International Journal of Mathematics and Statistics Invention (IJMSI)
E-ISSN: 2321 – 4767 P-ISSN: 2321 - 4759
www.ijmsi.org Volume 2 Issue 7 || July. 2014 || PP-28-36

To Determine Optimal Manpower Recruitment Size in Stochastic Model

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ABSTRACT: *In any organization the required staff strength is maintained through new recruitments. The exit of personnel from an organization is a common phenomenon, which is known as wastage. Many stochastic models dealings with wastage are found in Bartholomew and Forbes (1979). In this chapter it is assumed that persons are recruited for training and after training they are allotted to the group of working staff. Whenever the vacancy arises in the group of working staff then the trained individual are employed. At the same time if trained person are not put to employment, it results in greater expenditure because they are idle. The demand for persons at the works spot is of random character and highly fluctuating. So the optimal size to be recruited and trained is found out. In doing so the concept of setting the clock back to zero property by Raja Rao and Talwaker (1990) is used to depict the random variable which denotes the demand for manpower at the work spot.*

KEY WORDS: *Wastage, Recruitments, Stochastic model.*

I. INTRODUCTION

In any organization the required staff strength is maintained through new recruitments. The exit of personnel from an organization is a common phenomenon, which is known as wastage. Many stochastic models dealing with wastage are found in Bartholomew and Forbes (1979). A salient feature of the investigation is to determine the optimal length of time (0,T) and this cycle length is obtained under some specific assumptions using the concept of cumulative damage process of the reliability theory. For a detailed description and analysis of shock models one can refer to Ramanarayan (1977) who analyzed the system exposed to a cumulative damage process of shock. The use of compartment models in manpower planning is quite common.Consider a system which has two compartments C_1 and C_2. The size of C_1 is fixed as n. Transition of persons from C_1 to C_2 is allowed and in between there is a screening test to evaluate the competence of individuals to get into C_2. The compartment C_2 may be thought of as one consisting of person with greater skills, efficiency and administrative capabilities. These qualities are evaluated by screening test. The persons in C_1 are first recruited and kept in the reserved list. Assuming that they are given some training to improve their capabilities, keeping these persons in C_1 and training them involves a maintenance cost or reserve cost. Conducting the test, but with no persons getting entry to C_2 involve some cost namely screening test cost which is a total loss. In case no person gets selected and enters into C_2, the vacancies in C_2 remain unfilled and each such unfilled vacancy gives rise to some shortage cost in terms of loss of productivity. To make good this loss, recruitment of persons from outside to compartment C_2 is made on an emergency basis. The longer the time interval between the screenings tests the greater will be the cost of maintenance of persons in C_1 which turn increases the cost of shortage in C_2. Frequent screening tests result in higher test costs. With a view to minimize the above said costs, the optimal time interval namely T between successive screening tests is attempted here. The result has been applied on some special cases of distribution.

II. ASSUMPTIONS

[1] During the time interval (0,T) the demand for manhours is taken to be a random variable.
[2] Recruitment of personnel results in the generation of manpower in terms of manhours.
[3] Excess of manpower in terms of manhours realized by recruitments as well as shortage results in monetary loss for the organization.
[4] The distribution representing the demand undergoes a parametric change.

NOTATION

- c_1 : Cost of excess manpower per hour.
- c_2 : Cost of shortage of manpower per hour.
- X : A continuous random variable denoting the demand for manpower at the work spot in terms of manhours.
- F (.) : The p.d.f of X with corresponding c.d.f denoted as F(.).
- x_0 : A constant that denotes the truncation point of X.
- S : The number of manhours realized by recruitments and training.
- $\hat{S}$: The optimum value of S.

III. RESULTS

The expected cost per period (0, T) is given by

Based on SCBZ property,

If X $\sim f(x,\theta);\ x < x_0$(ie) $f(x,\theta) = \theta e^{-\theta x}$

$x \sim f(x,\theta^*); x > x_0$ (ie) $f(x,\theta^*) = \theta^* e^{-x\theta^*} e^{-x_0}$

Instead of threshold follows exponentiated exponential (EE) made an attempt to EE distribution

g $(x_0) = 2\lambda e^{-\lambda x_0}\,(1 - e^{-\lambda x_0})$. Let x_0 be a random variable with p .d .f g (.) then two cases

visualized.

