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A Fixed Point Theorem Using Common Property (E. A.) In PM Spaces

G. P. S. Rathore[1], Bijendra Singh[2], Kirty Chauhan[3]
[1]*College of Horticulture, Mandsaur, INDIA*
[2]*S.S. in Mathematics, Vikram University, Ujjain, INDIA*
[3]*Department of Mathematics, A T C, Indore, INDIA*

ABSTRACT: *Employing the common property (E. A.), we derive fixed point theorem in PM spaces via weakly compatible mapping. This result substantially improves available results.*

KEYWORDS : *Compatible maps, PM space, weakly commuting mappings, common property (E.A.).*

I. INTRODUCTION

Here it may be noted that the notion of compatible mapping is due to Jungck [6]. Pant [3] initiated the work area in non compatible mappings. Aamri and Moutawakil [2] introduced property (E. A.) and common property (E. A.), which is a an important generalization of mappings in metric space. Branciari [1] proved a fixed point result for a mapping satisfying an integral type inequality. There is various theorems focus on relatively more general integral type contractive conditions. In the sequel we derive a characterization of such definition where it is in linear form and use for obtaining some results on fixed points. In present paper we intend to utilize this relatively more natural concept to prove our theorem in PM spaces. The main purpose of this work is generalized known results in [4] and [5].

II. PRELIMINARIES

In this portion we give basic definitions which are used in the paper.
Basically a metric is a function that satisfies the minimal properties we might except of a distance.

Definition.2.1. A metric d on a set X is a function $d : X \times X \to [0, \infty)$ such that for all $x, y \in X$:
(i). $d(x, y) \geq 0$ and $d(x, y)= 0$ iff $x = y$,
(ii). $d(x, y) = d(y, x)$, (symmetry)
(iii). $d(x, y) \leq d(x, z) + d(z, y)$ (triangle inequality).
A metric space (X, d) is a set X with a metric d defined on X. It has a notion of the distance d(x, y) between every pair of points $x, y \in X$.
We can define many different metrics on the same set, but if the metric on X is clear from the context, we refer to X as a metric space and omit explicit mention of the metric d.

Definition.2.2 [5]. A pair (A, S) satisfy property (E. A.) if there exists a sequence $\{x_n\}$ in X such that
$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$, for $t \in X$.

Definition.2.3 [7]. Two pairs (A, S) and (B, T) of self mappings of a metric space (X, d) satisfy a common property (E. A.) if there exists two sequences $\{x_n\}$ and $\{y_n\}$ such that
$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = t$, for $t \in X$.

Definition.2.4 [8]. A pair (A, S) of self mappings of a non empty set X is said to be weakly compatible if the pair commutes on the set of coincidence points, that is, $Ap = Sp$ for some $p \in X$ implies that $ASp = SAp$.

III. MAIN RESULT

Our next theorem involves a continuous function ψ, which is said to be Ψ function if
(i). ψ is monotone increasing and continuous,
(ii). $\psi(t) > t$, for $\forall\, t \in (0, 1)$,
(iii). $\psi(1) = 1$, $\psi(0) = 0$,
(iv). $\psi : [0, 1] \to [0, 1]$.

And $\varnothing : R_+ \to R_+$ is a lebesgue integrable function which is non negative sum able such that
$1 > \int_0^{\varepsilon} \varnothing(t)\,dt > 0$ and $\int_0^{1} \varnothing(t)\,dt = 1$, where $0 < \varepsilon < 1$.

Theorem.3.1. Let A, B, S, T be self mappings of a metric space (X, d) satisfying following conditions
(i). $A(X) \subset T(X)$, $B(X) \subset S(X)$,
(ii). ψ is a Ψ function,
(iii). Pairs (A, S) and (B, T) share common property (E. A.),
(iv). S(X) and T(X) are closed subset of X,
(v). $\int_0^{d(Ap,Bq)} \varnothing(t)\,dt \geq \psi \int_0^{M(p,q)} \varnothing(t)\,dt$
Where, for $\forall$ p, q $\in$ X, n > 0, $\alpha \in (0, 2)$
$M(p, q) = \min\{d(Sp, Tq), d(Bq, Tq), d(Sp, Ap), d(Ap, Bq), d(Ap, Tq(\alpha n)), d(Bq, Sp((2-\alpha)n))\}$.
Thus the pairs (A, S) and (B, T) have a unique common fixed point provided that both pairs are weakly compatible.

