International Journal of Mathematics and Statistics Invention (IJMSI)
E-ISSN: 2321 – 4767 P-ISSN: 2321 - 4759
www.ijmsi.org Volume 2 Issue 8 || August. 2014 || PP-18-35

Analytical and Numerical Modelling Of Layer Effects on Far Field Microseismic Oscillations

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ABSTRACT : *We employ analytical techniques in designing numerical models of the layer effects on far field microseismic oscillations and the activities of wave train approaching the shoreline from a wide range of directions in the intermediate frequency range. We assume the elastic medium in the model earth to be damped and horizontally layered and the governing equations to be those that describe the small amplitude oscillations in such a medium. Therefrom, we obtain a relationship between the phenomenon of wave reflection along the shoreline and microseisms and thus estimate the distance from the shoreline over which the approaching shallow water waves are expected to acquire measurable bottom pressure, and further confirm that the distance is finite and proportional to wave period.*

KEYWORDS : *Intermediate Frequency Range, Microseism, Elastic Medium, Model Earth, Shoreline.*

I. INTRODUCTION

Microseisms, also known as micro-earth tremors are the continuous background noise on seismic records in the range from about 2 to 20 seconds (Hasselmann, 1963). The phenomena had been observed since the early days of seismology and the efficiency with which these waves are transmitted from the generating source to the far field, their polarization, subsequent detection and recording are quite remarkable and fairly well understood.Several mechanisms have been proposed to explain the origin of these background noises. Wiechart (1904) conceived it as surf breaking along coasts. Banerji (1930) suggested that the source of microseisms were the activities of large storms at sea. Gherzi (1932) proposed that air pressure fluctuations have a pumping action that could cause storm microseisms, that is, air pressure are transmitted into the ground and the resulting seismic vibrations propagated to great distances, away from the generating source. Bernard (1937), on his part suggested that standing waves are the cause of microseisms. Longuet-Higgins (1950) improved on Bernard's theory. He thereby demonstrated that the interference of gravity waves in the ocean could produce a second-order pressure effect that might be transmitted into the underlying seabed and further suggested that appropriate conditions for the process could occur around the centres of large cyclonic disturbances and also where waves are reflected from a coast.

One major difficulty in determining which mechanism explains the observed microseismic disturbances which is qualitatively in comparison with theory can be attributed to the fact that most of the theoretical analysis has been formulated in terms of the Green's function (Hasselmann, 1963). Hasselmann, thus, utilized statistical analysis to confirm the results of Longuet-Higgins and others. He also introduced the theory of high-phase velocity resonant energy transfer.Further, microseisms are essentially surface waves propagating in the direction parallel to the earth's surface and the associated energy trapped near the surface. Consequently, they could be detected at quite a distance from the generating source. **Interestingly**, this model has been able to calculate conclusively the layer depth within which the energy is trapped below the earth's surface.

An analysis of the energy spectrum of the seismic records in the range of microseisms frequencies clearly indicate two main peaks. It has been established conclusively that the two peaks are largely associated with two distinct activities related to the ocean waves. The lower peak corresponding to the primary frequency microseisms is associated with the first order effects of wave bottom pressure modulation as sea waves propagate through a sloping beach towards the shoreline (Hinde *et al* 1965; Darbyshire, 1950; Hasselmann, 1963; Okeke, 1972) and more recently (Goodman *et al* 1989, Trevorrow *et al* 1989; Okeke and Asor 1998, Okeke and Asor 2000). On the other hand, the upper frequency peak is associated with the double microseisms. This is so called for the microseisms frequencies in this band are double that of the generating sea waves. As determined by Longuet-Higgins (1950), the wave activities involved in this regard are the second order pressure effects.

These are energized through the nonlinear interactions among progressive sea waves moving in opposite directions. The phenomena are not affected by the depth of the water layer. Consequently, they are effective generating mechanism both in deep and shallow water areas. In our studies, we have included the activities of the generating water waves approaching the shoreline from a wide range of directions. Also, our elastic medium which is the model Earth is damped and horizontally layered with the governing equations being those that describe the small amplitude oscillations in such a medium. Our study also introduces a damping term in the governing equations representing the effect of the material inelasticity which we shall assume to be slight. This enables us to adopt the model due to Darbyshire and Okeke (1969) in which the damping term in the equation of motion is assumed to be proportional to the time rate of the change of material displacement components in the first normal incident theory. Previous calculations based on this model were quite close to the measurements of seismic events in the far field.

Further, there are a number of interesting and innovative publications on the evolution of the microseisms in the seafloor. Recent achievements in this area of geophysics owe a lot to the work of Yamamoto *et al* (1977, 1978) and recently, Trevorrow *et al* (1988, 1989). Information acquired therefrom had been effectively used in the study of such areas as the structural depth profile below the seabed. We have extended this analysis with identical calculations to the far field microseisms activities. Generally, previous investigators based their models on the theory of the homogenous earth without incorporating the effect of earth's layering in the numerical calculations. In this regard therefore, we have analysed the effects of earth's layering on far field micro-earth tremors while extending the normal incident theory of Darbyshire and Okeke (1969) to two dimensions.

II. GOVERNING EQUATIONS AND THEIR SPECIFICATIONS

The *x-axis* and *y-axis* are taken as perpendicular and along the shoreline respectively. The *z-axis* points vertically downwards with $z=0$ as the earth's surface, $t>0$ is the time with $t=0$ giving the onset of the geophysical activities involved in our subsequent studies.The behaviour of an isotropic solid is completely specified if μ and λ are given (μ is the modulus of elasticity, λ is the Lamé's constant). In particular, μ defines the strength of the layered elastic solid, hence, the most important parameter in our present investigation. ρ_s is the density of solid which, in the case of horizontally stratified elastic half-space, will be a function of z. Finally, the displacement components of the elastic half-space in response to the seismic events are U, V, W in the x, y and z directions respectively.

With these specifications, the governing equations are:

$$\rho_s \frac{\partial^2 U}{\partial t^2} = (\lambda + \mu)\frac{\partial \Delta}{\partial x} + \mu \nabla^2 U = 0 \tag{2.1}$$

$$\rho_s \frac{\partial^2 V}{\partial t^2} = (\lambda + \mu)\frac{\partial \Delta}{\partial y} + \mu \nabla^2 V = 0 \tag{2.2}$$

$$\rho_s \frac{\partial^2 W}{\partial t^2} = (\lambda + \mu)\frac{\partial \Delta}{\partial z} + \mu \nabla^2 W = 0 \tag{2.3}$$

where $\Delta = \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z}, \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

Differentiate both sides of equations (2.1) with respect to x, (2.2) with respect to y and (2.3) with respect to z and add, then,

$$\frac{\partial^2 \Delta}{\partial t^2} = \alpha^2 \nabla^2 \Delta \tag{2.4}$$

where Δ defines the wave of compression which moves with the speed α where $\alpha^2 = \frac{(\lambda + 2\mu)}{\rho_s}$.