Case (i) $S \geq x_0$

Let x_0 be a constant such that $S \geq x_0$, and the expected cost $E_1(C)$ is given by

$$E_1(C) = \int_0^\infty [C_1 \int_0^{x_0} (s-x) f(x,\theta)dx + C_1 \int_{x_0}^{s} (s-x) f(x,\theta^*)dx + C_2 \int_s^\infty (x-s) f(x,\theta^*)dx]\, g(x_0)d(x_0)$$

$$I_1 = C_1 \int_0^{x_0} (s-x) f(x,\theta)dx$$

$$I_2 = C_1 \int_{x_0}^{s} (s-x) f(x,\theta^*)dx$$

$$I_3 = C_2 \int_s^\infty (x-s) f(x,\theta^*)dx]$$

$$A = \int_0^\infty I_1 g(x_0)d(x_0)$$

$$B = \int_0^\infty I_2 g(x_0)d(x_0)$$

$$C = \int_0^\infty I_3 g(x_0)d(x_0)$$

$E_1(C) = \int_0^\infty I_1 g(x_0)d(x_0) + \int_0^\infty I_2 g(x_0)d(x_0) + \int_0^\infty I_3 g(x_0)d(x_0)$

$$E_1(C) = \frac{dA}{dS} + \frac{dB}{dS} + \frac{dC}{dS}$$

$$A = \int_0^\infty I_1 g(x_0)d(x_0)$$

$$I_1 = C_1 \int_0^{x_0} (s - x) f(x, \theta) dx$$

$$I_1 = C_1 \int_0^{x_0} (s - x)\, \theta e^{-\theta x}\, dx$$

After simplification, we get

$$I_1 = C_1 \left[S - (S - x_0)e^{-\theta x_0} + \frac{e^{-\theta x_0} - 1}{\theta} \right]$$

$$A = \int_0^\infty I_1 g(x_0)d(x_0)$$

$$A = \int_0^\infty \left\{ C_1 \left[S - (S - x_0)e^{-\theta x_0} + \frac{e^{-\theta x_0} - 1}{\theta} \right] \right\} . [\, 2\lambda e^{-\lambda x_0} - 2\lambda\, e^{-2\lambda x_0}\,] .d(x_0)$$

After simplification, we get

$$A = -C_1 \left[-s + \frac{2S\lambda}{\theta + \lambda} - \frac{2S\lambda}{(\theta + 2\lambda)} - \frac{2\lambda}{(\theta + \lambda)^2} + \frac{2\lambda}{(\theta + 2\lambda)^2} + \frac{1}{\theta}\left(1 - \frac{2\lambda}{\theta + \lambda} + \frac{2\lambda}{\theta + 2\lambda}\right) \right]$$

$$B = \int_0^\infty I_2 g(x_0)d(x_0)$$

$$I_2 = \; = C_1 \int_{x_0}^{s} (s - x) f(x, \theta^*) dx$$

$$I_2 = C_1 \int_{x_0}^{s} (s - x)\theta^* e^{-\theta^* x} e^{-x_0(\theta - \theta^*)} dx$$

After simplification, we get

$$I_2 = C_1\, e^{-x_0(\theta - \theta^*)} \left[\frac{e^{-\theta^* x}}{\theta^*} + e^{-\theta^* x_0} \left((S - x_0) - \frac{1}{\theta^*} \right) \right]$$

$$B = \int_0^\infty I_2 g(x_0)d(x_0)$$

$$B = \int_0^\infty C_1\, [\, e^{-x_0(\theta - \theta^*)} \left(\frac{e^{-\theta^* x}}{\theta^*} + e^{-\theta^* x_0} \left((S - x_0) - \frac{1}{\theta^*} \right) \right) [\, 2\lambda e^{-\lambda x_0} - 2\lambda e^{-2\lambda x_0}]. d(x_0)$$

After simplification, we get

$$B = 2\lambda C_1\left[\frac{e^{-\theta^* s}}{\theta^*[(\theta - \theta^*) + \lambda} - \frac{1}{(\theta+\lambda)^2} - \frac{S}{\theta+\lambda} - \frac{1}{\theta^*(\theta+\lambda)} - \frac{e^{-\theta^* s}}{\theta^*[(\theta-\theta^*) + 2\lambda} + \frac{1}{(\theta+2\lambda)^2} + \frac{S}{\theta+2\lambda} - \frac{1}{\theta^*(\theta+2\lambda)}\right]$$

$$C = \int_0^\infty I_3\, g(x_0) d(x_0)$$

$$I_3 = C_2 \int_s^\infty (x - s) f(x, \theta^*) dx]$$

$$I_3 = C_2 \int_S^\infty (x - S)\theta^* e^{-\theta^* x} e^{-x_0(\theta - \theta^*)} dx$$

