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Totally R*-Continuous and Totally R*-Irresolute Functions

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Abstract: *In this paper we present and study two new class of functions called the Totally R*continuous function and Totally R*-Irresolute Functions. Furthermore we obtain some of their basic properties.*

Keywords: *Totally R*-continuous function, Totally R*- Irresolute function, R*- space, R*-connected.*

I.INTRODUCTION

Continuity is one of the major research areas in Mathematical sciences. Many different forms of continuous functions have been introduced by Mathematicians over the years. Some of them are strongly continuous functions (Levine 1963), contra continuous functions (Dontchev1996), supra continuous functions (Mashhour 1983) and Jain introduced the totally continuous functions in 1980.

A new class of generalized closed sets named R*-closed sets was introduced by C.Janaki and Renu Thomas in 2012.The purpose of this paper is to study a new type of continuity called the totally R*-continuity along with totally R*-irresolute functions. We further study some of its properties.

Throughout this paper, the spaces X and Y always mean the topological spaces respectively. For $A \subset X$, the closure and the interior of A in X are denoted by cl(A) and int(A) respectively. Also the collection of all R*-open subsets of X is denoted by R*O(X).

II. PRELIMINARIES

Definition: 2.1. A subset A of a topological space (X, τ) is called

1. Semi open [5,13] if $A \subset cl(int(A))$
2. a regular open [3,17] if A = int(cl(A)) and regular closed [13] if A = cl(int(A)).

The intersection of all regular closed subset of (X, τ) containing A is called the regular closure of A and is denoted by rcl(A) .

Definition: 2.2. [1] A subset A of a space (X, τ) is called regular semi open set if there is a regular open set U such that $U \subset A \subset cl(U)$. The family of all regular semi open sets of X is denoted by RSO(X).

Lemma: 2.3. [7] In a space (X, τ), the regular closed sets, regular open sets and clopen sets are regular semi open.

Definition: 2.4. A subset of a topological space (X, τ) is called

1. a regular generalized (briefly rg-closed) [15] if $cl(A) \subset U$ whenever $A \subset U$ and U is regular open in X.
2. a generalized pre regular closed[8] (briefly gpr-closed) [3] if $pcl(A) \subset U$ whenever $A \subset U$ and U is regular open in X.
3. a regular weakly generalized closed[13] (briefly rwg-closed) [9] if $cl(int(A)) \subset U$ whenever $A \subset U$ and U is regular open in X.
4. a regular generalized weak closed [17] (briefly rgw-closed) [14] if $cl(int(A)) \subset U$ whenever $A \subset U$ and U is regular semi open in X.
5. a regular w-closed (briefly rw-closed) [1] if $cl(A) \subset U$ whenever $A \subset U$ and U is regular semi open in X.

The complements of the above mentioned closed sets are their respectively open set.

Definition: 2.5. [10] A subset A of a space (X,τ) is called R*-closed if $rcl(A) \subset U$ whenever $A \subset U$ and U is regular semi open in (X,τ) .We denote the set of all R*- closed sets in (X,τ) by R*C(X).

Definition: 2.6.[11] A subset of a topological space (X,τ) is said to be a R*-T_2 if for each pair of distinct points x and y of X, there exist R*-open subsets U and V in X such that $x \in U$ and $y \in V$ and $U \cap V = \phi$.

Definition: 2.7. A function f: $(X,\tau) \rightarrow (Y,\sigma)$ is called

1. totally continuous[9] if the inverse image of every open subset of Y is a clopen subset of X.
2. regular set connected[6,14] if $f^{-1}(V)$ is clopen in X for every regular open subset V of Y.
3. quasi irresolute[16] if the inverse image of every clopen subset of Y is clopen in X.

4. semi totally -continuous [2] if every f^{-1} (V) is clopen in X for every semi open set V in Y.
5. totally irresolute [19] if the inverse image of every open subset of Y is clopen in X.

