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Topological Entropy of Golden Mean Lookalike Shift Spaces

Khundrakpam Binod Mangang
Department of Mathematics & Computer Science, Mizoram University, Tanhril, Aizawl,Mizoram,India-796004.

ABSTRACT: *The aim of this paper is to calculate entropy of shift spaces which are lookalike to golden mean shift space. The golden mean shift space has alphabet of two members. We consider golden mean lookalike shifts by taking the alphabets having more than two members. We have found some interesting result that shift space having even number of alphabets have same entropy. Whereas shift space having odd numbers of alphabets are divided into two groups and each group has distinct entropy.*

KEYWORDS: *topological entropy, symbolic dynamics, irreducible matrix, golden mean shift, graph.*
AMS *subject classifications. 37B10, 37B40, 37B30, 37C25, 37M99.*

I. INTRODUCTION

We calculate the entropies of the shift space which lookalike the golden mean shift space. Before defining what a golden mean lookalike shift space is, we define briefly the terminology of symbolic dynamical systems. ***Alphabet*** is a finite set *A* of symbols. Elements of *A* are called *letters* and they will be typically denoted by *a*,*b*,*c*,..., or sometimes by digits like 0,1,2,3,… Although in real life sequences of symbols are finite, it is often extremely useful to treat long sequences as infinite in both directions (or bi-infinite) i.e., $\ldots x_{-2}x_{-1}x_0x_1x_2\ldots$ where $x_i \in \{a, b, c, \ldots\}$ or $\{0,1,2,3,\ldots\}$. The collection of all such sequence is denoted by A^Z , where Z denotes the set of integers.A ***block*** **(** ***or word*** **)** over *A* is a finite sequence of symbols from *A*. We will write blocks without separating their symbols by commas or other punctuation, so that a typical block over *A* = {*a*,*b*} looks like *aababbabbb*. It is convenient to include the sequence of non-symbols, called the *empty block (or empty word)* and denoted by $\mathcal{E}$. The central block of length $2k+1$ is denoted by $x_{[-k,k]}$, which is defined by $x_{[-k,k]} = x_{-k}x_{-k+1}\ldots x_0x_1\ldots x_{k-1}x_k$ .A collection of blocks over *A*, which are not allowed to occur in a subset X of A^Z are called ***Forbidden block*** of the subset X. A ***Shift map*** "σ" on full shift A^Z maps a point ***x*** to a point ***y*** = σ(***x***) whose i^{th} co-ordinate is $y_i = x_{i+1}$ .A***Shift space*** is a subset *X* of a full shift A^Z such that $X = X_F$, where *F* is a collection of forbidden blocks over A^Z. These forbidden blocks may be finite or infinite, but atmost countable. Full shift *X* is A^Z, where we can take *F* = ϕ, which means there are no constraints. Let *A* = { *e, f, g*} and let *X* be the set of point in the full *A*- shift for which *e* can be followed only by *e* or *f*, *f* can be followed only by *g* and *g* can be followed only by *e*. Therefore the forbidden block is$F = \{eg, gg, gf, fe\}$. If *X* is the set of all binary sequences, i.e., the alphabet is *A* = { 0,1} , so that between any two 1's there are an even number of 0's. Here $F = \{10^{2n+1}1 : n \geq 0\}$. This shift is called even shift. Let *X* be a set of all binary sequences with no two 1's next to each other. This shift is called ***Golden mean shift***. Thus Golden mean shift has forbidden block $F = \{11\}$. A ***shift of finite type*** is a shift space that can be described by a finite set of forbidden blocks, i.e., a shift space *X* having the form X_F for some finite set *F* of blocks. Golden mean shift is shift of finite type.Let *X* be a shift space over the alphabet *A*, and $A_X^{[N]} = B_N(X)$ be the collection of all allowed N-blocks in *X*. Form the full shift $\left(A_X^{[N]}\right)^Z$. Define the N-th higher block code $\beta_N : X \to \left(A_X^{[N]}\right)^Z$ by $\left(\beta_N(x)\right)_i = x_{[i,i+N-1]}$. Then the ***Nth higher block shift*** denoted by $X^{[N]}$ is the image $\beta_N(X)$ in the full shift over $A_X^{[N]}$. Let X be a subset of a full shift, and let $B_n(X)$ denote the set of all n −words that occur in points in X. The $Language$ of X is the collection $B(X) = \cup_{n=0}^{\infty} B_n(X)$. The full 2-shift has language $\{\varepsilon, 0,1,00,01,10,11,000,001,010,011,100, \ldots\}$. Let *X* be the full shift A^Z. A metric ρ is defined on $X = A^Z$ by