Proof. As both the pairs (A, S) and (B, T) share the common property (E. A.), there exist two sequences $\{x_n\}$, $\{y_n\} \in X$ such that

$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = t$, for t $\in$ X.
From predefined inequality, S(X) is closed subset of X. Therefore we can take a point u $\in$ X, such that Su = t.

If Au $\neq$ t from (v)

$$\int_0^{d(Au,By_n)} \varnothing(t)\,dt \geq \psi \int_0^{\min\{d(Su,Ty_n),d(By_n,Ty_n),d(Su,Au),d(Au,By_n),d(Au,Ty_n(\alpha n)),d(By_n,Su((2-\alpha)n))\}} \varnothing(t)\,dt$$

Applying $n \to \infty$

$$\int_0^{d(Au,t)} \varnothing(t)\,dt \geq \psi \int_0^{\min\{d(t,t),d(t,t),d(t,Au),d(Au,t),d(Au,t(\alpha n)),d(t,tu((2-\alpha)n))\}} \varnothing(t)\,dt$$

Let $\beta \in [-1, 1]$ and $\beta = 1-\alpha$ or $\alpha = 1-\beta$

$$\geq \psi \int_0^{\min\{d(t,t),d(t,t),d(t,Au),d(Au,t),d(Au,t((1-\beta)n)),d(t,t((1+\beta)n))\}} \varnothing(t)\,dt$$
$$\geq \psi \int_0^{\min\{0,0,d(t,Au),d(Au,t),d(Au,0),d(t,0)\}} \varnothing(t)\,dt$$
$$\geq \psi \int_0^{d(t,Au)} \varnothing(t)\,dt$$
$$\geq \int_0^{d(t,Au)} \varnothing(t)\,dt$$

This is contradiction. Hence Au = t.

As T(X) is also a closed subset of X, then there exist v $\in$ X, such that Tv = t.
If Bv $\neq$ t, then by (v) we obtain

$$\int_0^{d(Ax_n,Bv)} \varnothing(t)\,dt \geq \psi \int_0^{\min\{d(Sx_n,Tv),d(Bv,Tv),d(Sx_n,Ax_n),d(Ax_n,Bv),d(Ax_n,Tv(\alpha n)),d(Bv,Sx_n((2-\alpha)n))\}} \varnothing(t)\,dt$$

Taking $n \to \infty$

$$\int_0^{d(t,Bv)} \varnothing(t)\,dt \geq \psi \int_0^{\min\{d(t,t),d(Bv,t),d(t,t),d(t,Bv),d(t,t(\alpha n)),d(Bv,t((2-\alpha)n))\}} \varnothing(t)\,dt$$

$$\geq \psi \int_0^{\min\{d(t,t),d(Bv,t),d(t,t),d(t,Bv),d(t,t((1-\beta)n)),d(Bv,t((1+\beta)n))\}} \varnothing(t)\,dt$$

$$\geq \psi \int_0^{\min\{0,d(Bv,t),0,d(t,Bv),d(t,0),d(Bv,0)\}} \varnothing(t)\,dt$$

$$\geq \psi\int_0^{d(Bv,t)} \emptyset(t)\,dt$$
$$\geq \int_0^{d(Bv,t)} \emptyset(t)\,dt$$

This is contradiction, so that Bv = t.
If we combine all results Au = Su = Bv = Tv = t. This conclude that the pairs (A, S) and (B, T) have a point of coincidence u and v respectively.