Definition2.8. A function f: $(X,\tau)\rightarrow(Y,\sigma)$ is called

1. R*- continuous [10] if the inverse image of every closed subset of Y is R*-closed in X.
2. R*-irresolute [10] if the inverse image of every R*-closed subset of Y is R*-closed in X.
3. contra R*-continuous[11] if the inverse image of every open set in Y is R*-closed in X.
4. almost contra R*-continuous[11] if the inverse image of every regular open set of Y is R*-closed in X.

III. TOTALLY R*- CONTINUOUS FUNCTIONS

Definition: 3.1. A function f: X → Y is totally R*- continuous if the inverse image of every open subset of Y is a R*-clopen subset of X.

Theorem: 3.2. Every totally R*-continuous function is R*-continuous and contra R*-continuous.

Proof: Obvious

Remark: 3.3. Converse of the above theorem need not be true as seen in the following example.

Example: 3.4. Let X = {a, b, c} and τ = { X, φ ,{a},{b},{a, b},} and let Y= {a, b, c} σ = {Y, φ ,{b},{c},{b, c}}. Let f: X →Y be defined by f(a) = b, f(b) = c ,f(c) = a.Here the inverse image of every open set of Y is R*-open in X but not R*- clopen in X. Hence f is R*- continuous but not totally R*-continuous.

Remark: 3.5. Totally continuous function implies totally R*-continuous function but not conversely.

Example: 3.6. Let X = {a, b, c, d}= Y and τ = { X, φ ,{a},{b},{d},{a, b},{a, d},{b, d},{a, b, d},{a, c, d}} and let σ = {Y, φ ,{b},{a, c, d }}. Let f: X →Y be defined by f(a) = d, f(b) = a, f(c) = b, f(d) = c, Here f is totally R*-continuous function but not totally continuous.

Remark: 3.7. Totally continuous function implies R* - continuous function but not conversely.

Example: 3.8. Let X = {a, b, c} =Y and τ = {X, φ , {a},{c}, {a, b}{a, c}} and let σ = {Y, φ ,{c},{a, b}}. Let f: X →Y be defined by f (a) = a, f (b) = c,f (c) = b .
Here f is totally R*- continuous but not totally continuous.

Theorem: 3.9.

(a) Every totally R*- continuous mapping is totally rw-continuous.
(b) Every totally R*- continuous mapping is totally rg-continuous.
(c) Every totally R*- continuous mapping is totally rwg-continuous.
(d) Every totally R*- continuous mapping is totally rgw-continuous.
(e) Every totally R*- continuous mapping is totally gpr-continuous.

Proof: Straight forward.

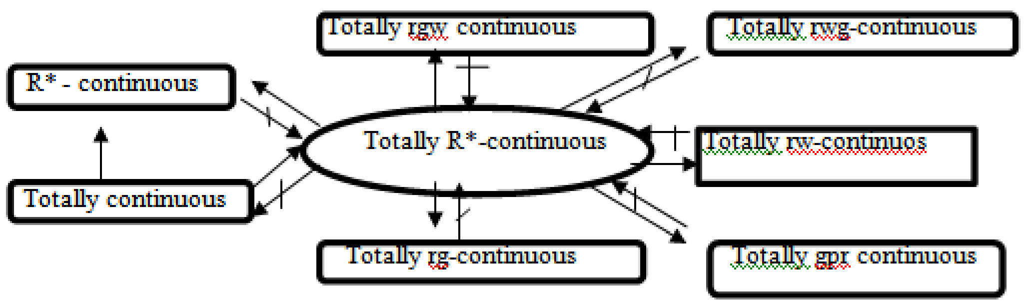
Remark: 3.10. The converses of the above need not be true as shown by following examples.