After simplification, we get

$$I_3 = C_2 e^{-x_0(\theta-\theta^*)} \frac{e^{-\theta^* x}}{e^*}$$

$$C = \int_0^\infty I_3\, g(x_0) d(x_0)$$

$$C = C_2 \int_0^\infty e^{-x_0(\theta-\theta^*)} \frac{e^{-\theta^* x}}{e^*}[2\lambda e^{-\lambda x_0} - e^{-2\lambda x_0}] d(x_0)$$

After simplification, we get

$$C = \frac{2\lambda C_2 e^{-\theta^* s}}{\theta^*[(\theta - \theta^x) + \lambda]} - \frac{2\lambda C_2 e^{-\theta^* s}}{\theta^*[(\theta - \theta^x) + 2\lambda]}$$

$$E_1(C) = \frac{dA}{dS} + \frac{dB}{dS} + \frac{dC}{dS}$$

$$\frac{dA}{dS} = \frac{d}{dS}[-C_1\left[-s + \frac{2S\lambda}{\theta+\lambda} - \frac{2S\lambda}{(\theta+2\lambda)} - \frac{2\lambda}{(\theta+\lambda)^2} + \frac{2\lambda}{(\theta+2\lambda)^2} + \frac{1}{\theta}\left(1 - \frac{2\lambda}{\theta+\lambda} + \frac{2\lambda}{\theta+2\lambda}\right)\right]]$$

$$\frac{dA}{dS} = \frac{2c_1\lambda}{\theta+2\lambda} - \frac{2c_1\lambda}{\theta+\lambda} + C_1$$

$$\frac{dB}{dS} = \frac{d}{dS}\left[2\lambda C_1\left[\frac{e^{-\theta^* s}}{\theta^*[(\theta-\theta^*) + \lambda} - \frac{1}{(\theta+\lambda)^2} - \frac{S}{\theta+\lambda} - \frac{1}{\theta^*(\theta+\lambda)} - \frac{e^{-\theta^* s}}{\theta^*[(\theta-\theta^*) + 2\lambda} + \frac{1}{(\theta+2\lambda)^2} + \frac{S}{\theta+2\lambda} - \frac{1}{\theta^*(\theta+2\lambda)}\right]\right]$$

$$\frac{dB}{dS} = 2\lambda C_1 e^{-\theta^* s}\left[\frac{1}{(\theta-\theta^*) + 2\lambda} - \frac{1}{(\theta-\theta^*) + \lambda}\right] + 2\lambda C_1\left[\frac{1}{\theta+2\lambda} - \frac{1}{\theta+\lambda}\right]$$

$$\frac{dC}{dS} = \frac{d}{dS}\left[\frac{2\lambda C_2 e^{-\theta^* s}}{\theta^*[(\theta-\theta^x) + \lambda]} - \frac{2\lambda C_2 e^{-\theta^* s}}{\theta^*[(\theta-\theta^x) + 2\lambda]}\right]$$

$$\frac{dC}{dS} = \frac{-2\lambda C_2 e^{-\theta^* s}}{(\theta - \theta^*) + \lambda} + \frac{2\lambda C_2 e^{-\theta^* s}}{(\theta - \theta^*) + 2\lambda}$$

$$E_1(C) = \frac{dA}{dS} + \frac{dB}{dS} + \frac{dC}{dS}$$

$$E_1(C) = C_1 + 4\lambda C_1 \left[\frac{1}{\theta + 2\lambda} - \frac{1}{\theta + \lambda}\right] + \left[\frac{2\lambda e^{-S\theta^*}}{(\theta - \theta^* + 2\lambda)} - \frac{2\lambda e^{-S\theta^*}}{(\theta - \theta^* + \lambda)}\right](C_1 + C_2)$$

IV. NUMERICAL ILLUSTRATION

On the basis of the numerical illustration under the assumption that $s>x_0$& the truncation point x_0 follows exponentiated exponential (EE) distribution the following conclusions given

TABLE 1

$\hat{S}$	$E_1(C)$
0.02	396.91
0.04	393.65
0.06	390.22
0.08	386.61
0.1	382.84

From the above the table we concluded if the truncation point x_0 follow EE distribution if $s>x_0$ then the solution of the equation θ and S increase with fixed value of C_1 = 100 and C_2 = 200 and for all the other values $E_1(c)$ decreases

