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Minimality and Equicontinuity of a Sequence of Maps in Iterative Way

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ABSTRACT : *Let (X, d) be a compact metric space and $f_n : X \to X$ a sequence of continuous functions such that (f_n) converge orbitally in the iterative way, to a function f. The equicontinuity, minimality and Transitivity of the limit function f have been studied.*

KEY WORDS AND PHRASES: *Topological Transitivity, Equicontinuous, Minimality, Orbital convergence in iterative way.*

I. INTRODUCTION

A dynamical system is a pair (X, f) in which X is a compact metric space and f is a continuous self map. Several authors have studied the dynamical properties inherited by the uniform limit f of a sequence (f_n) of continuous self maps. It has been shown in [7] that a sequence (f_n) of continuous and transitive maps that converge uniformly to f, is not necessarily topologically transitive. Recently many Mathematicians have studied topological transitivity of the uniform limit of a sequence of uniformly convergent transitive system. Tian and Chen [8] studied chaos of a sequence of time invariant continuous functions on a general metric space. The authors also introduced quite a few new concepts, such as chaos in the successive way in the sense of Devaney, chaos in the iterative way in the sense of Devaney. Bhaumik and Choudhury [1] have investigated turbulent maps and strongly transitive maps in general metric spaces which are not necessarily compact. It has been proved that if (f_n) is a sequence of continuous functions which is topologically transitive in the strongly iterative way in an infinite compact metric space (X, d) the uniform limit function in the iterative way is topologically mixing.

In [6] there is a sufficient condition so that the uniform limit is transitive. In [4] also there is a sufficient condition for the transitivity of the limit function. In [3] the concept of orbital convergent of a sequence (f_n) is defined . They have shown that if a sequence (f_n) of transitive system is orbitally convergent to f, then f is topologically transitive. They also have shown that if a sequence (f_n) of chain transitive dynamical system is convergent uniformly to f, then f is also chain transitive. Recently Mangang [5] has studied the minimality, equicontinuity of the limit function of an orbitally convergent sequence as well as uniform convergence sequence. In [2] the topological transitivity of sequence of transitive maps under group action has been studied. In [9] there is a sufficient condition for the transitivity of the uniform limit of a sequence of transitive system. In this paper, the topological transitivity, minimality and equicontinuity of the limit function of a sequence of functions which converges orbitally in the iterative way, have been investigated.

Definition 1.1. *Let (X, d) be a metric space. Let $f : X \to X$ be a continuous self map. The map f is said to be topologically transitive if for each pair of non empty subsets U and V, there exists some $n \in N$ such that $f^n(U) \cap V \neq \phi$.*

Let (X, f) be a dynamical system, the orbit $O(x, f)$ of a point $x \in X$ is defined by $O(x, f) = \{f^n(x) : n \in N\}$. A point $x \in X$ is called a transitive point if the orbit of x is dense in X, i.e. $\overline{O(x, f)} = X$. The set of all transitive points of f is denoted by $tr(f)$.

It is well known fact that if X is a compact metric space without isolated points and f is a continuous self map on it, then a single transitive point x of f is necessary and sufficient condition for f to be topological transitive.

Definition 1.2. *A dynamical system (X,f) is said to be equicontinuous if for all $x \in X$ and for every $\epsilon > 0$, there exists $\delta > 0$ such that $d(f^n(x), f^n(y)) < \epsilon$, for all $y \in B_\delta(x) = \{y \in X: d(x,y) < \delta\}$, and for all $n \in N$.*

Every isometry is an equicontinuous system because $d(f(x), f(y)) = d(x,y)$, implies that $d(f^n(x), f^n(y)) = d(x,y)$ for all $n \in N$

Definition 1.3. *Let (f_n) be a sequence of continuous self maps on a metric space (X,d). Then (f_n) is called orbitally convergent to a map $f: X \to X$ if for every $\epsilon > 0$, there exists $k \in N$ such that $d(f_n^m(x), f^m(x)) < \epsilon$, for all $x \in X$, for all $m \in N$ and for all $n \geq k$.*

Definition 1.4. *A dynamical system (X,f) is called a minimal dynamical system if $tr(f) = X$.*

A point $x \in X$ is said to be an equicontinuous point of (X,f) if for every $\epsilon > 0$, there exists $\delta > 0$ such that $d(f^n(x), f^n(y)) < \epsilon$, for all $y \in B_\delta(x)$, and for all $n \in N$.

Definition 1.5. *Let $f_n: X \to X$ be a sequence of continuous functions. Then $\{x, f_1(x), f_2 \circ f_1(x), f_3 \circ f_2 \circ f_1(x) \ldots\}$ is called orbit of the sequence (f_n) (starting at x) in the iterative way*[8].

We denote $f_k \circ f_{k-1} \circ \ldots \circ f_1(x)$ by $F_k(x)$ for all $k \geq 1$ and for all $x \in X$.