Example 3.11. Let X = {a, b, c, d} =Y and τ = {X, φ , {a},{c, d}, {a, c, d}} and let σ = {Y, φ ,{b},{a, b, d}}. Let f: X →Y be an identity mapping. Here f is totally rgw-continuous and totally rwg-continuous but not totally R*- continuous.

Example: 3.12. Let X = {a, b, c, d} =Y and τ = {X, φ , {a},{c, d}, {a, c, d}} and let σ = {Y, φ ,{c},{a, c }}. Let f: X →Y be an identity mapping. The function is totally gpr continuous but not totally R*-continuous.

Example: 3.13. Let X = {a, b, c, d} =Y and τ = {X, φ , {a}, {b}, {a,b},{a, b, c}, {a, b, d}} and let σ = {Y, φ ,{c},{d},{c, d}}. Let f: X →Y be an identity mapping. The function is totally rg and totally rw continuous but not totally R*-continuous.

Figure 3.14. The above discussions are implicated in the following diagram.



Definition: 3.15. A subset A of a topological space (X,τ) is called R*-connected if X cannot be written as the disjoint union of two non-empty R*-open subsets.
Theorem: 3.16. If f is totally R*-continuous map from a R*-connected space X onto another Space Y then Y is an indiscrete space.
Proof: Suppose Y is not an indiscrete space. Let A be a proper non-empty open subset of Y. Since f is totally R*-continuous. f^{-1}(A) is proper non-empty R*-clopen subset of X. Then X = f^{-1}(A)∪(f^{-1}(A))^c . Hence X is a union of two non-empty disjoint R*-open subsets. This is a contradiction. Thus Y is an indiscrete space.
Theorem: 3. 17. A space X is R*-connected if every totally R*-continuous function from a space X into any T_0 space Y is a constant map.
Proof: Suppose f: X → Y is totally R*-continuous function. Let Y be a T_0 space. Suppose f is not a constant map, Then we choose two points x and y in X such that f(x) ≠ f(y).Since Y is a T_0 space and f(x) and f(y) are distinct points of Y, then there exist an open set G say in Y containing f(x) but not f(y).Now f^{-1}(G) is R*-clopen subset of X since f is totally R*-continuous. Thus x ∈ f^{-1}(G) and y ∉ f^{-1}(G). Now X = f^{-1}(G)∪(f^{-1}(G))^c which is the union of two non-empty R*-clopen subsets of X. Hence X is not R*-connected, which is a contradiction to our hypothesis. Thus f is a constant map.
Theorem: 3.18. Let f: X → Y be totally R*-continuous and Y is a T_1 space. If A is a non-empty R*-connected subset of X then f(A) is a singleton.
Proof: Suppose f(A) is not a singleton. Let f(x_1) = y_1 ∈ A, f(x_2) = y_2 ∈ A. since y_1, y_2 ∈ Y and Y is a T_1 space , then there exist an open set G in Y (say) containing y_1 but not y_2. Since f is totally R*-continuous, Then f^{-1}(G) is R*- clopen set containing x_1 but not x_2. Now X = f^{-1}(G) ∪(f^{-1}(G))^c. Hence X is the union of two non-empty R*- open subsets, which is a contradiction. Thus f(A) is a singleton.
Theorem: 3.19. Let f: X → Y be totally R*-continuous injection. If Y is T_0 then X is R*-T_2.
Proof: Let x and y be any two distinct points of X. Since f is an injection, then f(x) ≠ f(y). Since Y is T_0 , there exist an open subset V of Y containing f(x) but not f(y).Then x ∈ f^{-1}(V) and y ∉ f^{-1}(V). Since f is totally R*-continuous, then f^{-1}(V) is R*-clopen subset of X. Assume A = f^{-1}(V) and B = (f^{-1}(V))^c. Here A and B are two disjoint R*- open subsets of X containing x and y respectively. Hence X is R*-T_2 space.