Case (ii) $S < x_0$

Let x_0 be a constant such that $S < x_0$, and the expected cost $E_2(C)$ is given by

$$E_2(C) = \int_0^\infty [C_1 \int_0^s (s-x) f(x,\theta) dx + C_2 \int_s^{x_0} (x-s) f(x,\theta) dx + C_2 \int_{x_0}^\infty (x-s) f(x,\theta^*) dx]\, g(x_0) d(x_0)$$

$$A_1 = C_1 \int_0^s (s-x) f(x,\theta) dx$$

$$B_2 = C_2 \int_s^{x_0} (x-s) f(x,\theta) dx$$

$$D_1 = C_2 \int_{x_0}^\infty (x-s) f(x,\theta^*) dx]$$

$$I_1 = \int_0^\infty A_1\, g(x_0) d(x_0)$$

$$I_2 = \int_0^\infty B_1\, g(x_0) d(x_0)$$

$$I_3 = \int_0^{\infty} D_1\, g(x_0) d(x_0)$$

$$E_2\,(C) = \int_0^{\infty} A_1\, g(x_0) d(x_0) + \int_0^{\infty} B_1\, g(x_0) d(x_0) + \int_0^{\infty} D_1\, g(x_0) d(x_0)$$

$$E_2\,(C) = \frac{dI_1}{ds} + \frac{dI_2}{ds} + \frac{dI_3}{ds}$$

$$I_1 = \int_0^{\infty} A_1\, g(x_0) d(x_0)$$

$$A_1 = C_1 \int_0^{s} (s - x) f(x, \theta)\, dx$$

$$A_1 = C_1 \int_0^{s} (s - x)\, \theta e^{-\theta x}\, dx$$

After simplification, we get

$$A_1 = C_1 \left[\frac{e^{-\theta s}}{\theta} + \; s - \frac{1}{\theta} \right]$$

$$I_1 = \int_0^{\infty} A_1\, g(x_0) d(x_0)$$

$$I_1 = C_1 \int_0^{\infty} \left\{ \frac{e^{-\theta s}}{\theta} + s - \frac{1}{\theta} \right\} . [\; 2\lambda e^{-\lambda x_0} - 2\lambda\, e^{-2\lambda x_0}\;] . d(x_0)$$

After simplification, we get

$$I_1 = C_1 \left[\frac{1 - e^{-\theta s}}{\theta} - s \right]$$

$$I_2 = \int_0^{\infty} B_1\, g(x_0) d(x_0)$$

$$B_1 \; = C_1 \int_s^{s_0} (x - s) f(x, \theta)\, dx$$

$$B_1 \; = C_1 \int_s^{s_0} (x - s) \theta e^{-\theta x} dx$$

After simplification, we get

$$B_1 \; = C_1 \left[-(x_0 - s) e^{-\theta x_0} - \frac{e^{-\theta x_0}}{\theta} + \frac{e^{-\theta s}}{\theta} \right]$$

$$I_2 = \int_0^{\infty} B_1\, g(x_0) d(x_0)$$

$$I_2 = C_2 \int_0^{\infty} \left[-(x_0 - s) e^{-\theta x_0} - \frac{e^{-\theta x_0}}{\theta} + \frac{e^{-\theta s}}{\theta} \right] . [\, 2\lambda e^{-\lambda x_0} - \; 2\lambda e^{-2\lambda x_0}] . d(x_0)$$

After simplification, we get

$$I_2 = 2C_2\left[\frac{e^{-\theta s}}{\theta} + \frac{\lambda}{(\theta+\lambda)^2} + \frac{S\lambda}{\theta+\lambda} - \frac{\lambda}{\theta\ (\theta+\lambda)} - \frac{e^{-\theta s}}{2\theta} - \frac{s\lambda}{2\lambda+\theta} + \frac{\lambda}{(\theta+2\lambda)^2} + \frac{\lambda}{\theta(\theta+2\lambda)}\right]$$

$$I_3 = \int_0^\infty D_1\, g(x_0) d(x_0)$$

$$D_1 = C_2 \int_{x_0}^{\infty} (x-s) f(x,\theta^*) dx]$$

$$D_1 = C_2 \int_{x_0}^{\infty} (x-S)\theta^* e^{-\theta^* x} e^{-x_0(\theta-\theta^*)} dx$$

After simplification, we get

$$D_1 = C_2 e^{-x_0\theta}.\ [x_0 - s + \frac{1}{\theta^*}]$$

$$I_3 = \int_0^\infty D_1\, g(x_0) d(x_0)$$

$$I_3 = C_2 \int_0^\infty e^{-x_0\theta}.[x_0 - s + \frac{1}{\theta^*}]\ [2\lambda e^{-\lambda x_0} - e^{-2\lambda x_0}] d(x_0)$$