$$\rho(x,y) = \begin{cases} 2^{-k} & if\ x \neq y\ and\ k\ is\ maximal\ so\ that\ x_{[-k,k]} = y_{[-k,k]}, and\ 1\ if\ x_0 \neq y_0 \\ 0 & if\ x = y \end{cases}.$$

Then it is well known that (X, ρ) is a compact metric space. The shift function σon X with respect to the above metric is a continuous function. Therefore (X, σ) is a dynamical system [4]. In this context, there is an equivalent definition of shift space, i.e., a shift space is a pair (A, σ) in which A is a closed invariant subset of X. Thus the name shift space comes from the shift function. A $Graph\ G$ consists of a finite set $\vartheta = \vartheta(G)$ of vertices (or states) together with a finite set $\varepsilon = \varepsilon(G)$ of edges. Each edge $e \in \varepsilon(G)$ starts at a vertex denoted by $i(e) \in \vartheta(G)$ and terminates at a vertex $t(e) \in \vartheta(G)$ (which can be the same as (e) and in this case the edge e is called a loop or self loop). We can always represent a shift of finite type by a directed graph. This representation can be done either with edges denoted as ε(G) or vertex ν(G) of a graph G. It is well known that every shift of finite type can be represented by a graph and vice versa [2].

The ***edge shift*** X_G or X_A of a graph G is the shift space over the alphabet $A = \varepsilon(G)$ specified by $X_G = X_A = \{\xi = (\xi_i)_{i \in Z} \in \varepsilon^Z : t(\xi_i) = i(\xi_{i+1})$ for all i ∈Z, ***i***(e) is the initial state of edge 'e' and ***t***(e) is the terminal state of edge 'e'}.

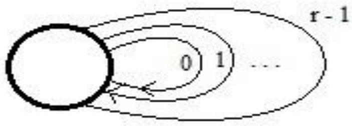

Fig. 1.Graph for full r-shift.

A graph G is ***irreducible*** if for every ordered pair of vertices *I* and *J* there is a path in G starting at *I* and terminating at *J*.If G is a graph with vertex set ν. For every *I*,*J*∈ν, let A_{IJ} denote the number of edges in G with the initial state *I* and terminal state *J*. Then the ***adjacency matrix*** of G is $A_G = [A_{IJ}]$.An equivalent definition of irreducible graph is that its adjacency matrix $A = [A_{IJ}]$ is irreducible, i.e., for each pair I, J of vertices, there exists a non-negative integer n such that $A_{IJ}^n > 0$.A shift of finite type is ***M-step*** if it can be described by a collection of forbidden blocks all of which have length M+1. If G is a graph with adjacency matrix A, then the associated edge shift $X_G = X_A$ is a 1-step shift of finite type [2].This can be proved as follows. Let $A = \varepsilon$ be the alphabet of X_G. Consider the finite collection

$$F = \{ef : e, f \in A, t(e) \neq i(f)\}$$

of 2-blocks over A. According to the definition, a point $\alpha \in A^Z$ lies in X_G exactly when no block of F occurs in α. This means that $X_G = X_F$,so that X_G has finite type. Since all blocks in *F* have length 2, X_F is 1-step.