IV. TOTALLY R*- IRRESOLUTE FUNCTIONS

Definition: 4.1. A function f: X → Y is
1. totally R*- irresolute if the inverse image of every R*- open subset of Y is a R*-clopen subset of X.
2. quasi R*- irresolute if the inverse image of every R*- clopen subset of Y is a R*-clopen subset of X.
3. contra R*- irresolute if the inverse image of every R*- open subset of Y is a R*-closed subset of X.

Theorem: 4.2. Every totally R*-irresolute function is R*-irresolute and contra R*-irresolute.
Proof: Obvious
Remark: 4.3. The converse of the above theorem need not be true as shown in following example.
Example: 4.4. Let X = {a, b,c}=Y and τ = { X, φ , {a},{b},{a,b}} and let σ = {Y,φ ,{b},{c},{b,c}}. Let f: X →Y be defined by f(a) = b, f(b) = c ,f(c) = a. Here the inverse image of every R*- open set of Y is R*-open in X but not R*- clopen in X. Hence f is R*- irresolute but neither totally R*- irresolute nor contra R*-irresolute.
Remark: 4.5 The concepts totally continuous and totally R*-irresolute are independent to each other shown by following examples.
Example: 4.6. Consider example 3.7.f is totally continuous but not totally R*-irresolute.
Example: 4.7. Let X = {a, b, c, d} and τ = { X, φ , {a},{b},{a, b },{b, c},{a, b, c},{b, c, d}} and let Y= {a, b, c, d} , σ = {Y, φ ,{a},{b},{a, b},{a, b, c},{a, b, d}.
Let f: X →Y defined as an identity mapping. f is totally R*- irresolute but not totally continuous.
Theorem: 4.8. Every totally R*- irresolute function is totally R*-continuous function.
Proof: Obvious
Theorem: 4.9. Every totally R*-irresolute function is quasi R*-irresolute function.
Proof: Let f: X → Y be a totally R*-irresolute function. Let V be a R*- clopen subset of Y. Since f is totally R*-irresolute, f^{-1}(V) is R*-clopen subset of X. Therefore, f is quasi R*-irresolute function.

