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Minimal M-*gs* Open and Maximal M-*gs* Closed Sets In Interior Minimal Space

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Abstract: *The main objective of this paper is to study the notions of Minimal 𝓜-GS Closed set, Maximal 𝓜-GS Open set, Minimal 𝓜-GS Open set and Maximal 𝓜-GS Closed set and their basic properties in Interior Minimal Space.*

Key Words: *Maximal GS-closed set, Maximal GS-open set, Minimal GS-closed set, Minimal GS-open set*

I. Introduction

One can define the topology on a set by using either the axioms for the closed sets or the Kuratowski closure axioms. Closed sets are fundamental objects in a topological space. In 1970, N. Levine initiated the study of so-called generalized closed sets. By definition, a subset A of a topological space X is called generalized closed if clA⊆U whenever A⊆U and U is open. This notion has been studied by many topologists because generalized closed sets are not only natural generalization of closed sets but also suggest several new properties of topological spaces. Nakaoka and Oda have introduced Minimal open sets[1] and Maximal open sets[2], which are subclasses of open sets. A.Vadivel and K.Vairamanickam [3] introduced Minimal rgα-open sets and Maximal rgα-open sets in topological spaces. S. Balasubramanian and P.A.S. Vyjayanthi [4] introduced Minimal v-open sets and Maximal v-open sets; Minimal v-closed sets and Maximal v-closed sets in topological spaces. Inspired by these developments, we further study a new type of closed and open sets namely Minimal 𝓜-gs Open sets, Maximal 𝓜-gs Closed sets. In this paper a space X means a Minimal space (X, 𝓜). The class of 𝓜-gs closed sets is denoted by 𝓜-gscl(X). For any subset A of X its 𝓜-interior, 𝓜-closure, 𝓜-ginterior and 𝓜-gclosure are denoted respectively by the symbols 𝓜-int(A), 𝓜-cl(A), 𝓜-gint(A) and 𝓜-gcl(A).

2. Preliminaries

Definition 1: Let A⊂X. (i) A point x∈A is an 𝓜-ginterior point of A if ∃ an 𝓜-gs open set G in X such that x∈G⊂A. (ii) A point is said to be an 𝓜-glimit point of A if for each 𝓜-gs open set U in X, U∩(A∼{x}) ≠ φ. (iii) A point x∈A is said to be an 𝓜-gisolated point of A if ∃ 𝓜-gs open set U in X such that U∩A = {x}.

Definition 2: Let A⊂ X. (i) A is said to be 𝓜-gdiscrete if each point of A is 𝓜-gisolated. The set of all 𝓜-gisolated points of A is denoted by 𝓜-giso(A). (ii) For any A⊂X, the intersection of all 𝓜-gs closed sets containing A is called the 𝓜-gs closure of A and is denoted by 𝓜-gscl(A).

Example 1: Let X={0,1,2}; 𝓜 = {ϕ, {0,1}, {0,2}, X}. Here 𝓜-gs open sets are ϕ, {0}, {0,1}, {0,2} and X. Now {0}∩X={0}. Therefore 0 is a 𝓜-isolated point of X. In example, 𝓜-gs open sets are ϕ, {0}, {0,1},{0,2} and X. Now {0}∩X={0} and {0} is the only 𝓜-gisolated point of X.

Example 2: Let X = {0, 1, 2}; 𝓜={ϕ, {0,1},{0,2},{1,2},X}. Here 𝓜-gs open sets are ϕ, {0,1}, {0,2},{1,2} and X. Here there is no 𝓜-isolated point of X.

3. Minimal 𝓜-gs Open Sets and Maximal 𝓜-gs Closed Sets

We now introduce Minimal 𝓜-gs open sets and Maximal 𝓜-gs closed sets in Minimal space as follows:

Definition 3: A proper nonempty 𝓜-gs open subset U of X is said to be a Minimal 𝓜-gs open set if the only 𝓜-gs open sets contained in U are φ and U.

In Example 1**,** {0} and {1} are both Minimal 𝓜-open sets and Minimal 𝓜-gs open sets but {2} is Minimal 𝓜-gs open but not Minimal 𝓜-open.