Further, differentiate both sides of equations (2.2) with respect to z and (2.3) with respect to y, then subtract, we obtain

$$\frac{\partial^2 \varpi_x}{\partial t^2} = \beta^2 \nabla^2 \varpi_x \tag{2.5}$$

$$\varpi_x = \frac{\partial W}{\partial y} - \frac{\partial V}{\partial z} \tag{2.6}$$

In the same way we obtain

$$\frac{\partial^2 \varpi_y}{\partial t^2} = \beta^2 \nabla^2 \varpi_y \tag{2.6a}$$

$$\varpi_y = \frac{\partial V}{\partial x} - \frac{\partial U}{\partial z} \tag{2.6b}$$

and

$$\frac{\partial^2 \varpi_z}{\partial t^2} = \beta^2 \nabla^2 \varpi_z \tag{2.6c}$$

$$\varpi_z = \frac{\partial W}{\partial x} - \frac{\partial U}{\partial y} \tag{2.6d}$$

The vector

$$\underline{\omega} = (\overline{\omega}_x, \overline{\omega}_y, \overline{\omega}_z) \tag{2.6e}$$

gives the wave of rotation in the elastic solid which moves with speed β where

$$\beta^2 = \frac{\mu}{\rho_s} \tag{2.6f}$$

It is to be noted however that, for surface waves, we assume the motion to be uniform with respect to the y-axis. We then introduce two scalar potentials ϕ and ψ for the displacements of the elastic solid. Thus, we have,

$$U = \frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial z}, \qquad V = \frac{\partial \phi}{\partial z} - \frac{\partial \psi}{\partial x} \tag{2.6g}$$

Using the above equation, with $\nabla_2^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}$, then

$$\Delta = \frac{\partial U}{\partial x} + \frac{\partial W}{\partial z} = \nabla_2^2 \phi \tag{2.7}$$

$$\varpi_y = \frac{\partial U}{\partial z} - \frac{\partial W}{\partial x} = \nabla_2^2 \psi \tag{2.8}$$

Equations (2.7) and (2.8) indicate that the scalar potentials ϕ and ψ are respectively related to the waves of compression and rotation. Thus, introducing (2.7) and (2.8) into (2.4) and (2.6) respectively, we obtain the wave equations in the form

$$\frac{\partial^2 \phi}{\partial t^2} = \alpha^2 \nabla_2^2 \phi \tag{2.9}$$

$$\frac{\partial^2 \psi}{\partial t^2} = \beta^2 \nabla_2^2 \psi \tag{2.10}$$

Details of the above equations are found in Bullen & Bolt (1985), Burridge (1976).

III. GRAVITY WAVES AND GROUND MOVEMENTS

A gravity wave is an oscillation caused by the displacement of an air parcel which is restored to its initial position by gravity. The lifting force is buoyancy, while the restoring force is gravity. In this consideration, we introduce a damping term into the governing equations to represent the effect of material inelasticity which we shall assume to be small since the oscillations take place near the Earth's surface (0<z<100m) and the variations in the elastic parameters are slight.

Introducing ϕ and ψ in equation (2.1) with (2.2) and using the same notations therein, we have the following system of equations:

$$P(k)e^{ik(x-ct)} = \lambda\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial z^2}\right)+2\mu\left(\frac{\partial^2\phi}{\partial z^2}-\frac{\partial^2\psi}{\partial x\partial z}\right)+\gamma\rho_s\frac{\partial}{\partial t}\left(\frac{\partial\phi}{\partial x}+\frac{\partial\psi}{\partial z}\right) \tag{3.1}$$

$$\mu\left(2\frac{\partial^2\phi}{\partial x\partial z}-\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial z^2}\right)+\gamma\rho_s\frac{\partial}{\partial t}\left(\frac{\partial\phi}{\partial z}-\frac{\partial\psi}{\partial x}\right)=0 \tag{3.2}$$

In (3.1), the term on the left hand side is the generating pressure field of the water waves; $P(k)$ being the amplitude spectrum of the bottom pressure. The wave number k and the phase speed c are such as to match those of the seismic trapped modes below the seabed. Hence, k and c will refer to both the generating water waves and the seismic response of the elastic half-space in the subsequent discussion.

The solutions of equations (3.1) and (3.2) are expressible in the form:

$$\phi(x,z,t) = A\exp[ik(rz+x-ct)] \tag{3.3}$$

$$\psi(x,z,t) = B\exp[ik(sz+x-ct)] \tag{3.4}$$

where A and B do not depend on space and time.

$$r^2 = \left(\frac{c^2}{\alpha^2}-1\right)$$

$$s^2 = \left(\frac{c^2}{\beta^2}-1\right) \tag{3.5}$$

The effect of damping term introduced in equations (3.1) and (3.2) is to make k and c complex with non-zero immaginary part. Thus, $k = k_0 + i\delta k$ and $c = c_0 + i\delta c$ but, $k_0 >> \delta k$ and $c_0 >> \delta c$

On the earth's surface and in the far field, the waveforms are free, hence, the equations (3.1) to (3.5) gives:

$$A[\beta^2(s^2-1)+\gamma c]+B[2\beta^2 s-\gamma sc]=0 \tag{3.6}$$

$$A[2r\beta^2-\gamma cs]+B[\beta^2(1-s^2)-\gamma c]=0 \tag{3.7}$$

Equations (3.6) and (3.7) above are consistent if

$$f(c)=(2\beta^2-\gamma c)^2 rs+[\beta^2(1-s^2)-\gamma c]^2=0 \tag{3.8}$$

Eliminating r and s in equation (3.8) using (3.5), then,

$$f(c)=(2\beta^2-\gamma c)^4[(\frac{\alpha_1 c}{\beta})^2-1]-[\beta^2(2-\frac{c^2}{\beta^2})-\gamma c]^4=0 \tag{3.9}$$

With ν as the medium's Poisson's constant, we introduce the following notations:

$$\alpha_1=\frac{\beta}{\alpha}=\frac{1-2\nu}{2-2\nu},\qquad \nu=\frac{\lambda}{2(\lambda+\mu)}\quad\text{and}\quad k_1=\frac{c}{\beta}$$

Thus, equation (3.9) gives

$$(2\beta-k_1\gamma)^4(\alpha_1^2k_1^2-1)-[\beta(2-k_1)-\gamma k_1]^4=0 \tag{3.10}$$

Rearranging equation (3.10) as an equation in k_1,

$$f(k_1)=k_1^6(\beta^4-\alpha_1^2\gamma^4)+4\gamma\beta k_1^5(\beta^2+2\gamma^2\alpha_1^2)-k_1^4[8\beta^4-\gamma^2\{6\beta^2(4\alpha_1^2-1)$$
$$-\gamma^2(\alpha_1^2+1)\}]+4\beta\gamma k_1^3[\gamma^2(2\alpha_1^2+1)+2\beta^2(4\alpha_1^2-3)]+k_1^2[8\beta^4(3-2\alpha_1^2)$$
$$+24\beta^2\gamma^2\alpha_1^2]+16\beta^2\gamma k_1(1-2\alpha_1)-16\beta^4(1-\alpha_1^2)=0 \tag{3.11}$$

In an undamped elastic medium ($\gamma = 0$), equation (3.11) reduces to the usual equation for the non-dispersive Rayleigh waves in elastic solid. In this case, the equation reduces to a cubic equation in k_1^2 which has been thoroughly analysed (Bullen and Bolt, 1985) to obtain the propagational properties of the surface waves for a range of values of ν. Equation (3.11) therefore, exemplifies the case of material dispersion in which k_1 and γ are coupled. So, attenuation term induces material dispersion into an otherwise non-dispersive Rayleigh surface waves in the elastic material (Okeke and Asor, 2000).

Equation (3.11) is a sixth order equation and so has six roots that are complex conjugate in the k_1-plane. It cannot be reduced to a cubic equation because it contains terms involving odd powers of k_1. However, quantitative analysis (Okeke and Asor, 2000), suggests that, $f(0) = -16\beta^2(1-\alpha_1^2) < 0$ since $0 < \alpha_1 < 1$ for surface waves. $f(1) > 0$ because $\gamma^3[\alpha_1^2(2\gamma - 4\beta) + (\gamma - 4\beta)] - 16\beta^2\gamma^2(1-\alpha_1^2) + \beta^3(4\gamma - \beta) < 0$ for each term in the bracket is negative since $\alpha_1 < 1$ and $\gamma < \frac{\beta}{4}$ because, γ is small. Thus, there is a root of 3.11 in $k_1 \in [0,1]$.

