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On πgr - Homeomorphisms in Topological Spaces.

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ABSTRACT: *The purpose of this paper is to introduce and study the concept of πgr -closed maps , πgr - homeomorphism ,πgrc - homeomorphism and obtain some of their characterizations.*

KEYWORDS: *πgr-closed map, πgr-open map,πgr-homeomorphism and πgrc- homeomorphism.*
Mathematics subject classification: *54A05, 54D10.*

I. INTRODUCTION

Levine [9]introduced the concept of generalized closed sets in topological spaces and a class of topological space called $T_{1/2}$-space. The concept of π-closed sets in topological spaces was initiated by Zaitsav[18] and the concept of πg-closed set was introduced by Noiri and Dontchev[4]. N.Palaniappan[16] studied and introduced regular closed sets in topological spaces. Generalized closed mappings, wg-closed maps ,regular closed maps and rg-closed maps were introduced and studied by Malghan[13],Nagaveni[14],Long[11] and Arokiarani[1] respectively.Maki et al [12] who introduced generalized homeomorphism and gC-homeomorphism which are nothing but the generalizations of homeomorphism in topological spaces. Devi et al [3]defined and studied generalized semi-homeomorphism and gsc homeomorphism in topological spaces. In 2013,Jeyanthi.V and Janaki.C [6] introduced and studied the properties of πgr-closed sets in topological spaces. Here we introduce and study the concepts of πgr- homeomorphisms ,πgrc -homeomorphism and their relations.

II. PRELIMINARIES

Throughout this paper, X , Y and Z denote the topological spaces (X,τ),(Y,σ) and (Z,η) respectively, on which no separation axioms are assumed. Let us recall the following definitions.

Definition:2.1
A subset A of a topological space X is said to be
[1] a semi -open [10] if $A\subset cl(int(A))$ and semi-closed if $int(cl(A))\subset A$
[2] a regular open[16] if A = int (cl(A)) and regular closed if A = cl(int(A))
[3] π- open [18] if A is the finite union of regular open sets and the complement of π- open set is π- closed set in X.
The family of all open sets [regular open, π-open, semi open] sets of X will be denoted by O(X)(resp. RO(X), πO(X), SO(X)]

Definition:2.2
A map f: X→Y is said to be
[1] continuous [10]if $f^{-1}(V)$ is closed in X for every closed set V in Y.
[2] Regular continuous (r-continuous) [16]if $f^{-1}(V)$ is regular-closed in X for every closed set
[3] V in Y.
[4] An R-map[2] if $f^{-1}(V)$ is regular closed in X for every regular closed set V of Y.
[5] πgr-continuous[7,8] if $f^{-1}(V)$ is πgr-closed in X for every closed set V in Y.
[6] πgr-irresolute[7,8] if $f^{-1}(V)$ is πgr-closed in X for every πgr -closed set V in Y.

Definition :2.3
A space X is called a πgr-$T_{1/2}$ space [7,8]if every πgr-closed set is regular closed.

Definition:2.4
A map f: X→Y is called
1.closed [13]if f(U) is closed in Y for every closed set U of X.

2.almost closed[17] if f(U) is closed in Y for every regular closed set U of X.
3.regular closed [11]if f(U) is regular closed in Y for every closed set U of X
4.rc-preserving [15]if f(U) is regular closed in Y for every regular closed set U of X.

Definition:2.5[6]
Let f : (X,τ)→(Y,σ) be a map. A map f is said to be
[1] πgr -open if f(U) in πgr-open in Y for every open set U of X.
[2] strongly πgr-open map (M-πgr-open)if f(V) is πgr-open in Y for every πgr-open set V in X.
[3] quasi πgr-open if f(V) is open in Y for every πgr-open set V in X.
[4] almost πgr-open map if f(V) is πgr-open in Y for every regular open set V in X.

Definition:2.6
A bijection f:X→ Y is called a homeomorphism [12]if f is both continuous and open.((i.e), f & f^{-1} are continuous)

III. πGR - HOMEOMORPHISMS

Definition:3.1
A bijection f:X→ Y is called
[1] πgr - homeomorphism if f is both πgr- continuous and πgr - open.((i.e), f & f^{-1} are πgr -continuous)
[2] πgrc - homeomorphism if f and f^{-1} are πgr- irresolute.

