\bar{x}, p) = \left[\frac{1}{g_i(\bar{x})} + \frac{f_i(\bar{x})}{g_i^2(\bar{x})}\right] K(\bar{x}, p),$$

$$\bar{d}(x, \bar{x}) = \left[\frac{1}{g_i(x)} + \frac{f_i(\bar{x})}{g_i(x).g_i(\bar{x})}\right]^{\frac{1}{2}} d(x, \bar{x}).$$

Proof: Since $f_i(x)$ and $-g_i(x)$ are higher-order (F, α, β, ρ, d)-convex at $\bar{x}$ with respect to the same function K, we have

$$f_i(x) - f_i(\bar{x}) \geqq F(x, \bar{x}; \alpha(x, \bar{x})\{\nabla f_i(\bar{x}) + \nabla_p K(\bar{x}, p)\})$$
$$+ \beta(x, \bar{x})\{K(\bar{x}, p) - p^T \nabla_p K(\bar{x}, p)\} + \rho d^2(x, \bar{x})$$

And

$$-g_i(x)+g_i(\bar{x}) \geqq F(x, \bar{x}; \alpha(x, \bar{x})\{-\nabla g_i(\bar{x})+\nabla_p K(\bar{x}, p)\})$$
$$+\beta(x, \bar{x})\{K(\bar{x}, p)-p^T\nabla_p K(\bar{x}, p)\}+\rho d^2(x, \bar{x}).$$

Also

$$\frac{f_i(x)}{g_i(x)} - \frac{f_i(\bar{x})}{g_i(\bar{x})} = \frac{1}{g_i(x)}\left[f_i(x) - f_i(\bar{x})\right] + \frac{f_i(\bar{x})}{g_i(x)g_i(\bar{x})}\left[-g_i(x)+g_i(\bar{x})\right].$$

Using the above inequalities and sublinearity of F, we get

$$\frac{f_i(x)}{g_i(x)} - \frac{f_i(\bar{x})}{g_i(\bar{x})} \geq \frac{1}{g_i(x)} F(x, \bar{x}; \alpha(x, \bar{x})\{\nabla f_i(\bar{x})+\nabla_p K(\bar{x}, p)\})$$
$$+\frac{1}{g_i(x)}(\beta(x, \bar{x})\{K(\bar{x}, p)-p^T\nabla_p K(\bar{x}, p)\}+\rho d^2(x, \bar{x}))$$
$$+\frac{f_i(\bar{x})}{g_i(x)g_i(\bar{x})} F(x, \bar{x}; \alpha(x, \bar{x})\{-\nabla g_i(\bar{x})+\nabla_p K(\bar{x}, p)\})$$
$$+\frac{f_i(\bar{x})}{g_i(x)g_i(\bar{x})}(\beta(x, \bar{x})\{K(\bar{x}, p)-p^T\nabla_p K(\bar{x}, p)\}+\rho d^2(x, \bar{x})).$$

$$= F(x, \bar{x}; \frac{\overline{\alpha(x, x)}}{g_i(x)}\{\nabla f_i(\bar{x})+\nabla_p K(\bar{x}, p)\})$$
$$+ F(x, \bar{x}; \alpha(x, \bar{x})\frac{f_i(\bar{x})}{g_i(x)g_i(\bar{x})}\{-\nabla g_i(\bar{x})+\nabla_p K(\bar{x}, p)\}$$
$$+\beta(x, \bar{x})\left[\frac{1}{g_i(x)}+\frac{f_i(\bar{x})}{g_i(x).g_i(\bar{x})}\right]\{K(\bar{x}, p)-p^T\nabla_p K(\bar{x}, p)\}$$
$$+\rho\left[\frac{1}{g_i(x)}+\frac{f_i(\bar{x})}{g_i(x).g_i(\bar{x})}\right] d^2(x, \bar{x}).$$
$$= F(x, \bar{x}; \alpha(x, \bar{x})\frac{g_i(\bar{x})}{g_i(x)}\{\nabla\frac{f_i(\bar{x})}{g_i(\bar{x})}+\left[\frac{1}{g_i(\bar{x})}+\frac{f_i(\bar{x})}{g^2_i(\bar{x})}\right]\nabla_p K(\bar{x},p)\})$$
$$+\beta(x, \bar{x})\frac{g_i(\bar{x})}{g_i(x)}\left[\frac{1}{g_i(\bar{x})}+\frac{f_i(\bar{x})}{g^2_i(\bar{x})}\right]\{K(\bar{x}, p)-p^T\nabla_p K(\bar{x}, p)\}$$
$$+\rho\left[\frac{1}{g_i(x)}+\frac{f_i(\bar{x})}{g_i(x).g_i(\bar{x})}\right] d^2(x, \bar{x}).$$

Therefore,

$$\frac{f_i(x)}{g_i(x)} - \frac{f_i(\bar{x})}{g_i(\bar{x})} \geq F(x, \bar{x}; \bar{\alpha}(x, \bar{x})\left[\nabla\frac{f_i(\bar{x})}{g_i(\bar{x})}+\nabla_p K(\bar{x}, p)\right])$$
$$+\bar{\beta}(x, \bar{x})\{\bar{K}(\bar{x}, p)-p^T\nabla_p \bar{K}(\bar{x}, p)\}+\rho \bar{d}^2(x, \bar{x}).$$

i.e., $\frac{f_i(x)}{g_i(x)}$ is higher-order $(F, \bar{\alpha}, \bar{\beta}, \rho, \bar{d})$-convex at $\bar{x}$ with respect to $\bar{K}$.

In view of Theorem 4, the results of Section 4 lead to the following duality relations between (MFP) and its Mond-Weir dual (MFD).