For $f(-1)$, we have

$$\beta^4 + 4\gamma\beta^3 + 6\beta^2\gamma^2(8\alpha_1^2 - 1) - 4\beta\gamma^3(4\alpha_1^2 + 1) - \gamma^4(2\alpha_1^2 + 1)$$
$$> \frac{10}{16}\beta^4 + 59\tfrac{7}{8}\beta^2\alpha_1^2\gamma^2 + 3\tfrac{31}{32}\beta^3\gamma$$
$$> 0$$

for $\gamma < \frac{\beta}{4}$ and introducing the realistic values of β and α_1.

Therefore, there is at least a root of equation (3.11) between $k_1 = 0$ and $k_1 = -1$. In brief, there are roots of equation (3.11) in the circle of unit radius $|k_1| < 1$ and none on the circumference $|k_1| = 1$.

In the studies involving surface waves, $|k_1| < 1$, so, the interest is roots of equation (3.11) in the circle. To do this, sequence $\{f_m(k_1)\}, \ m = 1,2,3,4,5$ of Sturm's function (Kurosh, 1980) are computed from the equation (3.11).

We now let $f_0(k_1) = f(k_1)$ as in equation (3.11); $f_m(k_1)$ be taken as the first derivative of $f_{n-1}(k_1), \ n = 1,2,3,4,5$. Further, let $c(0)$ be the number assigned to the changes of sign in these sequences when $k_1 = 0$. Attach an identical meaning to $c(1)$ when $k_1 = 1$. In this consideration, it is deduced that the difference $c(0) - c(1)$ in $0 < k_1 < 1$ depends on the assigned values of γ. From symmetry, identical conclusion applies for the difference $c(0) - c(-1)$ in $-1 < k_1 < 0$.

Consequently, if $0 \le \gamma \le \frac{\alpha_1^2\beta}{40}$, then $c(0) - c(1) = 1$ and there is only one real root in each of the interval $-1 < k_1 < 0$ and $0 < k_1 < 1$. In this case, propagating elastic waves are undamped. With regards to the Sturm's sequence, all the leading coefficients are positive. However, for higher values of γ in the range $\frac{\alpha_1^2\beta}{40} < \gamma < \frac{\beta}{4}$, the leading coefficients for $m = 2$ and $m = 3$ in the Sturm's sequence are negative and $c(0) - c(1) = 2$. Thus, in $|k_1| < 1$, there are four complex conjugate roots, one in each of the four quadrants of the k_1-plane. Consequently, this analysis convincingly proves that <u>seismic waves in an elastic solid are effectively damped if the attenuation coefficient γ inherent in the solid exceeds the value $\frac{\alpha_1^2\beta}{40}$</u>. Thus, it is concluded that, the effectiveness of the damping of elastic vibrations in elastic solid is a function of the strength of the solid material. Put differently, the more rigid a solid is, the greater is the damping of elastic vibrations passing through it. In practice, the upper limit of $\frac{\beta}{4}$ is never attained. In particular, the complex root in the first quadrant of k-plane for which $\mathrm{Re}(k_1) > 0,\ \mathrm{Im}(k_1) > 0$ corresponds to the observed damped seismic vibration.

We now apply this result to the microseismic signals recorded on land below which is made of fairly hard rock. With this earth's structure, the phase speed, c_0, of the seismic signal ranges from $1.1 km\sec^{-1}$ to $1.8 km\sec^{-1}$. Using the value $c_0 = 0.8\beta$, the corresponding value of γ is between $0.021 km\sec^{-1}$ to $0.04 km\sec^{-1}$. This range of values of γ is between $\frac{\alpha_1^2\beta}{40}$ and $\frac{\beta}{4}$ suggesting strongly that the microseismic signals propagating from the source to the recording station in the far field are damped appreciably.

The variation of this range of values of γ with depth is shown in Asor (2000). The uniformity of this range with depth is apparent. The calculations cover the case of the horizontally stratified earth for which the elastic parameters and density are functions of the z-co-ordinates only. Further, the calculations are confined to the shallow earth's layer below the surface.

IV. THE FREQUENCY SPECTRAL AMPLITUDE COMPONENTS OF MICROSEISMIC SIGNALS

An attempt is made to calculate the frequency spectral amplitude components of microseismic signals as functions of depth variation below the earth's surface. Identical studies had given rise to a number of useful results, (Trevorrow *et al*, 1991). However, the previous work in this direction (Yamamoto, 1978; Trevorrow *et al*, 1991) concerned seabed gravity waves induced seabed oscillations. Instead, our interest is in the far field seismic events. Thus, our model will concern the records obtained from a laboratory buried seismometer at a distance of 13km from the seashore. Extrapolating from the data for the seabed vertical profile (Trevorrow *et al*, 1989, 1991; Bullen and Bolt, 1985) of elastic shear modulus and other elastic parameters, we have calculated the corresponding density, compressional and shear wave speed respectively in the far field. Our results, Asor (2000) are in reasonable agreement with the locally observed data. In this consideration, equations (3.1) and (3.2) are to be expressible in terms of the related displacement components rather than the scalar potentials. Thus, we shall adopt the following representations:

$$U(x,z,t) = \overline{U}(z)e^{ik(x-ct)} \qquad 4.1$$

$$W(x,z,t) = \overline{W}(z)e^{ik(x-ct)} \qquad 4.2$$

Introduce 4.1 and 4.2 into 3.1 and 3.2, then, rearranging to obtain the following differential equations

$$\frac{d\overline{U}}{dz} = ik\left(\frac{c}{\beta^2}\right)\left(\gamma - \frac{\mu}{\rho_s c}\right)\overline{W} \qquad 4.3$$

$$\frac{d\overline{W}}{dz} = ik\left(\frac{c}{\alpha^2}\right)\left(\gamma - \frac{\lambda}{\rho_s c}\right)\overline{U} + \frac{P(k)}{\lambda + 2\mu} \qquad 4.4$$

Equations (4.3) and (4.4) combine to give the usual matrix form (Bullen & Bolt, 1985)

$$\frac{df}{dz} = \mathbf{Af} + \mathbf{g_0} \qquad 4.5$$

where

$$\mathbf{f} = \begin{bmatrix} \overline{U} \\ \overline{W} \end{bmatrix}; \quad \mathbf{g_0} = \begin{bmatrix} 0 \\ \frac{p(k)}{\lambda + 2\mu} \end{bmatrix}; \quad \mathbf{A} = ick\begin{bmatrix} 0 & \frac{1}{\alpha^2}\left(\gamma - \frac{\lambda}{\rho_s c}\right) \\ \frac{1}{\alpha^2}\left(\gamma - \frac{\lambda}{\rho_s c}\right) & 0 \end{bmatrix}$$

$$P(k) = \rho_w g a(k)\sec h(kd) \qquad 4.6$$

In equation (4.6), d is the depth of the shallow water layer measured from undisturbed level. Usually, in shallow water,

$$kd \to 0 \text{ and } \operatorname{sech} kd \to 1, \text{ thus,}$$
$$P(k) = \rho_w g a(k)$$

In this case, the pressure is hydrostatic being unaffected by the depth of the water layer. $a(k)$ is the amplitude spectrum of the exciting water wave. In the subsequent calculations, $a(k)$ will be expressed in terms of the observed wave periods, T rather than wave number component, k.