Remark: 4.10. The converse of the above theorem need not be true as shown in the following example.
Example: 4.11. Every quasi R*-irresolute function is need not be totally R*-irresolute function. Let X = {a, b, c} and τ = {X, φ , {a},{b}{a, b}} and let Y= {a, b, c}, σ = {Y, φ ,{b},{c},{b, c}}. Let f: X →Y be defined by f(a) = b, f(b) = c, f(c) = a. Here f is quasi R*- irresolute but not totally R*-irresolute.
Remark: 4.12. The concepts of regular set connected and totally R*-irresolute are independent to each other as shown by following examples.
Example: 4.13. Let X = {a, b, c} =Y and τ = { X, φ ,{b},{a, c }} and let σ = {Y, φ ,{a},{b},{a, b}}. Let f: X →Y be an identity mapping. The mapping is totally R*-irresolute but not regular set connected.
Example: 4.14. Let X = {a, b, c} and τ = {X, φ ,{a},{b, c}} and let Y= {a, b, c} , σ = {Y, φ ,{a}, {b},{a, b}Let f : X →Y be defined by f(a)= a, f(b) = c, f(c) = b. f is totally R*- irresolute but not regular set connected.
Definition: 4.15. A topological space (X,τ) is said to be a R*-space if every R*-open (R*-closed) set is open (closed).
Theorem: 4.16. If f: X → Y is totally R*-continuous function and Y is an R*- space, then f is totally R*-irresolute.
Proof: Let U be R*-open in Y. Since Y is an R*-space, U is open and f is totally R*-continuous, f^{-1}(U) is R*-clopen in X. Suppose U is R*-closed, then Y-U is R*-open in Y. By similar argument f^{-1}(Y-U) is R*- clopen in X. So $f^{-1}(Y)$ - f^{-1}(U) is R*-clopen in X. X- f^{-1}(U) is R*- clopen in X. Then f^{-1}(U) is R*- clopen in X and hence f is totally R*-irresolute.
Theorem: 4.17. If f: X → Y is totally R*-irresolute and g: Y → Z is R*-irresolute, then g o f is totally R*-irresolute.
Proof: Let U be R*-open subset of Z. Since g is R*-irresolute function, g^{-1}(U) is R*-open subset of Y. Since f is totally R*-irresolute function, $f^{-1}(g^{-1}(U))$ is R* -clopen in X. Hence g o f is totally R*-irresolute.
Theorem: 4.18. Let f: X → Y and g: Y → Z be two functions, such that f is quasi R*-irresolute and g is totally R*-irresolute, then g o f is totally R*-irresolute.
Proof: Let U be R*-open subset of Z. Since g is totally R*-irresolute function, g^{-1}(U) is R*-clopen subset of Y. Since f is quasi R*-irresolute function, $f^{-1}(g^{-1}(U))$ is R*-clopen in X. Hence g o f is totally R*-irresolute and contra R*-irresolute.
Theorem: 4.19. Let f: X → Y and g: Y → Z be two functions. Then, if f is totally R*-irresolute and g is R*-continuous then g o f is totally R*-continuous.
Proof: Let U be open subset of Z. Since g is R*-continuous function, g^{-1}(U) is R*-open subset of Y. Since f is totally R*-irresolute function, $f^{-1}(g^{-1}(U))$ is R*- clopen in X. Hence g o f is totally R*-continuous.
Theorem: 4.20. The composition of two totally R*-irresolute functions is totally R*- irresolute.
Proof: Let U be R*-open subset of Z,then g^{-1} (U) is R*-clopen subset of Y.Since f is totally R*-irresolute function.f^{-1} (g^{-1}(U)) is R*-clopen in X. Thus g o f is totally R*-irresolute.
Theorem: 4.21. If f: X → Y is totally R*-irresolute function and X is R*- space, then f is regular set connected.
Proof: Let U be regular open in Y. Since f is totally R*-irresolute, then f^{-1}(U) is R*- clopen subset of X. Since X is an R*-space, **t**hen f^{-1}(U) is clopen subset of X. Hence f is regular set connected.
Definition: 4.22. A function f: (X, τ) → (Y, σ) is called semi totally R*-continuous function if every f^{-1} (V) is R*-clopen in (X, τ) for every semi open set V in (Y, σ). **Theorem: 4.23.** A function f: (X, τ) → (Y, σ) is called semi totally R*-continuous function iff the inverse image of every semi-closed subset V of Y is R*- clopen in X. **Proof:** Let V be any semi closed set in Y. Then Y-V is semi open in Y. Since f is semi totally R* continuous function, f^{-1}(Y-V) is R*-clopen in X and so X-f^{-1}(V) is R*-clopen in X, then f^{-1} (V) is R*-clopen in X, on the other hand, if V is semi open in Y, then Y-V is semi-closed , by hypothesis f^{-1} (Y-V) is R*-clopen in X and so X- f^{-1}(V) is R*- clopen in X. Then f^{-1} (V) is R*-clopen in X, then f is semi totally R*-continuous.
Theorem: 4.24. Every semi totally R*- continuous function is totally R* continuous function.

Proof: Suppose f: (X, τ) → (Y, σ) is semi totally R* continuous. Let U be any open subset of Y. Since every open set is semi open, U is semi open. Since f is semi totally R* continuous, then

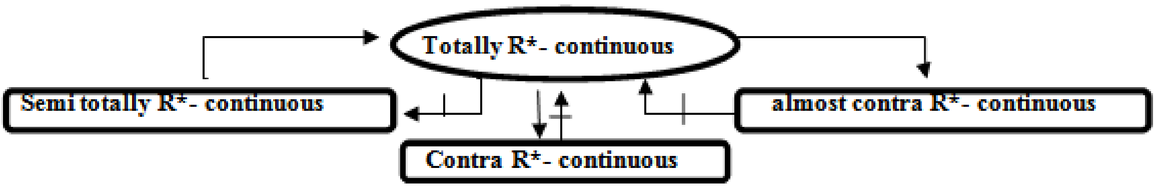
f^{-1}(U) is R*-clopen in Y. hence f is totally R*-continuous. **Remark: 4.25.** Converse of the above theorem need not be true as shown by following example.