Remark 1: Minimal 𝓜-open set and Minimal 𝓜-gs open set are independent to each other:

In Example 1, {0, 1} is Minimal 𝓜-open but not Minimal 𝓜-gs open and {0} is Minimal 𝓜-gs open but not Minimal 𝓜-open.

Theorem 1: (i) Let U be a Minimal 𝓜-gs open set and W be a 𝓜-gs open set. Then U ∩ W = φ or U⊂W. (ii) Let U and V be Minimal 𝓜-gs open sets. Then U ∩ V = φ or U = V.

Proof: (i) Let U be a Minimal 𝓜-gs open set and W be a 𝓜-gs open set. If U∩W≠φ, then U∩W⊂U. Since U is a Minimal 𝓜-gs open set, we have U∩W=U.

Therefore U⊂W.

(ii) Let U and V be Minimal $\mathcal{M}$-gs open sets. If $U \cap V \neq \varphi$, then $U \subset V$ and $V \subset U$ by(i). Therefore U = V.

Theorem 2: Let U be a Minimal $\mathcal{M}$-gs open set. If $x \in U$, then $U \subset W$ for any $\mathcal{M}$-gs open set W containing x.

Proof: Let U be a Minimal $\mathcal{M}$-gs open set and $x \in U$. Let W be any $\mathcal{M}$-gs open set containing x. Then by Theorem 1, $U \subset W$ since $U \cap W \neq \varphi$.

Theorem 3: Let U be a Minimal $\mathcal{M}$-gs open set. Then $U = \cap\{W: W$ is a $\mathcal{M}$-gs open set of X containing x$\}$ for any element x of U.

Proof: By theorem 2, $U \subset W$ for any $\mathcal{M}$-gs open set W containing x. So $U \subset \cap\{W: W$ is a $\mathcal{M}$-gs open set of X containing x$\}$. Also U is one such W. Therefore the intersection is contained in U. Hence $U = \cap\{W: W$ is a $\mathcal{M}$-gs open set of X containing x$\}$.

Theorem 4: Let V be a nonempty finite $\mathcal{M}$-gs open set. Then ∃ at least one (finite) Minimal $\mathcal{M}$-gs open set U such that $U \subset V$.

Proof: Let V be a nonempty finite $\mathcal{M}$-gs open set. If V is a Minimal $\mathcal{M}$-gs open set, we may set U=V. If V is not a Minimal $\mathcal{M}$-gs open set, then ∃ (finite) $\mathcal{M}$-gs open set V_1 such that $\varphi \neq V_1 \subset V$. If V_1 is a Minimal $\mathcal{M}$-gs open set, we may set $U=V_1$. If V_1 is not a Minimal $\mathcal{M}$-gs open set, then ∃ (finite) $\mathcal{M}$-gs open set V_2 such that $\varphi \neq V_2 \subset V_1$. Continuing this process, we have a sequence of $\mathcal{M}$-gs open sets $V \supset V_1 \supset V_2 \supset V_3 \supset \ldots \supset V_k \supset \ldots$ Since V is a finite set, this process must stop and finally we get a Minimal $\mathcal{M}$-gs open set $U = V_n \subset V$.

Definition 4: A Minimal space X is said to be $\mathcal{M}$-locally finite if each of its elements is contained in a finite $\mathcal{M}$-open set.

Theorem 5: Let U; U_λ be Minimal $\mathcal{M}$-gs open sets for any element $\lambda \in \Gamma$. If $U \subset \bigcup_{\lambda \in \Gamma} U_\lambda$, then ∃ an element $\lambda \in \Gamma$ such that $U = U_\lambda$.

Proof: Let $U \subset \bigcup_{\lambda \in \Gamma} U_\lambda$. Then $U \cap (\bigcup_{\lambda \in \Gamma} U_\lambda) = U$. That is $\bigcup_{\lambda \in \Gamma} (U \cap U_\lambda) = U$. Also by theorem 1, $U \cap U_\lambda = \varphi$ or $U = U_\lambda$ for any $\lambda \in \Gamma$. Then ∃ an element $\lambda \in \Gamma$ such that $U = U_\lambda$.

Theorem 6: Let U; U_λ be Minimal $\mathcal{M}$-gs open sets for any $\lambda \in \Gamma$. If $U \neq U_\lambda$ for any $\lambda \in \Gamma$, then $(\cup_{\lambda \in \Gamma} U_\lambda) \cap U = \varphi$.