In a perfectly damped elastic medium where the elastic parameters and density are assumed uniform with depth, equation (4.5) can easily be integrated to give

$$\mathbf{f}(z) = (\mathbf{f_0} + \mathbf{g_0 B})e^{-\mathbf{A}z} - \mathbf{B g_0} \qquad 4.7$$
$$\mathbf{f}(z_0) = \mathbf{f_0}; \quad \mathbf{B} = \mathbf{A}^{-1}$$

where $\mathbf{f}(z_0)$ is the column matrix representative of the observed microseismic amplitude in the far field and $z = z_0$, z_0 is the depth of the burial of the seismometer fault.

Further the eigenvalues of the matrix $\mathbf{A}$ are $\pm\nu$ where

$$\nu = \frac{ick}{\alpha\beta}\left[\left(\gamma - \frac{\lambda}{\rho_s c}\right)\left(\gamma - \frac{\mu}{\rho_s c}\right)\right]^{1/2} \qquad 4.8$$

If one assumes that Poisson's relations apply (which is justified in the present case), $\alpha^2 = 3\beta^2$. Also, $c = c_0 + i\delta, \ \delta << c_0$. δ is the time decay factor representing the damping effect.

Thus,

$$\frac{1}{c} = \frac{c_0 - i\delta}{c_0^2} + O(\delta^2) \qquad 4.9$$

Using 4.9, we obtain

$$\operatorname{Re}(\nu) = \frac{k\lambda\delta^2}{\alpha\beta c_0^2 \rho_s}; \quad \operatorname{Im}(\nu) = \frac{kc_0}{\alpha\beta}\left(\gamma - \frac{\lambda}{\rho_s c_0}\right); \quad \alpha = \beta\sqrt{3} \qquad 4.10$$

$\operatorname{Re}(\nu)$ gives the variations of the vibrations with depth. The presence of δ^2 (which is quite small) in the numerator of the term suggests low rate of energy decay with depth below the earth's surface.

In a simplified case, we assume that the elastic parameters μ, λ and ρ_s are independent of the vertical co-ordinate. Equation (4.5) is now a linear first order differential equation with constant matrix coefficients. The solution given by equation (4.7) is not very efficient numerically. Instead, we propose the following solutions (Okeke and Asor, 1999),

$$\begin{bmatrix} \overline{U} \\ \overline{W} \end{bmatrix} = \frac{\delta k}{\rho_s c}\left\{ \beta^{-1}\left(\gamma - \frac{\mu}{\rho_s c}\right)\begin{bmatrix} \left(\gamma - \frac{\mu}{\rho_s c}\right) \\ 1 \end{bmatrix} e^{+\nu z} + \alpha^{-1}\left(\gamma - \frac{\lambda}{\rho_s c}\right)\begin{bmatrix} 1 \\ \alpha^{-1}\left(\gamma - \frac{\lambda}{\rho_s c}\right) \end{bmatrix} e^{-\nu z} \right\} + \mathbf{g_0 B}$$

4.11

4.11 simplified to

$$\begin{bmatrix}\overline{U}\\ \overline{W}\end{bmatrix} = \frac{\delta}{kc_0}\left\{\begin{bmatrix}b_1^2\\ 1\end{bmatrix}\sin\left(\frac{kc_0 b_1 z}{\alpha}\right) + \begin{bmatrix}1\\ b_2^2\end{bmatrix}\cos\left(\frac{kc_0 b_2 z}{\beta}\right)\right\}e^{-b_3 z} + \mathbf{g_0 B} \qquad 4.12$$

where

$$b_1 = \alpha^{-1}\left(\gamma - \frac{\mu}{\rho_s c_0}\right); \quad b_2 = \beta^{-1}\left(\gamma - \frac{\lambda}{\rho_s c_0}\right); \quad b_3 = \mathrm{Re}(\nu)$$

Equation (4.12) seems to have depicted the local pattern of the decoupled compressional and shear waves respectively; each of which is subjected to the depth decay. The decay depicted by this model is strongly dependent on the non-zero imaginary part of the phase velocity, c, introduced by the damping term γ in our fundamental equations for elastic half space.

In general, the elastic parameters,

$$\rho_s = \rho_s(z),$$
$$\mu = \mu(z) \quad \text{and}$$
$$\lambda = \lambda(z).$$

However, in the multi-layered half-space, the region below the earth's surface is structurally assumed to consist of horizontally parallel slabs in <u>welded contact</u>. The simplified situation implies that the region within each slab is homogeneous and elastic parameters constant.

We now introduce the propagator matrix $\mathbf{P(z, z_0)}$ defined in relation to the displacement column matrix $\mathbf{f(z)}$ as

$$\mathbf{f(z) = P(z, z_0)f(z_0)} \qquad 4.13$$

Thus,

$\mathbf{f(z_0) = P(z_0, z_0)f(z_0)}$ and $\mathbf{P(z, z_0)} = I$ where I is an identity $z \; x \; z$ matrix. Also,

$$\mathbf{P(z_1, z_2) = P^{-1}(z_2, z_1)}$$

which is a simple form of inverse matrix.

To determine $\mathbf{P(z, z_0)}$, we substitute equation (4.13) into the homogeneous form of equation (4.5) to obtain

$$\frac{d}{dz}P(z, z_0) = A(z)P(z, z_0) \qquad 4.14$$

The complete solution of equation (4.14) is given by

$$P(z, z_0) = \exp\left[\int_{z_0}^{z} A(z')dz'\right] \qquad 4.15$$

since $P(z_0, z_0) = I = P(z_0, z_1)P(z_1, z_0)$. Thus, $P(z_0, z_1) = P^{-1}(z_1, z_0)$ and

$$\det P(z, z_0) = \exp\left\{\int_{z_0}^{z} A^T(\xi)d\xi\right\} \qquad 4.16$$

We now obtain the solution of equation (4.5) when $f(z_0) = f_0$ is given by multiplying the equation by $P^{-1}(z, z_0)$, regarded as the integrating factor, i.e.

$P^{-1}(z,z_0)\frac{d}{dz}f(z)-P^{-1}(z,z_0)A(z)f(z)=P^{-1}(z,z_0)g_0(z)=P(z_0,z)g_0(z)$ That is,

$$\frac{d}{dz}[P^{-1}(z,z_0)f(z)]=P(z_0,z)g(z_0) \text{ or}$$

$$\frac{d}{dz}[P(z_0,z)f(z)]=P(z_0,z)g(z_0)$$

$$P(z_0,z)f(z)=\int_{z_0}^{z}P(z,\xi)g_0(\xi)d\xi+f(z_0) \qquad 4.17$$

For the boundary value problem,

$$f(z)=P(z,z_0)f(z_0)+P^{-1}(z_0,z)\int_{z_0}^{z}P(z,\xi)g_0(\xi)d\xi$$

$$=P(z,z_0)f(z_0)+\int_{z_0}^{z}P^{-1}(z_0,z)P(z,\xi)g_0(\xi)d\xi \qquad 4.18$$

But, $P^{-1}(z_0,z)P(z,\xi)=P(z,z_0)P(z,\xi)=P(z_0,\xi)$

Exchanging z and z_0,

$$P^{-1}(z,z_0)P(z_0,\xi)=P(z_0,z)P(z_0,\xi)=P(z,\xi)$$

Thus,

$$f(z)=P(z,z_0)f(z_0)+\int_{z_0}^{z}P(z,\xi)g_0(\xi)d\xi \qquad 4.19$$

But from the definition,

$$P(z,\xi)=\exp(\int_{z}^{\xi}A(y)dy$$

So, equation (5.19) becomes

$$f(z)=P(z,z_0)f(z_0)+\int_{z_0}^{z}g_0(\xi)e^{\int_{z}^{\xi}A(y)dy}d\xi \qquad 4.20$$