Theorem1.1[2]***:*** *If X is a M-step shift of finite type then there is a graph G such that* $X^{[M+1]} = X_G$.

A ***path*** $\pi = e_1e_2\ldots..e_m$ on a graph is a finite sequence of edges e_i from G such that $t(e_i) = i(e_{i+1})$ for $1 \le i \le m-1$. The ***path*** $\pi = e_1e_2 \ldots\ldots\ldots e_m$ starts at vertex $i(\pi) = i(e_1)$ and terminates at vertex $t(\pi) = t(e_m)$, and π is a path from $i(\pi)$ to $t(\pi)$. The ***length*** of $\pi = e_1e_2 \ldots\ldots e_m$ is $|\pi| = m$, the number of edges it traverses.A ***cycle*** is a path that starts and terminates at the same vertex.

Proposition 1.2[2]***:*** *Let G be a graph with adjacency matrix A, and let m ≥ 0.(i) The number of paths of length m from I to J is* $(A^m)_{IJ}$, *the I,J^th entry of* A^m. *(ii) the number of cycles of length m in G is the trace of* A^m, $tr(A^m)$, *and this equals the number of points in* X_G *with period m.*

Let B be an ***r*** x ***r*** matrix of 0's and 1's, or equivalently the adjacency matrix of a graph G such that between any two vertices there is atmost one edge. The ***vertex shift*** $\hat{X}_B = \hat{X}_G$ is the shift space with alphabet *A* = { 1,2,3, ... , r } defined by $\hat{X}_B = \hat{X}_G = \{x = (x_i)_{i \in Z} \in A^Z : Bx_ix_{i+1} = 1$, for all i ∈*Z*}

Example1.3:$B = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$. The vertex shift $\hat{X}_B$ is the golden mean shift.

Proposition 1.4.[2]**:***(i) Up to renaming of symbols, the 1-step shifts of finite type are the same as the vertex shifts(ii)Up to renaming of symbols, every edge shift is a vertex shift (on a different graph). (iii) If X is a M-step shift of finite type, there is a graph G such that* $X^{[M]} = \hat{X}_G$ *and* $X^{[M+1]} = X_G$.

Let *X* be a shift space. Entropy measures the complexity of *X*. $|B_n(X)|$ denotes the number of n-blocks appearing in points of *X*. It gives us the idea of complexity of *X*. The ***topological entropy*** of a shift space X is defined as $h(X) = \lim_{n\to\infty} \frac{1}{n} \log |B_n(X)|$[2]. One can see the definition of topological entropy of a continuous map in [6]. Baldwin and Slaminka [1], estimated topological entropy as the logarithmic growth rate of the one dimensional variation of n-composition map of T with n. Their method was generalized by Nwehouse and Pignataro [5].

Gary, F., et al., give a rigorous upper bound for the entropy of a multidimensional system with respect to a fixed partition [3]. If X has alphabet A, then $|B_n(X)| \leq |A|^n$. Hence $\frac{1}{n}\log |B_n(X)| \leq \log|A|$ for all n. So, $h(X) \leq \log|A|$. It is well known that $0 \leq h(X) < \infty$. Let $X = X_{[r]}$ be a full r-shift, then $|B_n(X)| = r^n$. Then $\log|B_n(X)| = n\log r$. Therefore we have $h(X) = \lim_{n\to\infty}\frac{1}{n}\log |B_n(X)| = \log r$. If G has k – vertices and there are exactly r-edges starting at each vertices, then $|B_n(X)| = k.r^n$,so that $\log|B_n(X)| = \log k + n\log r$.Hence, we see that here also $h(X) = \log r$.(Full r-shift is the case where k = 1). In the calculation of entropy for shift space, we shall be using the theorem 1.5 & theorem 1.6. A precise proof is available in [2].