Proof: Suppose that $(\bigcup_{\lambda \in \Gamma} U_\lambda) \cap U) \neq \varphi$. That is $\bigcup_{\lambda \in \Gamma} (U_\lambda \cap U) \neq \varphi$. Then ∃ an element $\lambda \in \Gamma$ such that $U \cap U_\lambda \neq \varphi$. By theorem1, we have $U = U_\lambda$, which contradicts $U \neq U_\lambda$ for any $\lambda \in \Gamma$. Hence $(\bigcup_{\lambda \in \Gamma} U_\lambda) \cap U = \varphi$.

We now introduce Maximal $\mathcal{M}$-gs closed sets in Minimal spaces as follows.

Definition 5: A proper nonempty $\mathcal{M}$-gs closed set $F \subset X$ is said to be Maximal $\mathcal{M}$-gs closed set if any $\mathcal{M}$-gs closed set containing F is either X or F. In Example 1, {1, 2} is Maximal $\mathcal{M}$-gs closed but not Maximal $\mathcal{M}$-closed.

Remark 3: Every Maximal $\mathcal{M}$-closed set is a Maximal $\mathcal{M}$-gs closed set.

Theorem 7: A proper nonempty subset F of X is Maximal $\mathcal{M}$-gs closed set if and only if X\F is a Minimal $\mathcal{M}$-gs open set.

Proof: Let F be a Maximal $\mathcal{M}$-gs closed set. Suppose X\F is not a Minimal $\mathcal{M}$-gs open set. Then ∃ a $\mathcal{M}$-gs open set $U \neq X \backslash F$ such that $\varphi \neq U \subset X \backslash F$. That is $F \subset X \backslash U$ and X\U is a $\mathcal{M}$-gs closed set, which is a contradiction since F is a Maximal $\mathcal{M}$-gs closed set. Therefore X\F is a Minimal $\mathcal{M}$-gs open set. Conversely, let X\F be a Minimal $\mathcal{M}$-gs open set. Suppose F is not a Maximal $\mathcal{M}$-gs closed set. Then ∃ a $\mathcal{M}$-gs closed set $E \neq F$ such that $F \subset E \neq X$. That is $\varphi \neq X \backslash E \subset X \backslash F$ and X\E is a $\mathcal{M}$-gs open set, which is a contradiction since X\F is a Minimal $\mathcal{M}$-gs open set. Therefore F is a Maximal $\mathcal{M}$-gs closed set.

Theorem 8: (i) Let F be a Maximal $\mathcal{M}$-gs closed set and W be a $\mathcal{M}$-gs closed set. Then $F \cup W = X$ or $W \subset F$. (ii) Let F and S be Maximal $\mathcal{M}$-gs closed sets. Then $F \cup S = X$ or F = S.

Proof: (i) Let F be a Maximal $\mathcal{M}$-gs closed set and W be a $\mathcal{M}$-gs closed set. Suppose $F \cup W \neq X$. Then $F \subset F \cup W$. Therefore $F \cup W = F$ and so WF. (ii) Let F and S be Maximal $\mathcal{M}$-gs clo⊂sed sets. If $F \cup S \neq X$, then we have $F \subset S$ and $S \subset F$ by (i). Therefore F = S.

Theorem 9: Let F_α, F_β, F_δ be Maximal $\mathcal{M}$-gs closed sets such that $F_\alpha \neq F_\beta$. If $F_\alpha \cap F_\beta \subset F_\delta$, then either $F_\alpha = F_\delta$ or $F_\beta = F_\delta$

Proof: If $F_\alpha \neq F_\delta$, then

$F_\beta \cap F_\delta = F_\beta \cap (F_\delta \cap X)$

$= F_\beta \cap (F_\delta \cap (F_\alpha \cup F_\beta)$ (by thm. 8)

$= F_\beta \cap ((F_\delta \cap F_\alpha) \cup (F_\delta \cap F_\beta))$

$= (F_\beta \cap F_\delta \cap F_\alpha) \cup (F_\beta \cap F_\delta \cap F_\beta)$

$= (F_\alpha \cap F_\beta) \cup (F_\delta \cap F_\beta)$ (since $F_\alpha \cap F_\beta \subset F_\delta$)

$= (F_\alpha \cup F_\delta) \cap F_\beta$

$= X \cap F_\beta$ (by theorem8)

$= F_\beta$.