Proposition :3.2
If a mapping f : X → Y is πgr -closed, then for every subset A of X, πgr- cl f(A) ⊂ f(cl(A))

Proof:
Suppose f is πgr -closed and let A ⊂ X .Then f(cl(A)) is πgr - closed in (Y, σ). We have f(A) ⊂ f(cl(A)). Then πgr -cl(f(A)) ⊂ πgr -cl [f(cl(A))] = f(cl(A))
⇒ πgr -cl (f(A)) ⊂ f(cl(A))

Theorem :3.3
Let f : X → Y and g : Y → Z be two mappings such that their composition g ∘ f : X → Y be a πgr - closed map.Then
[1] f is continuous and surjective, then g is πgr- closed.
[2] g is πgr- irresolute and injective, then f is πgr - closed.
[3] f is πgr- continuous, surjective and X is a πgr-$T_{1/2}$- space, then g is πgr - closed.

Proof :
(i)Let V be a closed set of Y. Since f is Continuous, $f^{-1}(V)$ is closed in X. Since (g ∘ f) is πgr -closed in Z, (g ∘ f) ($f^{-1}(V)$) is πgr- closed in Z.
⇒ g(f($f^{-1}(V)$)) = g(V) is πgr - closed in Z.(Since f is surjective)

ie, for the closed set V of Y, g(V) is πgr- closed in Z.
⇒ g is a πgr - closed map.

(ii)Let V be a closed set of X .Since (g ∘ f) is πgr - closed ,(g ∘ f) (V) is πgr- closed in Z. Since g is πgr - irresolute,g^{-1}[(g ∘ f)(V)] is πgr - closed in Y.
⇒ g^{-1}[g(f(V))] is πgr closed in Y

⇒ f(V) is πgr - closed in Y. Hence f is a πgr - closed map.

(iii)Let V be a closed set of Y
Since f is πgr - continuous, $f^{-1}(V)$ is πgr - closed in X for every closed set V of Y.Since X is πgr -$T_{1/2}$- space, $f^{-1}(V)$ is regular closed in X and hence closed in X. Now, as in (i), g is a πgr- closed map.
(iv)Let V be a closed set of Y
Since f is πgr - continuous, $f^{-1}(V)$ is πgr- closed in X.
Since X is πgr -$T_{1/2}$- space, $f^{-1}(V)$ is regular closed in X and hence closed in X.
Now, the proof as in (i), g is a πgr- closed map.

Proposition :3.4
Let $f : X \to Y$ and $g : Y \to Z$ be πgr - closed maps and Y is a πgr-$T_{1/2}$- space, then their composition $g \circ f$: $X \to Z$ is a πgr - closed map.

Proof :
Let $f : X \to Y$ be a closed map. Then for the closed set V of X, f(V) is πgr - closed in Y. Since Y is a πgr -$T_{1/2}$ space, f(V) is regular closed in Y and hence closed in Y.Again, since g is a πgr - closed map, g(f(V)) is πgr - closed in Z for the closed set f(V) of Y .

$\Rightarrow$ $(g \circ f)$ (V) is πgr - closed in Z for the closed set V of X .

$\Rightarrow$ $(g \circ f)$ is a πgr -closed map.

Proposition :3.5
Let $f : (X, \tau) \to (Y, \sigma)$ be a closed map and $g : (Y, \sigma) \to (Z,\eta)$ be a πgr - closed map, then their composition $g \circ f : (X, \tau) \to (Z,\eta)$ is πgr - continuous.

Proof : Let V be a closed set of X. Since f is a closed map, f (V) is closed in Y.
Again, since g is a πgr - closed map, g(f(V)) is a πgr - closed in Z .

$\Rightarrow$ $(g \circ f)$ (V) is πgr - closed in Z for the closed set V of X .

$\Rightarrow$ $(g \circ f)$ is πgr - closed map.

Proposition:3.6
Let $f : X \to Y$ be a πgr- closed map, $g : Y \to Z$ be a closed map, Y is πgr -$T_{1/2}$- space, then their composition $(g \circ f)$ is a closed map.
Proof :
Let V be a closed set of X. Since f is a πgr - closed map, f(V) is πgr - closed in Y for every closed set V of X. Since Y is a πgr -$T_{1/2}$- space, f(V) is regular closed hence closed in Y.Since g is a closed map, then g(f(V)) is closed in Z.