Definition 1.6. *Let $f_n: X \to X$ be a sequence of continuous functions. Let $f: X \to X$ be a continuous function and let $\epsilon > 0$ be a small number. We say that sequence (f_n) converge uniformly to f in the iterative way if there exists positive integer M such that $d(F_n(x), f(x)) < \epsilon \; \forall \; n \geq M$ and for all $x \in X$.*

Definition 1.7. *Let (f_n) be a sequence of continuous self maps on X. We say that (f_n) is orbitally convergent in the iterative way to a map $f: X \to X$ if for every $\epsilon > 0$, there exists $k \in N$ such that $d(F_n(x), f^m(x)) < \epsilon$, for all $x \in X$, for all $m \geq 0$ and for all $n \geq k$.*

Definition 1.8. *Let $f_n: X \to X$ be a sequence of continuous functions. If, for any two non-empty open subsets U and V of X, there exists a positive integer k such that $F_k(U) \cap V \neq \phi$ then the sequence of functions (f_n) is said to be topologically transitive on X in the iterative way.*

Definition 1.9. *A point $x \in X$ is said to be transitive in iterative way if the iterative orbit $\{F_k(x): k \in N\}$ is dense in X.*

II. MAIN RESULTS

Theorem 2.1. *Let (f_n) be a sequence of minimal functions in iterative way which converges orbitally in iterative way to f. Then f is minimal.*

Proof. $f_n \to f$, orbitally in iterative way, then for a given $\epsilon > 0$, there exists a positive integer M such that
(1) $d(F_k(x), f^n(x)) < \epsilon/2$ for all $k \geq M$, for all $x \in X$ and for all n.
Let $z \in X$ be a point. Since (f_n) is a sequence of minimal functions in iterative way, there exists k such that
(2) $F_k(x) \in B_{\epsilon/2}(z)$ i.e. $d(F_k(x), z) < \frac{\epsilon}{2}$
Now by the triangular inequality of metric, we have

$d(f^n(x), z) \leq d\big(f^n(x), F_k(x)\big) + d(F_k(x), z) < \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon$ (Using (1) and (2))
$\therefore d(f^n(x), z) < \epsilon \Rightarrow f^n(x) \in B_\epsilon(z)$. This shows that f is minimal.

Theorem 2.2. *f_n is a sequence which converge to f orbitally in iterative way. Suppose $x \in X$ is a transitive point in iterative way, then f is transitive.*

Proof. Let $\epsilon > 0$ be a small number, then there exists M such that $d(F_k(x), f^n(x)) < \frac{\epsilon}{2}$ for all $k \geq M$, for all n. Let $z \in X$ be any point. Since x is a transitive point in iterative way, there exists k such that $F_k(x) \in B_{\epsilon/2}(z)$. This implies that $d(F_k(x), z) < \epsilon/2$. Therefore by triangular inequality of metric, we have $d(f^n(x), z) < \epsilon/2 + \epsilon/2 = \epsilon$. This shows that $f^n(x) \in B_\epsilon(z)$. Therefore x is a transitive point of f and consequently f is transitive.

Definition 2.3. *Let $f_n: X \to X$ be a sequence of continuous functions on X. The sequence (f_n) is said to be equicontinuous in the iterative way if for all $x \in X$ and for every $\epsilon > 0$, there exists $\delta > 0$ such that $d(F_n(x), F_n(y)) < \epsilon$, for all $y \in B_\delta(x) = \{y \in X : d(x, y) < \delta\}$, and for all $n \in N$.*

Theorem 2.4. *Let (X, d) be a compact metric space and suppose that $f_n: X \to X$ is a sequence of equicontinuous functions in the iterative way. If (f_n) converges orbitally in the iterative way to a function f, then f is equicontinuous.*

Proof. (f_n) is an orbitally convergent sequence in the iterative way to f. Therefore for all $x \in X$ and for all $\epsilon > 0$, there exists $k \in N$ such that $d\big(F_n(x), f^m(x)\big) < \epsilon/3$, for all $n \geq k$, and for all $m \geq 0$. In particular, we have

(A) $d(F_k(x), f^m(x)) < \epsilon/3$, for all $m \geq 0$.

Also

(B) $d(F_k(y), f^m(y)) < \epsilon/3$, for all $m \geq 0$.

(f_n) is equicontinuous in the iterative way, therefore for each $\epsilon > 0$, there exists $\delta > 0$ such that

(C) $d(F_k(x), F_k(y)) < \epsilon/3$, for all $k \geq 0$ and for all $y \in B_\delta(x)$.

From (A), (B) and (C), we have

$$d(f^m(x), f^m(y)) \leq d(F_k(x), f^m(x)) + d(F_k(x), F_k(y)) + d(F_k(y), f^m(y))$$
$$< \frac{\epsilon}{3} + \frac{\epsilon}{3} + \frac{\epsilon}{3} = \epsilon$$

Hence f is equicontinuous.

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