Therefore $F_\beta = F_\delta$.

Theorem 10: Let F_α, F_β and F_δ be distinct Maximal $\mathcal{M}$-gs closed sets . Then $(F_\alpha \cap F_\beta) \not\subset (F_\alpha \cap F_\delta)$.
Proof: Suppose $(F_\alpha \cap F_\beta) \subset (F_\alpha \cap F_\delta)$. Then $(F_\alpha \cap F_\beta) \cup (F_\delta \cap F_\beta) \subset (F_\alpha \cap F_\delta) \cup (F_\delta \cap F_\beta)$. That is,
$(F_\alpha \cup F_\delta) \cap F_\beta \subset F_\delta \cap (F_\alpha \cup F_\beta)$.
Since by theorem 8, $F_\alpha \cup F_\delta = X$ and
$F_\alpha \cup F_\beta = X$, we get $F_\beta \subset F_\delta$.
From the definition of Maximal $\mathcal{M}$-gs closed set, we get $F_\beta = F_\delta$. We arrive at a contradiction since F_α, F_β and F_δ are distinct.
Therefore $(F_\alpha \cap F_\beta) \not\subset (F_\alpha \cap F_\delta)$.

Theorem 11: Let F be a Maximal $\mathcal{M}$-gs closed set and x be an element of F. Then $F = \cup \{ S: S$ is a $\mathcal{M}$-gs closed set containing x such that $F \cup S \neq X\}$.
Proof: Since F is one such S, we have $F \subset \cup\{ S: S$ is a $\mathcal{M}$-gs closed set containing x such that $F \cup S \neq X\} \subset F$. By Theorem 8, each $S \subset F$.Therefore we have $F=\cup\{ S: S$ is a $\mathcal{M}$-gs closed set containing x such that $F \cup S \neq X\}$.

Theorem 12: Let F be a Maximal $\mathcal{M}$-gs closed set. If x is an element of $X\backslash F$, then $X\backslash F \subset E$ for any $\mathcal{M}$-gs closed set E containing x.
Proof: $E \not\subset F$ for any $\mathcal{M}$-gs closed set E containing x. Then $E \cup F = X$, by Theorem 8. Therefore $X\backslash F \subset E$.

4. Minimal $\mathcal{M}$-gs Closed set and Maximal $\mathcal{M}$-gs Open set

We now introduce Minimal $\mathcal{M}$-gs closed sets and Maximal $\mathcal{M}$-gs open sets in Minimal space as follows:

Definition 6: A proper nonempty $\mathcal{M}$-gs closed subset F of X is said to be a Minimal $\mathcal{M}$-gs closed set if any $\mathcal{M}$-gs closed set contained in F is φ or F. In Example 3**,** {1} and {2} are both Minimal $\mathcal{M}$-closed sets and Minimal $\mathcal{M}$-gs closed sets.

Remark 4: Minimal $\mathcal{M}$-closed and Minimal $\mathcal{M}$-gs closed set are independent to each other. In Example 1, {0, 2} and {1,2} are Minimal $\mathcal{M}$-closed but not Minimal $\mathcal{M}$-gs closed and {2} is Minimal $\mathcal{M}$-gs closed but not Minimal $\mathcal{M}$-closed.

Definition 7: A proper nonempty $\mathcal{M}$-gs open $U \subset X$ is said to be a Maximal $\mathcal{M}$-gs open set if any $\mathcal{M}$-gs open set containing U is either X or U. In Example 1,{0,1} is Maximal $\mathcal{M}$-gs open but not Maximal $\mathcal{M}$-open and {0,2} and {1,2} are Maximal $\mathcal{M}$-open but not Maximal $\mathcal{M}$-gs open .