$\Rightarrow$ $(g \circ f)$ (V) is closed in Z for every closed set V of X and hence $(g \circ f)$ is a closed map.

Remark:3.7
a)Homeomorphism and πgr -homeomorphism are independent concepts.
b)Homeomorphism and πgrc -homeomorphism are independent concepts.

Example:3.8
(For both (a) and (b))
(i)Let X= { a,b,c}=Y,τ = { ϕ, X, {b} {b,c} {a,b}},σ = { ϕ ,Y,{a},{b},{a,b},{a,c}}.Let $f : X \to Y$ be an identity map. Here the inverse image of open subsets in Y are πgr-open in X and for every open set U of X, f(U) is πgr-open in Y. Hence f is a πgr - homeomorphism .Also,f and f^{-1} are πgr-irresolute and hence f is a πgrc-homeomorphism.
But inverse image of open subsets in Y are not open in X and inverse image of open set U in X is not open in Y. Hence f is not a homeomorphism . Thus πgr-homeomorphism and πgrc-homeomorphism need not be a homeomorphism.
(ii)Let X={a,b,c,d}=Y, τ ={ ϕ ,X,{c},{d},{c,d},{b,d},{a,c,d},{b,c,d},σ ={ ϕ ,Y,{a},{d},{a,d}, {c,d},{ a,c,d},{a,b,d}}.Let $f : X \to Y$ be defined by f(a) = b, f(b) = c, f(c) =a, f(d) = d. Here the inverse image of open sets in (Y, σ) are open in (X, σ) and the image of open sets in X are open in Y. Hence f is a homeomorphism .But the inverse image of open sets in (Y, σ) are not πgr-open in (X, σ) and also the image of open sets in X are not πgr-open in Y. Hence f is not a πgr - homeomorphism . Also, here f and $f^{-1}$are not πgr-irresolute and hence not a πgrc-homeomorphism.

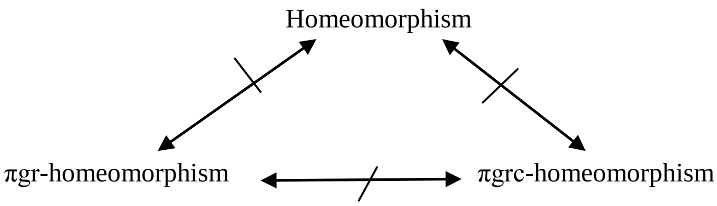
Remark:3.9
The concepts of πgrc - homeomorphism and πgr- homeomorphism are independent.

Example:3.10
a)Let X ={a,b,c}=Y,τ = { ϕ ,X,{b},{a,b}},σ={ ϕ ,Y,{b}}.Let $f : X \to Y$ be an identity map.

Here the both f and f^{-1} are πgr- irresolute and not πgr –continuous . Hence πgrc - homeomorphism need not be a πgr -homeomorphism.

b) Let X ={a,b,c}=Y, τ = { ϕ ,X,{a},{b},{a,b}},σ={ ϕ ,Y,{b}}. Let f : X → Y be an identity map. Here the both f and f^{-1} are πgr- continuous and not πgr- irresolute . Hence πgr -homeomorphism need not be a πgrc-homeomorphism.

The above discussions are summarized in the following diagram:



Remark :3.11

We say the spaces (X,τ) and (Y,σ) are πgr -homeomorphic (πgrc-homeomorphic)if there exists a πgr-homeomorphism(πgrc- homeomorphism) from (X, τ) onto (Y, σ) respectively . The family of all πgr-homeomorphism and πgrc-homeomorphisms are denoted by πgrh(X, τ) and πgrch(X, τ).

Proposition :3.12

For any bijection f : (X, τ) →(Y, σ), the following statements are equivalent.

[1] f is a πgr - open map

[2] f is a πgr - closed map

[3] f^{-1} : Y → X is πgr - continuous .

Proof :

(i) ⇒ (ii) :- Let f be a πgr - open map. Let U be a closed set in X. Then X – U is open in X

By assumption, f(X - U) is πgr - open in Y.
ie, Y – f(X – U) = f(U) is πgr - closed in Y. ie, for a closed set U in X, f(U) is πgr - closed in Y.Hence f is a πgr - closed map.