Next, we subdivide the shallow layer below the earth's surface into twenty parallel slabs that are in welded contact and each is of thickness 5m. Each subdivision is assumed to be homogeneous within which elastic parameters λ,μ and density, ρ are assumed to be constant. Regarding $z=z_0$ as the earth's surface, the depth of the slabs below $z=z_0$ is respectively $z=z_1,z_2,...,z_{20}$. Thus, for $z_s\le z\le z_{s+1}$, $s=1,2,...,20$

$$P(z_s,z_{s+1})=\exp[-A(z)(z_{s+1}-z_s).$$

So, for $z_s\le z\le z_{s+1}$, equation (5.20) gives

$$f(z) = P(z_s, z_{s+1})\, f(z_0) + \int_{z_s}^{z_{s+1}} g_0(z') \exp[-A(z')(z_{s+1} - z_s)] dz' \tag{4.21}$$

In numerical computation of the surface displacement components of the layer, we have used the Sylvester's interpolation formula (Bullen & Bolt, 1985) to obtain for each slab $z_s \le z \le z_{s+1}$,

$$\exp[A(z)(z_{r+1} - z_r)] = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \tag{4.22}$$

where

$P_{11} = Cosh(\nu_s h_s);\quad P_{12} = (\nu_s h_s)^{-1} Sinh(\nu_s h_s);\quad P_{21} = (\nu_s \mu_s) Sinh(\nu_s h_s);\quad P_{22} = Cosh(\nu_s h_s);$ an
$h_s = z_{s+1} - z_s$

d ν_s is the value of ν in $z_s \le z \le z_{s+1}$. Similar results also hold for μ_s.

V. WAVE INTERACTION WITH THE SHORELINE

In this consideration, we investigate the geophysical phenomenon which give rise to the intermediate frequency range of the microseismic frequency. This frequency range is constantly observed in the series of analysed gravity water waves and microseisms energy spectrum. In previous attempts, Darbyshire and Okeke (1969) proposed a model of normal incident and reflected waves on a rocky coastline. Okeke (1972, 1985) improved on this by assuming that the angle of incidence ranges from 0 to $\pi/2$. However, the reflected wave energy was neglected in the computation. Okeke and Asor (2000) finally generalized the two successful attempts. The last generalized theory is now used to study the phenomena of the observed micro-scale seismic oscillations in the range of the intermediate frequency.

In this study, the technique initiated by Darbyshire and Okeke (1969) will be exploited and further generalized. Let the subscripts i and r refer to the incident and reflected wave components along the coastline respectively. Let $\Delta k = k_i - k_r$ be the wave number difference. We now divide Δk in n sub-divisions each of width δk_p. Thus, $\Delta k = n\delta k_p$

For the incident modes, the spectral amplitudes for the sub-divisions are $h_1, h_2, ..., h_n$ and for the reflected modes, they are $g_1, g_2, ..., g_n$.

In this study, $h_i = h_i(R, \omega, \theta_i)$ and $g_r = g_r(R, \omega, \theta_r)$

The definitions incorporate the angle of incidence θ_i and that of reflection θ_r. Generally, they are usually regarded as equal. The resultant spectral amplitude is obtained by the convolution of the two spectral components. That is, $\sum_{i=1}^{n} \sum_{r=1}^{n} h_i g_r \delta_{ir}$

where $\begin{matrix} \delta_{ir} = 1 & \text{when } r = i \\ \delta_{ir} = 0 & \text{when } r \ne i \end{matrix}$ and δ_{ir} is the Kronecker delta. Physically, $r = i$ corresponds to the case of constructive interference, $r \ne i$ that of destructive interference.

This study concentrates only on the case of constructive interference, so, when, $r = i$, $g_r = h_r R_f$ where R_f is the reflection coefficient and $\bar{\theta}$ is taken as the ensemble average for angles of incidence and reflection. Thus,

$$\sum_{i=1}^{n} g_i h_i = \sum_{i=1}^{n} R_f g_i^2 \text{ and}$$

$$\sum_{i=1}^{n} R_f \, g_i^2 = R_f S_1(\omega, K_0, \bar{\theta}) \; n\delta K_p \, , \; K_0 >> K_m . \qquad 5.1$$

K_0 is the wave number of gravity water wave mode, K_m is the low wave number component of the water wave which is small enough to resonate the seismic modes of the seabed, $K = K_0 \cup K_m$, i.e. $K \in [-\infty, \infty]$. $S_1(\omega, K_0, \bar{\theta})$ is the spectral amplitude.

To a reasonable degree of accuracy, the power spectrum of a system is proportional to the square of the amplitude spectrum. Thus for $-\infty < K_0 < \infty$,

$$S_p(K_0, \omega, \bar{\theta}) = R_f S_1^2(K_0, \omega, \bar{\theta}) n\delta K_p \qquad 5.2$$

$S_p(K_0, \omega, \bar{\theta})$ is the power spectrum of the sea wave. The inequality immediately before equation (5.2) implies that both high and low phase velocity wave number components arising from the linear modulation of the gravity (water) wave bottom pressure are now activated (Hasselmann, 1963).

Our model sea wave is that which approaches a shoreline at an angle $\bar{\theta}$. Here, $\bar{\theta}$, is measured from the line normal to the shoreline. The sea bottom is uniformly sloping but not necessarily parallel to the shoreline. The constant α, is the gradient of slope. Then, following Okeke (1972, 1985), the wave bottom pressure in this study takes the form

$$P_{33} = \rho_w \; g\sqrt{\frac{d}{R}} \, J_0\left(2\omega\sqrt{\alpha\frac{R}{g}}\right)\cos\frac{\bar{\theta}}{2}\cos\omega t. \qquad 5.3$$

$0 < \bar{\theta} < \frac{\pi}{2}$, ρ_w = water density, R is the radial distance measured from the centre of the generating source, d is the width of the shelf which includes the breaking zone as measured from the shoreline, J_0 is a zero order Bessel function of the first kind, g is the acceleration due to gravity.

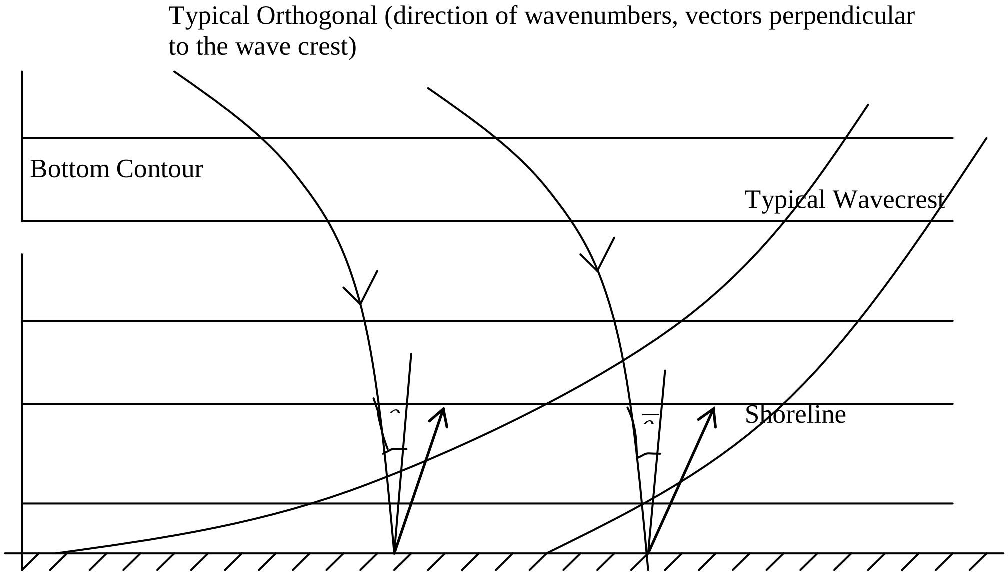

Fig.1 Reflection of waves along the shoreline….