Theorem 13: A proper nonempty subset U of X is Maximal $\mathcal{M}$-gs open iff $X\backslash U$ is a Minimal $\mathcal{M}$-gs closed .
Proof: Let U be a Maximal $\mathcal{M}$-gs open set. Suppose $X\backslash U$ is not a Minimal $\mathcal{M}$-gs closed set. Then $\exists$ $\mathcal{M}$-gs closed set $F \neq X\backslash U$ such that $\varphi \neq F \subset X\backslash U$.
Therefore $U \subset X\backslash F$ and $X\backslash F$ is a $\mathcal{M}$-gs open set with $X\backslash F \neq X$ and $X\backslash F \neq U$.
This contradicts the maximality of U.
Conversely, let $X\backslash U$ be a Minimal $\mathcal{M}$-gs closed set. Suppose U is not a Maximal $\mathcal{M}$-gs open set. Then $\exists$ $\mathcal{M}$-gs open set $V \neq U$ such that $U \subset V \neq X$. That is, $\varphi \neq X\backslash V \subset X\backslash U \neq X\backslash V$ and $X\backslash V$ is a $\mathcal{M}$-gs closed set, contradicting the minimality of $X\backslash U$.

Theorem 14: (i) Let E be a Minimal $\mathcal{M}$-gs closed set and F be a $\mathcal{M}$-gs closed set. Then $E\cap F=\varphi$ or $E\subset F$. (ii) Let E and F be Minimal $\mathcal{M}$-gs closed sets. Then $E\cap F= \varphi$ or $E=F$.
Proof: (i) If $E\cap F\neq \varphi$, then $E\cap F \subset E$ and $E\cap F$ is a $\mathcal{M}$-gs closed set and so $E\cap F=E$. Therefore $E \subset F$.
(ii) Let E and F be Minimal $\mathcal{M}$-gs closed sets. If $E\cap F \neq \varphi$, then $E \subset F$ and $F \subset E$ by(i). Therefore E = F.

Theorem 15: Let F be a nonempty finite $\mathcal{M}$-gs closed set. Then $\exists$ at least one (finite) Minimal $\mathcal{M}$-gs closed set E such that $E \subset F$.
Proof: If F is not a Minimal $\mathcal{M}$-gs closed set, then $\exists$ (finite) $\mathcal{M}$-gs closed set F_1 such that $\varphi \neq F_1 \subset F$.
If F_1 is not a Minimal $\mathcal{M}$-gs closed set, then $\exists$ (finite) $\mathcal{M}$-gs closed set F_2 such that $\varphi \neq F_2 \subset F_1$. Continuing this process, we get a sequence of $\mathcal{M}$-gs closed sets $F\supset F_1\supset F_2\supset F_3\supset.....\supset F_k\supset$ ...... Since F is a finite set, this process must stop and we get a Minimal $\mathcal{M}$-gs closed set $E = F_n$ for some positive integer n.

Theorem 16: Let F; F_λ be Minimal $\mathcal{M}$-gs closed sets for any element $\lambda\in\Gamma$. If $F \subset \cup_{\lambda\in\Gamma}F_\lambda$, then $\exists$ an element $\lambda\in\Gamma$ such that $F = F_\lambda$.
Proof: Let $F \subset \cup_{\lambda\in\Gamma}F_\lambda$. Then $F\cap(\cup_{\lambda\in\Gamma}F_\lambda) = F$. That is $\cup_{\lambda\in\Gamma}(F\cap F_\lambda) = F$. Also by Theorem 14, $F\cap F_\lambda = \varphi$ or $F = F_\lambda$ for any $\lambda\in\Gamma$. Hence $\exists$ an element $\lambda\in\Gamma$ such that $F = F_\lambda$.

Theorem 17: Let F; F_λ be Minimal $\mathcal{M}$-gs closed sets for any $\lambda\in\Gamma$. If $F \neq F_\lambda$ for any $\lambda\in\Gamma$, then $(\cup_{\lambda\in\Gamma}F_\lambda) \cap F = \varphi$.
Proof: Suppose $(\cup_{\lambda\in\Gamma}F_\lambda)\cap F \neq \varphi$. That is $\cup_{\lambda\in\Gamma}(F_\lambda\cap F) \neq \varphi$. Then $\exists$ an element $\lambda\in\Gamma$ such that $F\cap F_\lambda\neq \varphi$. By Theorem14, we have $F = F_\lambda$, which contradicts $F \neq F_\lambda$ for any $\lambda \in \Gamma$. Hence $(\cup_{\lambda\in\Gamma}F_\lambda)\cap F = \varphi$.

Theorem 18: A proper nonempty subset V of X is a Maximal $\mathcal{M}$-gs open set iff $X\backslash V$ is a Minimal $\mathcal{M}$-gs closed set.