(ii) ⇒ (i) :- let V be a closed set in X .By (ii), f(V) is πgr - closed in Y and f(V) = $(f^{-1})^{-1}(V)$

⇒ $f^{-1}(V)$ is πgr - closed in Y for the closed set V in Y

⇒ f^{-1} is πgr -continuous.

(iii) ⇒ (ii) :- let V be open in X .By (iii), $(f^{-1})^{-1}(V)$ = f(V) ie, f(V) in πgr - open in Y

Hence f is a πgr -open map.

Proposition :3.13

Let f : X→Y be a bijective πgr- continuous map. Then the following are equivalent.

[1] f is a πgr -open map.

[2] f is a πgr- homeomorphism.

[3] f is a πgr - closed map.

(i) ⇒ (iii) also (iii) ⇒ (i)

f is a πgr- closed map ⇒ f^{-1} is πgr - continuous.

Then by part (i) and by the above argument together implies f is a homeomorphism and hence (ii) holds.

Proposition : 3.14

For any bijection f : (X, τ)→(Y, σ) the following statements are equivalent.

[1] f^{-1} : Y →X is πgr - irresolute .

[2] f is an M- πgr- open map .
[3] f is a M- πgr- closed map.

Proof : (i) ⟹ (ii) :Let U be a πgr - open set in Y .

By (i), $(f^{-1})^{-1}(U) = f(U)$ is πgr - open in Y.
ie, For the πgr - open set U, f(U) is πgr- open in Y
⟹ f is an M - πgr -open map.

(ii) ⟹(iii): Let f be an M- πgr- open map

let V be πgr - closed set in X .Then X – V is πgr - open in X. Since f is an M- πgr- open map, f(X – V) is πgr- open in Y..
ie, f(X – V) = Y – f(V) is πgr - open in Y .ie, f(V) is πgr-closed in Y and hence f is an M- πgr- closed map.
(iii) ⟹ (i): let V be πgr- closed in X .By (iii), f(V) is πgr - closed in Y.Since f^{-1} is Y ⟶ X be a mapping and is a bijection. Again we say that for f(V), πgr - closed in Y, its inverse image $(f^{-1})^{-1}$ (V) is πgr - closed in Y.Hence f^{-1} is πgr- irresolute .

Remark : 3.15
Composition of two πgr -homeomorphisms need not be a πgr -homeomorphism.

Example:3.16
Let X = Y = Z={a,b,c},τ={ ϕ ,X,{a},{b},{a,b}},σ = { ϕ ,Y,{a},{a,b}},η={ ϕ ,Z,{c}}. Let us define the mapping f: X→Y by f(a) = b, f(b) = a, f(c) = c and g:Y→Z by g(a) = b,g(b) = a,
g(c) = c. Here f and g are πgr-homeomorphisms but (g ∘ f) is not πgr-continuous and not πgr-open.
ie, $(gof)^{-1}$ {c} = {c} is not πgr -open in X
Hence composition of two πgr - homeomorphism is not always be a πgr-homeomorphism.

Theorem:3.17
The composition of two πgrc - homeomorphism is a πgrc-homeomorphism .
Proof :
let f : (X, τ) →(Y, σ) and g : (Y, σ) → (Z,η) be two πgrc - homeomorphic functions.
Let F be a πgr - closed set in Z. Since g is a πgr - irresolute map, $g^{-1}(F)$ is πgr - closed in (Y, σ).Since f is a πgr - irresolute map, $f^{-1}(g^{-1}(F))$ is πgr - closed in X.
⟹ $(gof)^{-1}$ (F) is πgr - closed in X

⟹ (gof) is πgr - irresolute.

Let G be a πgr - closed set in (X,τ).Since f^{-1} is πgr - irresolute, $(f^{-1})^{-1}(G)$ is πgr - closed in (Y, σ).ie, f(G) is πgr - closed in (Y, σ)
Since g^{-1} is πgr - irresolute, $(g^{-1})^{-1}$ (f(G)) = g(f(G)) is πgr- closed in Z
∴ g(f(G)) = (gof) (G) is πgr - closed in Z.
⟹ $(gof)^{-1}$ (G) is πgr - closed in Z.

This shows that $(gof)^{-1}$: Y → Z is πgr - irresolute.
Hence (gof) is πgrc- homeomorphism.

Theorem :3.18
Let (Y, σ) be πgr-$T_{1/2}$- space. If f : X → Y and g : Y → Z are πgr- homeomorphism, then g ∘ f is a πgr- homeomorphism.