After simplification, we get

$$I_3 = 2C_2\left[\frac{\lambda}{(\theta+\lambda)^2} - \frac{S\lambda}{\theta+\lambda} - \frac{\lambda}{\theta^*\ (\theta+\lambda)} + \frac{s\lambda}{2\lambda+\theta} + \frac{\lambda}{(\theta+2\lambda)^2} + \frac{\lambda}{\theta^*(\theta+2\lambda)}\right]$$

$$E_2(C) = \frac{dI_1}{ds} + \frac{dI_2}{ds} + \frac{dI_3}{ds}$$

$$\frac{dI_1}{ds} = \frac{d}{ds}\left[C_1\left[\frac{1-e^{-\theta s}}{\theta} - s\right]\right]$$

$$\frac{dI_1}{ds} = C_1\left[e^{-\theta s} - 1\right]$$

$$\frac{dI_2}{ds} = \frac{d}{ds}\left[2C_2\left[\frac{e^{-\theta s}}{\theta} + \frac{\lambda}{(\theta+\lambda)^2} + \frac{S\lambda}{\theta+\lambda} - \frac{\lambda}{\theta\ (\theta+\lambda)} - \frac{e^{-\theta s}}{2\theta} - \frac{s\lambda}{2\lambda+\theta} + \frac{\lambda}{(\theta+2\lambda)^2} + \frac{\lambda}{\theta(\theta+2\lambda)}\right]\right]$$

$$\frac{dI_2}{ds} = 2\lambda C_2\left[\frac{1}{\theta+\lambda} - \frac{1}{\theta+2\lambda}\right] - C_1 e^{-\theta s}$$

$$\frac{dI_3}{ds} = \frac{d}{ds}\left[2C_2\left[\frac{\lambda}{(\theta+\lambda)^2} - \frac{S\lambda}{\theta+\lambda} - \frac{\lambda}{\theta^*\ (\theta+\lambda)} + \frac{s\lambda}{2\lambda+\theta} + \frac{\lambda}{(\theta+2\lambda)^2} + \frac{\lambda}{\theta^*(\theta+2\lambda)}\right]\right]$$

$$\frac{dI_3}{ds} = 2C_2\lambda\left[\left[\frac{1}{\theta+2\lambda} - \frac{1}{\theta+\lambda}\right]\right]$$

$$E_2(C) = \frac{dI_1}{ds} + \frac{dI_2}{ds} + \frac{dI_3}{ds}$$

$$E_2(C) = C_1\left[e^{-\theta s} - 1\right] + 2\lambda C_2\left[\frac{1}{\theta+\lambda} - \frac{1}{\theta+2\lambda}\right] - C_1 e^{-\theta s} + 2C_2\lambda\left[\left[\frac{1}{\theta+2\lambda} - \frac{1}{\theta+\lambda}\right]\right]$$

$$E_2(C) = e^{-\theta s}(C_1 - C_2) - C_1$$

Any value of s which satisfies above equation for a given value of C_1, C_2 and θ is the optimal S namely $\hat{s}$

V. NUMERICAL ILLUSTRATION

On the basis of the numerical illustration under the assumption that $s<x_0$& the truncation point x_0 follows exponentiated exponential (EE) distribution the following conclusions given,

TABLE 2

$\hat{S}$	$E_2(C)$
1	136.78
2	101.83
3	100.01
4	100
5	100

TABLE 3

$\hat{S}$	$E_2(C)$
0.2	196.07
0.4	185.21
0.6	169.76
0.8	152.72

If θ increases S increases for fixed value of $C_1 = 100$ $C_2 = 200$, $E_2(C)$ decrease as a small inventory of manpower is suggested (table2 & 3).

TABLE 4

$\hat{S}$	$E_2(C)$
1	336.78
2	301.83
3	300.01
4	300
5	300

TABLE 5

$\hat{S}$	$E_2(C)$
0.2	396.07
0.4	385.21
0.6	369.76

0.8	352.72

If θ increases S increases for fixed value of $C_1 = 300$ $C_2 = 400$, $E_2\ (C)$ decrease as a small inventory of manpower is suggested (table 5 & 6).

TABLE 6

$\hat{S}$	$E_2\ (C)$
1	536.78
2	501.83
3	500.01
4	500
5	500

TABLE 7

$\hat{S}$	$E_2\ (C)$
0.2	596.07
0.4	585.21
0.6	569.76
0.8	552.72

If θ increases S increases for fixed value of $C_1 = 500$ $C_2 = 600$, $E_2\ (C)$ decrease as a small inventory of manpower is suggested (table 6 & 7).

From the above all the table we concluded if the truncation point $\mathbf{x_0}$ follow EE distribution if $S<x_0$ then the solution of the equation θ and S increase with C_1 increases and C_2 increases and for all the other values $E_2\,(c)$ decreases.

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