By the definition of weakly compatible mapping
At = ASu = SAu = St
Bt = BTv = TBv = Tt

If At ≠ t, then by (v) we get

$$\int_0^{d(At,Bv)} \emptyset(t)\,dt \geq \psi\int_0^{\min\{d(St,Tv),d(Bv,Tv),d(St,At),d(At,Bv),d(At,Tv(\alpha n)),d(Bv,St((2-\alpha)n))\}} \emptyset(t)\,dt$$

Using $n \rightarrow \infty$

$$\int_0^{d(At,t)} \emptyset(t)\,dt \geq \psi\int_0^{\min\{d(At,t),d(t,t),d(At,At),d(At,t),d(At,t(\alpha n)),d(t,At((2-\alpha)n))\}} \emptyset(t)\,dt$$

$$\geq \psi\int_0^{\min\{d(At,t),d(t,t),d(At,At),d(At,t),d(At,t((1-\beta)n)),d(t,At((1+\beta)n))\}} \emptyset(t)\,dt$$
$$\geq \psi\int_0^{\min\{d(At,t),0,0,d(At,t),d(At,0),d(t,0)\}} \emptyset(t)\,dt$$
$$\geq \psi\int_0^{d(At,t)} \emptyset(t)\,dt$$
$$\geq \int_0^{d(At,t)} \emptyset(t)\,dt$$

This is contradiction, therefore At = t and so St = t.
Finally if Bt ≠ t, then by (v) we get

$$\int_0^{d(At,Bt)} \emptyset(t)\,dt \geq \psi\int_0^{\min\{d(St,Tt),d(Bt,Tt),d(St,At),d(At,Bt),d(At,Tt(\alpha n)),d(Bt,St((2-\alpha)n))\}} \emptyset(t)\,dt$$

Let $n \rightarrow \infty$

$$\int_0^{d(t,Bt)} \emptyset(t)\,dt \geq \psi\int_0^{\min\{d(t,Bt),d(Bt,Bt),d(t,t),d(t,Bt),d(t,Bt(\alpha n)),d(Bt,t((2-\alpha)n))\}} \emptyset(t)\,dt$$

$$\geq \psi\int_0^{\min\{d(t,Bt),d(Bt,Bt),d(t,t),d(t,Bt),d(t,Bt((1-\beta)n)),d(Bt,t((1+\beta)n))\}} \emptyset(t)\,dt$$
$$\geq \psi\int_0^{\min\{d(t,Bt),0,0,d(t,Bt),d(t,0)),d(Bt,0)\}} \emptyset(t)\,dt$$
$$\geq \psi\int_0^{d(t,Bt)} \emptyset(t)\,dt$$
$$\geq \int_0^{d(t,Bt)} \emptyset(t)\,dt$$

This is contradiction, so that Bt = t and Tt = t.
i.e. At = St = Bt = Tt = t

Now we can say that t is a common fixed point of A, S , B and T.

Uniqueness: To show the uniqueness of fixed point suppose that there are two fixed points t and w such that t ≠ w.

By (v)

$$\int_0^{d(At,Bw)} \emptyset(t)\,dt \geq \psi\int_0^{\min\{d(St,Tw),d(Bw,Tw),d(St,At),d(At,Bw),d(At,Tw(\alpha n)),d(Bw,St((2-\alpha)n))\}} \emptyset(t)\,dt$$

Applying $n \to \infty$

$$\int_0^{d(t,w)} \emptyset(t)\, dt \geq \psi \int_0^{\min\{d(t,w), d(w,w), d(t,t), d(t,w), d(t,w(\alpha n)), d(w,t((2-\alpha)n))\}} \emptyset(t)\, dt$$
$$\geq \psi \int_0^{\min\{d(t,w), d(w,w), d(t,t), d(t,w), d(t,w((1-\beta)n)), d(w,t((1+\beta)n))\}} \emptyset(t)\, dt$$
$$\geq \psi \int_0^{\min\{d(t,w), 0, 0, d(t,w), d(t,0), d(w,0)\}} \emptyset(t)\, dt$$
$$\geq \psi \int_0^{d(t,w)} \emptyset(t)\, dt$$
$$\geq \int_0^{d(t,w)} \emptyset(t)\, dt$$

This is contradiction. So that t = w.
This concludes the proof the existence of unique common fixed point of mappings.

IV. CONCLUSION

In this paper through Theorem 3.1 we introduce the new concept of common fixed point in case of integrable function and common property (E. A.).

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