The components of the Fourier-Bessel coefficients corresponding to equation (5.3) are

$$CH(K,\bar{\theta},\omega)=\frac{\alpha\, d^{-\frac{1}{2}}}{(K_0+K_m)^{\frac{1}{2}}}J_0\left[\frac{\omega^2\alpha\, g^{-1}}{K_0+K_m}\right]\cos\frac{\bar{\theta}}{2} \quad 5.4$$

$$SH(K,\bar{\theta},\omega)=\frac{\alpha\, d^{-\frac{1}{2}}}{(K_0-K_m)^{\frac{1}{2}}}J_0\left[\frac{\omega^2\alpha\, g^{-1}}{K_0-K_m}\right]\cos\frac{\bar{\theta}}{2} \quad 5.5$$

$$K=\begin{cases}K_0, & \text{in the range of gravity water waves.}\\ K_m, & \text{in the range of low wave number modes}\end{cases}$$

In the range of very low frequency or primary frequency modes,

$$J_0\left[\frac{\omega^2\alpha\, g^{-1}}{K_0+K_m}\right]\rightarrow J_0\left[\frac{\omega^2\alpha\, g^{-1}}{K_0-K_m}\right]\rightarrow 1 \quad 5.6$$

Okeke (1972) utilized the above approximations in the calculations involving the range of primary frequency microseisms. However, in the intermediate range, the whole expressions in the equations (5.4) and (5.5) will be used in the on-going calculations. Thus, with variation of one percent in the wave number, the amplitude spectral density is defined by

$$S_1^2(K_0,\omega,\bar{\theta})=\left[CH^2(K_0,\omega)+SH^2(K_0,\omega)\right]\cos^2\frac{\bar{\theta}}{2}=\frac{2\alpha^2 d^{-1}}{K_0}J_0^2\left[\frac{\omega\,\alpha\, g^{-1}}{K_0}\right]\mathrm{Cos}^2\frac{\bar{\theta}}{2} \quad 5.7$$

Equation (5.7) suggests that the spectral density favours the moderate breaker zone and a rather gently sloping beach. The equation gives the power of pressure wave per unit wave number in the water layer, which in the present study is inversely proportional to the gravity wave number of the exciting source, i.e. proportional to the wavelength of the exciting source (i.e. shallow water swell). This conclusion is quantitatively in agreement with the observed behaviour of microseisms and the generating sea waves.

VI. STRESS WAVES IN AN ELASTIC AND HOMOGENOUS HALF SPACE

In this section, we review the base equations governing the evolutions of stress waves in an elastic and homogeneous half space. Here, the components of the ground displacement in response to the passage of seismic oscillations are thus usually given by

$$U_R=\frac{\partial\phi}{\partial R}+\frac{\partial^2\psi}{\partial R\,\partial z} \quad 6.1$$

$$U_z=\frac{\partial\phi}{\partial z}-\frac{\partial^2\psi}{\partial R^2}-\frac{1}{R}\frac{\partial\psi}{\partial R} \quad 6.2$$

where U_R is the radial component of displacement whilst U_z is the vertical component. ϕ and ψ are still scalar potential functions associated with the compressional wave with speed α and shear wave with speed β respectively. z as before is the vertical coordinate with the related radial distance represented by R.

Take $\phi=\mathrm{Re}\{\phi_0(R,z)e^{i\omega t}\}$ and $\psi=\mathrm{Re}\{\psi_0(R,z)e^{i\omega t}\}$

then, ϕ_0 and ψ_0 respectively satisfy the following equations

$$\left(\frac{\partial^2}{\partial R^2}+\frac{1}{R}\frac{\partial}{\partial R}+\frac{\partial^2}{\partial z^2}+\frac{\omega^2}{\alpha_0^2}\right)\phi_0=0 \quad 6.3$$

$$\left(\frac{\partial^2}{\partial R^2}+\frac{1}{R}\frac{\partial}{\partial R}+\frac{\partial^2}{\partial z^2}+\frac{\omega^2}{\beta_0^2}\right)\psi_0=0 \quad 6.4$$

where $K^2=K_{\alpha_0}{}^2+\frac{\omega^2}{\alpha_0^2}=K_{\beta_0}^2+\frac{\omega^2}{\beta_0^2}$ 6.5

In the present consideration, K now refers to seismic mode wave number which, effectively, is the same as K_m in the previous section.

Using equation (6.5), the integral representations of the solutions of equations (6.3) and (6.4) are respectively:

$$\phi_0 = \int_0^{\infty} A(K) J_0(KR) K e^{-K_{\alpha 0} z} K \qquad 6.6$$

$$\psi_0 = \int_0^{\infty} B(K) J_0(KR) K e^{-K_{\beta 0} z} dK \qquad 6.7$$

$A(K)$ and $B(K)$ are wave number amplitude spectrum respectively. In this study, we are interested in the far field vertical component of the ground movements, $U_z(R,z)$ induced by the micro-scale seismic events. Consequently, introducing equations (6.6) and (6.7) into (6.2), we obtain

$$U_z(R,z,t) = \mathrm{Re}\left\{ e^{-i\omega t} \int_0^{\infty} K^2 \left[K_{\alpha_0} A(K) e^{-K_{\alpha_0} z} + K^2 B(K) e^{-K_{\beta_0} z} \right] J_0(KR) dK \right\} \qquad 6.8$$

$A(K)$ and $B(K)$ are determined using the boundary conditions at the seabed, i.e. at $z = 0$. These are

(a) The vanishing of tangential stress which gives

$$2 K_\alpha A(K) - \left(2K^2 - \frac{\omega^2}{\beta^2} \right) B(K) = 0 \qquad 6.9$$

(b) The vertical stress component is to be balanced by the generating bottom pressure associated with high phase velocity component of the generating water waves, that is

$$\left(\lambda + \rho_s \lambda' \frac{\partial}{\partial t} \right) \Delta^2 \phi + 2 \left(\mu + \rho_s \mu' \frac{\partial}{\partial t} \right) \frac{\partial U_z}{\partial z} = \int_0^{\infty} K P_{33}(\omega, R, t, \bar{\theta}) J_0(K R_0) e^{i\omega t} dK \qquad 6.10$$

In equation (6.9), the only terms not yet defined are λ' and μ'. Therefore, λ' and μ' represent the effect of imperfection in the elastic half space and indicate the extent of damping in the half space.

Solving equations (6.9) and (6.10), we obtain

$$B(K) = 2 K_\alpha \frac{P_{33}}{\rho_s \Delta(K)} ; A(K) = \frac{\left(2K^2 - \frac{\omega^2}{\beta^2} \right) P_{33}}{\rho_s \Delta(K)}$$

where

$$\Delta(K,\omega) = (\beta^2 + i\omega\lambda') \left\{ \left(2K^2 - \frac{\omega^2}{\alpha^2} \right) \left(2K^2 - \frac{\omega^2}{\beta^2} \right) - 4 K^2 K_\alpha K_\beta \right\} \qquad 6.11$$

is the Rayleigh function (Bullen and Bolt, 1985).
Take

$$F(K,\omega) = \left(\frac{\beta}{\omega} \right)^2 \Delta(K,\omega) \qquad 6.12$$

Equation (6.12) is also the Rayleigh function which is multiplied by an empirical factor of $(\beta/\omega)^2$. This factor drops out in the subsequent calculations. The exception is however in the computation of the variation of the frequency spectral width $\delta\omega(z)$ with depth where the dependence of material rigidity and frequency is more apparent.