Example: 4.26. Let X = {a, b, c} and τ = {X, φ ,{a},{b},{a, b},} and let
Y= {a, b, c}, σ = {Y, φ ,{b},{a,c}}.Let f: X →Y be the identity mapping. Here the inverse image of every open set of Y is R*- clopen in X but inverse image of every semi open set of Y is not R*- clopen in X .Hence f is totally R*-continuous but not semi totally R*- continuous.

Theorem: 4.27. Every totally R* continuous function is almost contra R*- continuous function.

Proof: Let V be regular open in Y. By hypothesis f^{-1} (V) is R*- clopen in X. Hence f is almost contra R* continuous functions.

Figure: 4.28. The above discussed relationships are represented by the following diagram.



References

[1] S.S. Benchalli and R.S.Wali,On RW-closed sets in topological spaces,Bull of Malaysian Mathematical Soc(2)30(2)(2007),99-110 .
[2] S.S.Benchalli and Umadevi I Neeli,Semi Totally continuous functions,International forum,Vol 6,no 10,2011,479-492 .
[3] D..E. Camaron, Properties of S-closed spaces, Proc. Amer math.soc 72(1978),581-586
[4] J. Dontchev, On Door spaces, Indian J. Pure appl Math., 26(1995), 873-81.
[5] J. Dontchev, M. Ganster and I. L. Reilly, More on almost s- continuity Indian J. Pure Math., 41(1995), 139-146.
[6] Erald Ekici, Acta Mathemathica Hungarica,(2005)Vol 107,Issue 3,193 -206.
[7] G.L.Garg and D.Sivaraj,On sc-compact and S-closed spaces,Boll.Un.Math.Ital 6 (3B)(1984),321-332.
[8] Y.Gnanambal, On generalizes preregular closed sets in topological spaces, Indian J.Pure Appl. Math 28(1997)351-360.
[9] R. C Jain, Ph.D Thesis, Meerut University, Meerut, India, 1980.
[10] C.Janaki and Renu Thomas, "On R*-closed sets in topological Spaces"Int. journal of Math Archive-3(8),2012, 3067-3074.
[11] C.Janaki and Renu Thomas, "Contra R*- continuous and Almost contra R*-continuous functions " Int. journal of Math Mathematics, Volume 9, Issue 1 (Nov. – Dec. 2013), PP 63-69.
[12] L.Levine,Semi open sets and semi continuity in topological spaces,Amer Math monthly,10(1963)36-41.
[13] N.Nagaveni, Studies on Generalization of Homeomorphisms in Topological Spaces, Ph.D Thesis Bharathiar Univ.Coimbatore, 1999.
[14] N. Palaniappan and K. C Rao, Regular generalized closed sets, Kyungpook Math, J., 33(1993), 211-219.
[15] Saeid Jafari and Takashi Noiri,On α-quasi- irresolute functions, Rend. Gre. Mat. Palermo, Sevie II,Tomo L(2001),137-152.
[16] Sanjay Mishra, On Regular Generalized Weakly-closed sets in Topological Spaces, Int. Journal of Math Analysis, Vol.6,(2012).no 39,1939-1952.
[17] M. H Stone, application of theory of Boolean rings to the general topology, Trans. A. M. S 41(1937), 375-481.
[18] P. Thangavelu and J. Maheshwari, totally irresolute functions, the mathematics Education, Vol. XL, No. 1(2006), 7-13.