Proof:
If f : X → Y and g : Y → Z be two πgr- homeomorphism. Let U be an open set in (X, τ). Since f is πgr- open map, f(U) is πgr- open in Y.
Since Y is a πgr-$T_{1/2}$- space, f(U) is regular open in Y and hence open in Y.
Also, since g is πgr- open map, g(f(U)) is πgr- open in Z.

Hence (gof) (U) = g([f(U)] is πgr- open in Z for every open set U of X.

⇒ (g ∘ f) is a πgr- open map.

Let U be a closed set in Z .
Since g is πgr- continuous, $g^{-1}(U)$ is πgr- closed in Y.
Since Y is a πgr-$T_{1/2}$- space, every πgr- closed set in Y is regular closed in Y and hence closed in Y.

⇒ $g^{-1}(V)$ is regular closed in Y and hence closed in Y.

Since f is πgr - continuous, $f^{-1}[g^{-1}(V)]$ is πgr- closed set in X
$(gof)^{-1}$ (V) is πgr-closed in X for every closed set V in Z.

⇒ (gof) is πgr-continuous and hence (gof) is a πgr-homeomorphism.

Remark: 3.19
Even though πgr- homeomorphism and πgrc- homeomorphism are independent concepts, we have the following results(theorem 3.20 and theorem 3.21)

Theorem:3.20
Every πgr- homeomorphism from a πgr-$T_{1/2}$- space into another πgr-$T_{1/2}$- space is a homeomorphism.

Proof :
let f : X → Y be a πgr - homeomorphism. Then f is bijective, πgr- open and πgr- continuous map. Let U be an open set in (X, τ).Since f is πgr- open and Y is πgr-$T_{1/2}$- space, f(U) is πgr- open in Y.Since Y is a πgr-$T_{1/2}$- space, every πgr-open set is regular open in Y

⇒ f(U) is Regular open and hence open in Y.

⇒ f is an open map.

Let Y be a closed set in (Y, σ). Since f is πgr- continuous, $f^{-1}(V)$ is πgr-closed in X. Since X is a πgr-$T_{1/2}$ -space, every πgr - closed set is regular closed and hence closed in X.
Therefore, f is continuous.
Hence f is a homeomorphism.

Theorem:3.21
Every πgr- homeomorphism from a πgr-$T_{1/2}$- space into another πgr-$T_{1/2}$- space is a πgrc- homeomorphism.
Proof :
Let f : X → Y be a πgr - homeomorphism
Let U be πgr- closed in Y. Since Y is a πgr-$T_{1/2}$- space, every πgr-closed set is regular closed and hence closed in Y.

⇒ U is closed in Y.

Since f is πgr- continuous, $f^{-1}(U)$ is πgr- closed in X.
Hence f is a πgr- irresolute map.
Let U be πgr- open set in X.
Since X is a πgr-$T_{1/2}$- space, U is Regular open and hence open in X.
Since f is a πgr- open map, f(U) is πgr- open set in Y.
$(f^{-1})^{-1}$ = f ie, $(f^{-1})^{-1}$ (U) = f(U) is πgr- open in Y
Hence inverse image of (f^{-1}) is πgr- open in Y for every πgr- open set U of X and hence f^{-1} is πgr- irresolute.
Hence f is πgrc- homeomorphism.

Remark :3.22
Here, we shall introduce the group structure of the set of all πgrc- homeomorphism from a topological space (X, τ) onto itself and denote it by πgrch-(X,τ).

Theorem :3.23
The set πgrch-(X,τ) is a group under composition of mappings.

Proof :

We know that the composition of two πgrch(X,τ) is again a πgrch(X,τ).ie, For all f, g ∈ πgrch(X,τ), $g \circ f \in$ πgrch(X, τ).We know that the composition of mappings is associative, the identity map belongs to πgrch(X,τ) acts as an identity element. If f∈ πgrch(X,τ),then $f^{-1} \in$ πgrch(X, τ) such that $f \circ f^{-1} = f^{-1} \circ f = I$ and so inverse exists for each element of πgrch(X, τ).
Hence πgrc- homeomorphism (X, τ) is a group under the composition of mappings.

Theorem :3.24
Let f : (X, τ) →(Y, σ) be a πgrc- homeomorphism. Then f induces an isomorphism from the group πgrch(X,τ) onto the group πgrch(Y, σ).