(MFD) Maximize $\frac{f_i(u)}{g_i(u)}$

Subject to $\sum_{i=1}^{k} \mu_i [\nabla f_i(u) - \lambda_i \nabla g_i(u)] + \sum_{j=1}^{m} v_j \nabla h_j(u) = 0$

$$\sum_{i=1}^{k} \mu_i [f_i(u) - \lambda_i g_i(u)] \geq 0,$$

$$\sum_{j=1}^{m} v_j^T \nabla h_j(u) \geq 0,$$

$$u \in X, \mu_i \geq 0, \lambda_i \geq 0, v_j \geq 0, \mu_i \in R^n, \lambda_i \in R^n, v_j \in R^m.$$

Theorem5 (Weak Duality)**.** Let x and (u,v) be feasible solutions of (MFP) and (MFD) respectively.Let
(i) f_i and $-g_i$ be higher-order (F, α, β, ρ_1, d)-convex at u with respect to K,
(ii) $v^T h$ be higher-order (F, $\bar{\alpha}$, $\bar{\beta}$, ρ_2, $\bar{d}$)-convex at u with respect to $-\bar{K}$, where $\bar{\alpha}$, $\bar{\beta}$, $\bar{K}$ and $\bar{d}$ are as given in Theorem 4, and
(iii) $\rho_1 + \rho_2 \geqq 0$.
Then $\frac{f_i(x)}{g_i(x)} \geq \frac{f_i(u)}{g_i(u)}$

Theorem 6 (Strong Duality)**.** Let $\bar{x}$ be an optimal solution of the problem (MFP) and let h satisfya constraint qualification. Further, let Theorem 5 hold for the feasible solution $\bar{x}$ of (MFP) andall feasible solutions (u, v) of (MFD). Then there exists a $\bar{v} \in R^m_+$such that
($\bar{x}$, $\bar{v}$) is an optimalsolution of (MFD).

VI. WOLFE DUALITY

The Wolfe dual of (MP) and (MFP) are respectively

(MWD) Maximize $\sum_{i=1}^{k} \mu_i \phi_i(u) + \sum_{j=1}^{m} v_j^T h_j(u)$

Subject to $\sum_{i=1}^{k} \mu_i \phi_i(u) + \sum_{j=1}^{m} v_j \nabla h_j(u) = 0$

$$u \in X, \mu_i \geq 0, v_j \geq 0, \mu_i \in R^n, v_j \in R^m$$

(WMFD) Maximize $\sum_{i=1}^{k} \mu_i [f_i(u) - \lambda_i g_i(u)] + \sum_{j=1}^{m} v_j^T h_j(u)$

Subject to $\sum_{i=1}^{k} \mu_i [\nabla f_i(u) - \lambda_i \nabla g_i(u)] + \sum_{j=1}^{m} v_j \nabla h_j(u) = 0$

$$u \in X, \mu_i \geq 0, \lambda_i \geq 0, v_j \geq 0, \mu_i \in R^n, \lambda_i \in R^n, v_j \in R^m$$

Now we state duality relations for the primal problems (MP) and (MFP) and their Wolfe duals(MWD) and (WMFD) respectively. Their proofs follow as in Section 4.

Theorem 7 (Weak Duality)**.** Let x and (u, v) be feasible solutions of (MP) and (MWD) respectively.
Let
(i) $\phi_i$be higher-order (F, α, β, ρ_1, d)-convex at u with respect to K,
(ii) v^Thbe higher-order (F, α, β, ρ_2, d)-convex at u with respect to −K, and
(iii) $\rho_1 + \rho_2 \geqq 0$.
Then$\phi_i(x) \geq \phi_i(u) + v^T h(u)$.

Theorem 8 (Strong Duality)**.** Let $\bar{x}$ be an optimal solution of the problem (MP) and let h satisfya constraint qualification. Further, let Theorem 7 hold for the feasible solution $\bar{x}$ of (MP) andall feasible solutions (u, v) of (MWD). Then there exists a $\bar{v} \in R^m_+$such that ($\bar{x}$, $\bar{v}$) is an optimalsolution of (MWD) and the optimal objective function values of (MP) and (MWD) are equal.

Theorem9 (Weak Duality)**.** Let x and (u,v) be feasible solutions of (MFP) and (WMFD) respectively.
Let
(i) $f_i$and $-g_i$ be higher-order (F, α, β, ρ_1, d)-convex at u with respect to K,
(ii) $v^T h$ be higher-order (F, $\bar{\alpha}$, $\bar{\beta}$, ρ_2, $\bar{d}$)-convex at u with respect to $-\bar{K}$, where $\bar{\alpha}$, $\bar{\beta}$, $\bar{K}$ and $\bar{d}$ are as given in Theorem 4, and
(iii) $\rho_1 + \rho_2 \geqq 0$.
Then $\frac{f_i(x)}{g_i(x)} \geq \frac{f_i(u)}{g_i(u)} + v^T h(u)$.

Theorem 10 (Strong Duality)**.** Let $\bar{x}$ be an optimal solution of the problem (MFP) and let h satisfya constraint qualification. Further, let Theorem 9 hold for the feasible solution $\bar{x}$ of (MFP) and allfeasible solutions (u, v) of (WMFD). Then there exists a $\bar{v} \in R^m_+$such that
($\bar{x}$, $\bar{v}$) is an optimalsolution of (WMFD) and the optimal objective function values of (MFP) and (WMFD) are equal.

VII. CONCLUSION

In this paper a new concept of generalized convexity has been introduced. Under this generalized convexity we have established sufficient optimality conditions and duality results for amultiobjective programming problem.These duality relations lead to duality in multiobjective fractionalprogramming.

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