Proof: Let V be a Maximal $\mathcal{M}$-gs open set. Suppose X\V is not a Minimal $\mathcal{M}$-gs ciosed set. Then ∃ $\mathcal{M}$-gs ciosed set F ≠ X\V such that φ ≠ F ⊂ X\V. That is, V⊂ X\F and X\F is a $\mathcal{M}$-gs open set with X\F ≠ X and X\F ≠ V, contradicting the maximality of V. Conversely, let X\V be a Minimal $\mathcal{M}$-gs closed set. Suppose V is not a Maximal $\mathcal{M}$-gs open set. Then ∃ $\mathcal{M}$-gs open set U ≠ V such that V⊂ U ≠ X. That is φ ≠ X\U ⊂ X\V ≠ X\U and X\U is a $\mathcal{M}$-gs ciosed set, contradicting the minimality of X\V.

Theorem 19: (i) Let V be a Maximal $\mathcal{M}$-gs open set and U be a $\mathcal{M}$-gs open set. Then U∪V=X or U ⊂ V. (ii) Let U and V be Maximal $\mathcal{M}$-gs open sets. ThenU∪V=X or U=V.

Proof:(i)Suppose U∪V≠X.SinceV⊂U∪V, we get U∪V=V, by maximality of V.
Hence U ⊂ V.
(ii) Let U and V be Maximal $\mathcal{M}$-gs open sets. If U∪V≠X, then we have U⊂V and V⊂U, by (i). Therefore U = V.

Theorem 20: Let V_α, V_β, V_δ be Maximal $\mathcal{M}$-gs open sets such that $V_\alpha \neq V_\beta$. If $V_\alpha \cap V_\beta \subset V_\delta$, then either $V_\alpha = V_\delta$ or $V_\beta = V_\delta$.

ProoV: Given $V_\alpha \cap V_\beta \subset V_\delta$. If $V_\alpha \neq V_\delta$, then $V_\beta \cap V_\delta = V_\beta \cap (V_\delta \cap X)$
$= V_\beta \cap (V_\delta \cap (V_\alpha \cup V_\beta)$, by theorem.19
$= V_\beta \cap ((V_\delta \cap V_\alpha) \cup (V_\delta \cap V_\beta))$
$= (V_\beta \cap V_\delta \cap V_\alpha) \cup (V_\beta \cap V_\delta \cap V_\beta)$
$= (V_\alpha \cap V_\beta) \cup (V_\delta \cap V_\beta)$,since $V_\alpha \cap V_\beta \subset V_\delta$
$= (V_\alpha \cup V_\delta) \cap V_\beta$
$= X \cap V_\beta$
$= V_\beta$
Since V_β and V_δ are Maximal $\mathcal{M}$-gs open sets, we get $V_\beta = V_\delta$.

Theorem 21: Let V_α, V_β and V_δ be distinct Maximal $\mathcal{M}$-gs open sets. Then $(V_\alpha \cap V_\beta) \not\subset (V_\alpha \cap V_\delta)$.

ProoV: Suppose $(V_\alpha \cap V_\beta) \subset (V_\alpha \cap V_\delta)$. Then $V_\alpha \cap V_\beta) \cup (V_\delta \cap V_\beta) \subset (V_\alpha \cap V_\delta) \cup (V_\delta \cap V_\beta)$
That is, $(V_\alpha \cup V_\delta) \cap V_\beta \subset (V_\alpha \cup V_\beta) \cap V_\delta$.
That is, $X \cap V_\beta \subset X \cap V_\delta$.
Hence $V_\beta \subset V_\delta$. From the definition of Maximal $\mathcal{M}$-gs open set it follows that $V_\beta = V_\delta$ which is a contradiction to the fact that V_α, V_β and V_δ are distinct. Therefore $(V_\alpha \cap V_\beta) \not\subset (V_\alpha \cap V_\delta)$.

Theorem 22: Let V be a Maximal $\mathcal{M}$-gs open set. If x is an element of X\V, then X\V ⊂ U for any $\mathcal{M}$-gs open set U containing x.

Proof: U ⊄ V for any $\mathcal{M}$-gs open set U containing x. Then U∪V =X by Theorem 19. Therefore X\V⊂ U.

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