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Modification of matrix method in computing equation of A line between two points

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ABSTRACT: *There are many applications of determinant in computation. This paper gives modified matrix method in computing equation of a line between two points. The method proofs that if two slopes are collinear, then the slopes can be expressed in determinant form of a matrix with any arbitrary positive integer on the first row.*

KEYWORDS: *collinear, determinant.*

I. INTRODUCTION

Equation of a straight line follows a matrix pattern because of its gradient. With this notion we can trace the necessity of matrix to equation of straight line between two points. Thus, matrix has a deeper root in geometry. In [1], sometimes we are given the gradient, m, of a straight line passing through a given point (x_1, y_1) and we are required to find its equation. In this case, we use the form $y - y_1 = m(x - x_1)$
In other word, from [2], [3], Let $A = (x_1, y_1)$ be one of the given points and $B = (x_2, y_2)$ be the other point, let $C = (x, y)$ be any other point on the line between A and B. Then,
Slope of $AC = \frac{y-y_1}{x-x_1}$ and Slope of $AB = \frac{y_2-y_1}{x_2-x_1}$

Since A, B, C are collinear then the two slopes $(AC \text{ and } AB)$ are equal.
Hence, the equation of a straight line between two points is $\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$ (1)

In [4], the determinant formula for equation of a straight line is $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$ (2)

From equation (2), I decided to let each row contains identical (similar) coordinates and 1(I also discovered that an arbitrary number 2,3,4 or higher number can also be used) as elements in the first row.
The modified matrix formula is given as

$$\begin{vmatrix} 1 & 1 & 1 \\ x & x_1 & x_2 \\ y & y_1 & y_2 \end{vmatrix} = 0 \quad \text{or} \quad \begin{vmatrix} 2 & 2 & 2 \\ x & x_1 & x_2 \\ y & y_1 & y_2 \end{vmatrix} = 0 \quad \text{or} \quad \begin{vmatrix} 3 & 3 & 3 \\ x & x_1 & x_2 \\ y & y_1 & y_2 \end{vmatrix} = 0 \qquad (3)$$

Picking the matrix with arbitrary 1(matrix with other arbitrary number can also be used) and getting the determinant, we will have

$$+1\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} - 1\begin{vmatrix} x & x_2 \\ y & y_2 \end{vmatrix} + 1\begin{vmatrix} x & x_1 \\ y & y_1 \end{vmatrix} = 0 \qquad (4)$$

It is important to note that if one picks other matrix with arbitrary number different from one, one will get the same equation if fully expressed.
We will proof that our modified matrix formula is equal to the conventional formula of the equation of a straight line
From equation (1) we have $\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$, cross multiply
$(y - y_1)(x_2 - x_1) = (x - x_1)(y_2 - y_1)$
Expand to have
$y(x_2 - x_1) - y_1(x_2 - x_1) = x(y_2 - y_1) - x_1(y_2 - y_1)$
$y(x_2 - x_1) - y_1x_2 + x_1y_1 = x(y_2 - y_1) - x_1y_2 + x_1y_1$

$$yx_2 - yx_1 - y_1x_2 + \cancel{x_1y_1} - xy_2 - xy_1 + x_1y_2 - \cancel{x_1y_1} = 0$$

$yx_2 - yx_1 - y_1x_2 - xy_2 - xy_1 + x_1y_2 = 0$
By grouping we have $(x_1y_2 - xy_1) - (xy_2 - yx_2) + (xy_1 - yx_1) = 0$ (5)

We could see that $(x_1y_2 - xy_1) = \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}$, $(xy_2 - yx_2) = \begin{vmatrix} x & x_2 \\ y & y_2 \end{vmatrix}$ and $(xy_1 - yx_1) = \begin{vmatrix} x & x_1 \\ y & y_1 \end{vmatrix}$

Hence, equation (5) is equal to equation (4)

That is, $(x_1y_2 - xy_1) - (xy_2 - yx_2) + (xy_1 - yx_1) = +\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} - \begin{vmatrix} x & x_2 \\ y & y_2 \end{vmatrix} + \begin{vmatrix} x & x_1 \\ y & y_1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ x & x_1 & x_2 \\ y & y_1 & y_2 \end{vmatrix}$

(6)

Examples

We will consider the following examples from [5]

1.) Find the equation of the straight line joining the points (2,4) and (-3,5)

Solution

Let $(x_1, y_1) = (2,4)$; $(x_2, y_2) = (-3,5)$ and (x, y) be any point that divides the two points

Using conventional method, we have $\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$

$$\frac{y-4}{x-2} = \frac{5-4}{-3-2}$$

$$\frac{y-4}{x-2} = \frac{1}{-5}$$

$$-5(y-4) = 1(x-2)$$

$$-5y + 20 = x - 2$$

Thus, $x + 5y = 22$

Using the modified matrix method, we have

$$+\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} - \begin{vmatrix} x & x_2 \\ y & y_2 \end{vmatrix} + \begin{vmatrix} x & x_1 \\ y & y_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} 2 & 4 \\ -3 & 5 \end{vmatrix} - \begin{vmatrix} x & -3 \\ y & 5 \end{vmatrix} + \begin{vmatrix} x & 2 \\ y & 4 \end{vmatrix} = 0$$

$$10 + 12 - 5x - 3y + 4x - 2y = 0$$

$$-x - 5y + 22 = 0$$

Thus, equation of the line is $x + 5y = 22$

2.) Compute the equation of the straight line joining the points (2,3) and (6,-3)

Solution

Let $(x_1, y_1) = (2,3)$; $(x_2, y_2) = (6,-3)$ and (x, y) be any point that divides the two points

Using conventional method, we have $\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$

$$\frac{y-3}{x-2} = \frac{-3-3}{6-2}$$

$$\frac{y-3}{x-2} = \frac{-6}{4}$$

$$4(y-3) = -6(x-2)$$

$$4y - 12 = -6x + 12$$

Thus, $4y + 6x = 24$

Using the modified matrix method, we have

$$+\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} - \begin{vmatrix} x & x_2 \\ y & y_2 \end{vmatrix} + \begin{vmatrix} x & x_1 \\ y & y_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} 2 & 3 \\ 6 & -3 \end{vmatrix} - \begin{vmatrix} x & 6 \\ y & -3 \end{vmatrix} + \begin{vmatrix} x & 2 \\ y & 3 \end{vmatrix} = 0$$

$$-6 - 18 + 3x + 6y + 3x - 2y = 0$$

$$4y + 6x - 24 = 0$$

Thus, $4y + 6x = 24$

II. CONCLUSION

In conclusion, I showed that equation of a straight line can be completely represented by matrix and solved by determinant method. That is, combinations of slopes that are collinear actually represent determinants

REFERENCES

[1]. K.A. Stroud, engineering mathematics (New York, Palgrave Publisher Limited, 2001).
[2]. Kalejaiye A.O., et al, further mathematics 2 (Longman Nigeria Plc, 1991).
[3]. Tuttuh Adegun, et al, further mathematics project 2 (Ibadan, NPS Publisher, 2009).
[4]. www.en.m.wikipedia.org/wiki/linear_equation
[5]. Femi Ajayi, et al, introductory university mathematics (Lagos, JAS Publisher, 1998).