In the areas outside the shallow water zone, $P_{33} = 0$. Thus, if equations (6.9) and (6.10) are to be consistent, $F(K,\omega)$ must vanish identically. Consequently, in verifying the wave number,

$$F(K,) = F(K',) + K\frac{\partial F}{\partial K}(K,)\rfloor_{K=K'} + (\delta K)^2 = 0 \qquad 6.13$$

From which

$$\delta K \cong \frac{-F(K',\omega)}{\frac{\partial F}{\partial K}(K',\omega)} \qquad 6.14$$

where K' is the value of K for which equation (6.13) is satisfied.

In the evaluation of the numerator of equation (6.14), we work in terms of the group velocity, V, of the seismic modes. Thus,

$$\frac{\partial F}{\partial K} = \frac{\partial F/\partial V}{\partial K/\partial V} = \frac{C_m}{K_m}\frac{\partial F}{\partial V} = 65.3\frac{\partial F}{\partial V} \qquad 6.15$$

(Here, C_m = 2.8 km sec^{-1}). When K' = 0.30km^{-1} (wave length of about 20.9km), gives

$$\delta K = 3.1 \times 10^{-4}\ \text{km}^{-1} \qquad 6.16$$

Equation (6.16) gives a result which suggests that the spectrum (energy and amplitude) of the underlying elastic solid is highly peaked.

However, wave energy or amplitude spectrum is usually calculated in terms of frequencies rather than wave numbers. Thus, if $\delta\omega$ is the frequency bandwidth shown in figure 2 below, from equation (6.13),

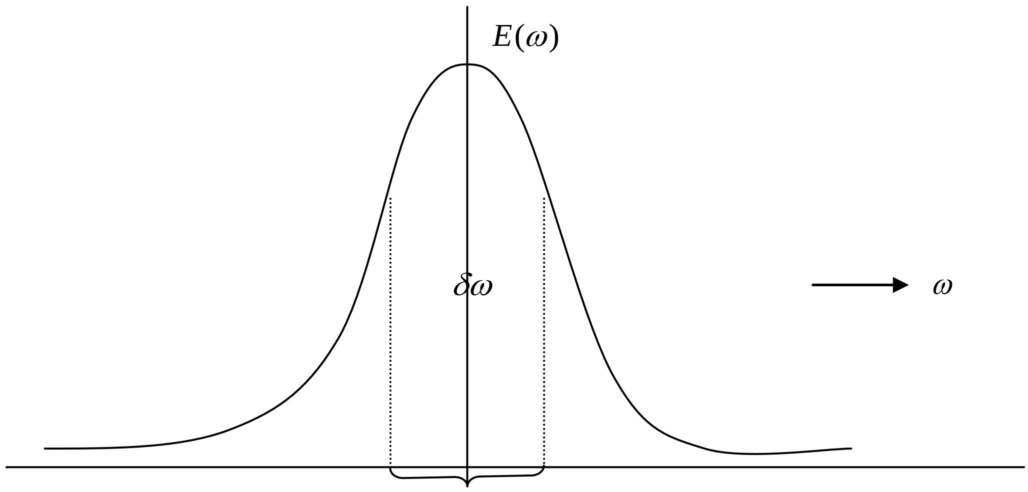

Figure 2. Sketch of Wave Energy as a function of frequency, ω, (in Hz)

$$F(K,\omega+\delta\omega) = 0 \qquad 6.17$$

we obtain approximately that,

$$F(K,\omega+\delta\omega) = F(K,\omega) + \delta\omega\frac{\partial F}{\partial\omega} + (\delta\omega)^2 = 0$$

and then, as $\delta\omega \to 0$

$$\delta\omega = \frac{-F(K,\omega)}{\partial F(k,\omega)/\partial\omega}$$

$$= \frac{k^2\beta^2}{\omega}\left[\frac{96-128\left(c/\beta\right)^2+16\left(c/\beta\right)^4-\left(c/\beta\right)^6}{256-64\left(c/\beta\right)^2+6\left(c/\beta\right)^4}\right] \qquad 6.18$$

$$= \left(\frac{4\pi^2\beta^2}{\lambda_m^2\omega_m}\right)\left(\frac{96-128\alpha_1^2+16\alpha_1^4-\alpha_1^6}{256-64\alpha_1^2+6\alpha_1^4}\right)$$

$\alpha_1 = \frac{c}{\beta} < 1,$ $\delta f = \frac{\delta\omega}{2\pi}$ (In Hz.) , λ_m and ω_m are the dominant wavelength and peak frequency respectively.
Equation (6.18) is here used in studying the shallow layer below the earth's surface. So, we have neglected the effect of λ' which gives the rate of decay of seismic vibrations in the horizontal direction.

In a horizontally stratified shallow structure below the earth's surface, $\beta = \beta(z)$ and $\rho_s = \rho_s(z)$. Therefrom, $\delta\omega = \delta\omega(z)$. Further, λ_m is about 30km and β is about 1.8km/sec in the upper earth's layer made of soft rock. Thus, despite the factor β^2 on the numerator of the right hand side of equation (6.18), $\delta\omega(z) << 1$. The inequality applies at all depths below the earth's surface over which microseismic signals are detectable. However, the factor β^2 suggests the strong dependence of $\delta\omega(z)$ on the layer rigidity and the peak wavelength λ_m.

The foregoing statement is confirmed by the numerical calculations depicting the vertical profile of $\delta\omega(z)$. The data source is the shear velocity $\beta(z)$ and density $\rho_s(z)$ vertical structures extrapolated from the reference shear wave velocity profile (Bullen and Bolt, 1985; Yamamoto and Torii, 1986; Trevorrow and Yamamoto, 1991). Thus, figure 2, compares well with the records from the local data. The computed values of δf as function of z are shown in Asor (2000).

If $z = 1.1m$ and the period is 8seconds, then, $(\delta\omega)^2 = 16.9x10^{-8}(rad\sec^{-1})^2$. These data are those frequently used for theoretical calculations involving the peak energy of the solid vertical displacement in response to the passage of the seismic events. Hence, $z = 1.1m$ suggests the likely depth of burial of a land-based seismometer. Calculations from equation (6.8) further verify that $\delta\omega(z)$ is a decreasing function of the material rigidity for $\delta\omega = O\left(\frac{1}{\alpha_1^2 T_m^2}\right)$, where T_m is the period of the peak, $T_m = 2\pi/\omega_m$.

Consequently, the computed values of δf as function of z depicts the form of the vertical structure of the elastic medium to a depth of about $100m$ below the earth's surface in the locality (Trevorrow *et. al.*, 1989).

Now,

$\mu' = \gamma\alpha^2 = \gamma\left(\frac{\lambda+2\mu}{\rho_s}\right)$, $\lambda' = \frac{\gamma\mu}{\rho_s}$ for the damping coefficient, γ (Okeke, 1972).

Eventually, equation (6.8) takes the form

$$U_z(R,\omega)\exp(-0.11\gamma\, t) = 2\int_0^{\infty}\left[\frac{K\, J_0\ \ (KR)P_{33}(K,\omega,\theta)}{\rho_s F(K,\omega)}\right]dK \qquad 6.19$$

$K = K_0 \cup K_m, \quad 20km^{-1} \le K_0 \le 100\,km^{-1}, \quad 0.1 \le K_m \le 0.4km^{-1}$

For large R, we use the asymptotic form of $J_0(KR)$ which is

$$J_0(KR) = \left(\frac{2}{\pi\, KR}\right)^{\frac{1}{2}} cos\left(KR - \frac{\pi}{4}\right) \qquad 6.20$$

Applying the stationary phase method in seismology (Ewing *et. al.*, 1957) to equation (6.19) using equation (6.20), then,

$$U_z(R,\omega)\exp(-0.11\gamma\, t) = \frac{2\, P_{33}\ (K_0,\omega)}{\rho_s}\left(\frac{2}{\pi\, R}\right)^{\frac{1}{2}}\sum_k\left(\frac{\sqrt{K}}{\partial F/\partial K}\right)\delta(K - K_m) \qquad 6.21$$

where $\delta(K - K_m)$ is the delta function. $\sum_k$ implies that the summation is over all possible values of k in the spectrum. However, the contribution to equation (6.21) will come from those values of K that are the roots of $F(K,\omega) = 0$.