Proof :
We define a map,f_* : πgrch(X,τ) → πgrch(Y, σ) by $f_*(k) = f \circ k \circ f^{-1}$ every k ∈ πgrch(X,τ)
Then f_* is a bijection and also for all $k_1, k_2 \in$ πgrc- homeomorphism (X, τ)

$$f_*(k_1 \circ k_2) = f \circ (k_1 \circ k_2) \circ f^{-1} = (f \circ k_1 \circ f^{-1}) \circ (f \circ k_2 \circ f^{-1}) = f_*(k_1)\ o\ f_*(k_2)$$

Hence f_* is a homeomorphism and so it is an isomorphism induced by f.

Theorem :3.25
πgrc-homeomorphism is an equivalence relation in the collection of all topological spaces.

Proof :
Reflexivity and symmetry are immediate and transitivity follows from the fact that the composition of πgr-irresolute maps is πgr-irresolute.

Proposition :3.26
For any two subsets A and B of (X, τ)
[1] If A ⊂ B, then πgr- cl (A) ⊂ πgr- cl (B)
[2] πgr- cl (A∩B) ⊂ πgr- cl (A) ∩ πgr- cl (B)

Theorem :3.27
If f : (X, τ) → (Y, σ) is a πgrc- homeomorphism and suppose πgr-closed set of X is closed under arbitrary intersections, then πgr-cl(f^{-1}(B)) = f^{-1}(πgr- cl(B) for all B ⊂ Y.

Proof :
Since f is a πgrc-homeomorphism, f and f^{-1} are πgr - irresolute.
Since f is π-irresolute, πgr - cl (f(B)) is a πgr - closed set in (Y, σ), f^{-1}[πgr-cl (f(B)] is πgr - closed in (X, τ).
Now, f^{-1}(B) ⊂ f^{-1} (πgr - cl f(B))
and πgr - cl (f^{-1}(B)) ⊂ f^{-1} (πgr - cl(B)) → ①
Again, since f is a πgrc- homeomorphism, f^{-1} is πgr -irresolute. Since πgr - cl (f^{-1}(B)) is πgr - closed in X, $(f^{-1})^{-1}$ [πgr - cl (f^{-1}(B))]= f (πgr - cl (f^{-1}(B)) is πgr - closed in Y.
Now, B ⊂ $(f^{-1})^{-1}(f^{-1}$(B))
⊂ $(f^{-1})^{-1}$ (πgr - cl (f^{-1}(B))
= f (πgr - cl (f^{-1}(B))
So, πgr - cl (B) ⊂ f (πgr - cl (f^{-1}(B))
∴ f^{-1}(πgr -cl(B)) ⊂ πgr - cl (f^{-1}(B)) → ②
From ① & ②, the equality πgr-cl(f^{-1}(B)) = f^{-1}(πgr- cl(B) holds and hence the proof.

Corollary :3.28
If f : X → Y is a πgrc-homeomorphism, then πgr - cl (f(B)) =f(πgr - cl (B)) for all B ⊂ X.

Proof :
Since f : X → Y is a πgrc- homeomorphism, f^{-1} : Y→ X is a πgrc- homeomorphism.
By previous theorem,
πgr - cl (($(f^{-1})^{-1}$(B)) = $(f^{-1})^{-1}$ (πgr -cl(B)) for all B ⊂ X
πgr -cl (f(B)) = f(πgr -cl(B))

Corollary :3.29

If f : X →Y is a πgrc - homeomorphism, then f(πgr - int (B)) = πgr -int(f(B)) for all B ⊂ X

Proof :

For any set B ⊂ X, πgr -int (B) = $[\pi gr - cl(B^C)]^C$

By previous corollary, we obtain

f (πgr -int (B)) = f $[\pi gr - cl(B^C)^C]$

= $[f(\pi gr - cl(B^C)]^C$

= $[\pi gr - cl(f(B^C)]^C$

= $[\pi gr - cl(f(B^C)]^C$

= πgr - int(f(B))

Corollary :3.30

If f : X → Y is a πgrc- homeomorphism, then f^{-1}(πgr -int (B)) = πgr -int(f^{-1}(B)) for all B ⊂ Y

Proof :

If f^{-1} : Y → X is also a πgrc - homeomorphism, the proof follows by using corollary 3.29.

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