We now evaluate the amplitude spectrum in the K-plane for the left and right hand sides of equation (6.21). The convolution theorem applied to Hankel's transform is used to evaluate the power associated with product on the right-hand side. However, in terms of the amplitude spectrum,

$$S_u(K,\omega,\theta) = S_p(K_0,\omega,\theta)H(K_p,\omega) \qquad 6.22$$

$H(K_p,\omega)$ is the spectrum of the transfer function obtained from

$$\left(\frac{2}{\pi\, R}\right)^{\frac{1}{2}}\sum_k\left(\left(\frac{\sqrt{K}}{\partial F/\partial K}\right)\delta(K - K_p)\right)$$

Using a sampling property of the delta function with support at $K = K_p$,

$$H^2(K_p,\omega) = \left[\frac{2K}{\pi R}\left(\frac{\partial F}{\partial K}\right)^{-2}\right]_{K = K_p} \qquad 6.23$$

$$\frac{\partial F}{\partial K} = 4K\left(\frac{\beta}{\omega}\right)^2\left(\beta^2 + i\omega\lambda'\right)\left[4K^2 - \omega^2\left(\frac{1}{\beta^2} + \frac{1}{\alpha^2}\right) - 2K_\alpha K_\beta \qquad 6.24$$

with K_α as the wave number associated with compressional wave and K_β that associated with wave of rotation.

The spectrum expressed by equation (6.23) is strongly peaked when $K_p = K_m$ with δK as the width. However, due to the damping factor, the spectral height is still finite and inversely proportional to R.

Equation (5.7) which gives the spectral density contains d, the shelf width. Because the goal of this study is the quantitative evaluation of the gravity waves (water) induced seismic activities in the far field, a realistic estimate of d as a function of wave period is necessary. In this consideration, we take $\Delta\omega$ as the angular frequency change between two successive maxima in the spectra of the incident and reflected beach waves. Using some of the relations for the shallow water sea waves, the characteristic linear wave speed, $c_0 = \sqrt{gh_0}$, h_0 being the depth of the water layer measured from the undisturbed free surface, $\omega^2 = K_0^2 c_0$. Thus,

$\Delta K_0 = \frac{\Delta\omega}{\sqrt{gh_0}} = 0.0081K_0 = \frac{0.051}{L_0}$; Δk_0 is the corresponding change in K_0 between two successive maxima in the wave number spectrum, $L_0 = 2\pi / K_0$

With $\omega = 2\pi f$, where $f = 0.11Hz,\ c_0 = 15m\sec^{-1},\ h_0 = 22m,\ \Delta K_0 = 1.4x10^{-4}km^{-1}$, correspondingly, $d = 0.002\pi / \Delta K_0$. The relationship is one of the most acclaimed outcome of this study. It strongly suggests that the shelf width varies linearly as the wave period or wavelength of the propagating swell. If wave period is $8s$, then, $d = 45km$. The value agrees with that obtained by computing the orthogonal spacing, the corresponding group velocity, V_g, and thence, the wave bottom pressure. The data are from a refraction diagram for an $8s$ water wave (Darbyshire and Okeke, 1969; Kinsman, 1965). In this study, d is the distance from the shoreline (seaward) where the wave bottom pressure is appreciable enough to contribute significantly to the generation of microseisms in the shallow water zone.

VII. DISCUSSION AND CONCLUSION

Equation (6.22) computes the relative energies of microseisms and associated sea waves with R now assigned the value of 13km. This represents the average distance of a seismometer on the land measured from the ocean bottom seismic source, near the coastline. In previous calculations R_f was taken to be 1/30 corresponding to that of Savarensky and others at Lake Yussi-Kul (Darbyshire and Okeke, 1969). In their investigation, the coastline was assumed to be rocky. However, in this study, theoretical calculation (Jackson, 1962) gives the mean value of R_f =1/38. This value allows for the finite angle of incident and reflection; thus, it seems more realistic. The calculations from this study are shown in Okeke and Asor (2000). On the whole, this model represents an improvement on our two previous investigations and further suggests that the phenomenon of wave reflection along coastlines contributes significantly to the spectral distortions observed in the intermediate frequency range of the spectrum. Finally, this theoretical model concerned the problem of the microseismic wave field generated by the activities of random pressure waves acting on the fluid/solid interface. The microseisms originating from this process propagate to the far field recording station in the form of guided elastic surface waves as expressed by equation (6.8). Along this guide, it is assumed that the mean elastic parameters are generally constant. However, any slight variation associated with these is reasonably accounted for by the introduction of damping factors in the governing equations for the elastic modes.

In addition however, the denominator of each of equations (6.19) and (6.23) contains $\rho_s(z)$ and $\beta_0(z)$, hence the energy ratio of microseismic and gravity waves will depend on the depth below the earth's surface. Consequently, we now divide the region below the earth’s surface into 20 parallel subdivisions. These are given by $z = 1,2,5,10,\ldots,100m$. Using the vertical earth’s structure as data input and finite difference method, the calculations which resulted in the energy ratio are repeated at each subdivisions for the specified wave periods. The calculations were simplified by the replacement of the quantity $\frac{\partial F}{\partial V}$ in equation (5.3) by $\frac{\partial F}{\partial z} / \frac{\partial V}{\partial z}$ and also assuming that the layer between two subdivisions is homogeneous.

We thus show that the layers with low shear strength generally corresponds to those with high energy ratio. Consequently, the energy ratios are apparently decreasing function of the depth below the earth’s surface. This development is more at depths below 70m. Our calculations further suggests that the energy ratio is vanishingly small at about a depth of 100m and below.

We also mention that, because of the presence of $\rho_s(z)$ in the denominator of the energy density ratio, the depth variation of the latter does not closely follow that of the spectral bandwidth. In the range of the low and intermediate frequency, appreciable microseisms are generated by the high phase velocity components of the seafloor pressure fluctuations associated with the propagating shallow water gravity waves. It is not in doubt that these components pressure modes possess sufficient energy adequate enough to effectively resonate the seismic modes within the seabed considering the intense wave activity that frequently dominates the shallow water areas. Equation (6.22) governs this process with $S_p(K_0, \omega, \bar{\theta})$ as the functional representative of the water wave energy spectrum. On the other hand, $H_p(K_0, \omega)$ is the coupling function whose role is to communicate the gravity wave energy to the seismic modes.

However, the double frequency microseisms are not related to the linear modulation of the seafloor randomly distributed seawave bottom pressure fluctuations. Instead, the energy input in this case is derived from non-linear interactions among the components of the seawaves. The amplitude and energy spectra of the interacting seawaves in shallow water need to be derived. Successful attempts had been made by Darbyshire and Okeke (1969). More promising is the model developed by Okeke (1978). This is a one-dimensional solution and it only needs a generalization to two-dimensions to produce the desired result.As already stated, the computed results arising from this study are nearer to the observed than previous attempts. However, these results could be significantly improved if the energy of the second order wave effects is incorporated.

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