Theorem 1.5(Perron Frobenius theorem)*Let the matrix $A \neq 0$ be an irreducible matrix. Then A has a positive eigenvector V_A with corresponding eigenvalue $\lambda_A > 0$ that is both geometrically and algebraically simple. If λ is another eigenvalue of A, then $|\lambda| \leq \lambda_A$. Any positive eigenvector for A is a positive multiple of V_A .*
The greatest positive real eigen value is called Perron eigen value. If G is an irreducible graph, then by $\lambda_{A(G)}$ we denote the corresponding Perron eigen value of the adjacency matrix $A(G)$.

Theorem 1.6 *(i) If G is irreducible graph, then $h(X_G) = \log \lambda_{A(G)}$. (ii) If X is irreducible M- step shift of finite type and G is the essential graph for which $X_{[M+1]} = X_G$, then $h(X) = \log\lambda_{A(G)}$.*

Now we define the golden mean lookalike shifts. Let the alphabet be $A = \{0,1,2\}$ and the corresponding forbidden block be $F = \{11,22,12,21\}$, then the corresponding shift space $X = X_F$ is called golden mean lookalike shift (GMLS3) with alphabet $A = \{0,1,2\}$. Similarly we define GMLS for higher alphabet $A = \{0,1,2,3\}$ and so on.The aim of this paper is to calculate entropy for various GMLS. We have found some interesting result that shift space having even number of alphabets have same entropy. Whereas shift space having odd numbers of alphabets are divided in to two groups and each group has distinct entropy.

II. ENTROPY OF GMLS

2.1 **GMLS3 with alphabet $A = \{0,1,2\}$**

Let $A = \{0,1,2\}$ be an alphabet and the corresponding forbidden block be $F = \{11,22,12,21\}$. Then the resulting shift is a golden mean lookalike shift (GMLS3)with alphabet $A = \{0,1,2\}$. The corresponding graph is shown in Fig.3.1.

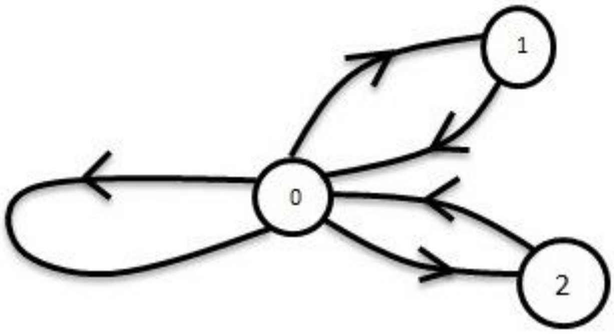

Fig. 2. Graph of GMLS3 with alphabet $A = \{0,1,2\}$

The Adjacency Matrix is given by

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

and $$A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

therefore A is irreducible.

The characteristic equation $|A - \lambda I| = 0$, is given by $\lambda - \lambda(\lambda^2 - \lambda - 1) = 0$ and the Eigen values are $\lambda = 0, -1, 2$. The PerronEigen value is 2, therefore the required entropy is $entropy_{GMLS3} = \log 2$.

2.2 **GMLS4 with alphabet $A = \{0,1,2,3\}$:**

Let $A = \{0,1,2,3\}$ be an alphabet and the corresponding forbidden block be $F = \{11,22,33,12,21,13,31,23,32\}$. Then the resulting shift is a golden mean lookalike shift (GMLS4)with alphabet $A = \{0,1,2,3\}$.

Adjacency matrix$A$ is given by

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \text{ and } A^2 = \begin{bmatrix} 4 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix},$$ therefore A is irreducible. The characteristic equation $|A - \lambda I| = 0$, is given by $\lambda^2(-\lambda^2 + \lambda + 3) = 0$. The eigen values are $\lambda = 0, 0, \frac{1-\sqrt{13}}{2}, \frac{1+\sqrt{13}}{2}$ and the Perron Eigen value is $\frac{1+\sqrt{13}}{2}$ therefore the entropy is $entropy_{GMLS4} = log\,\frac{1+\sqrt{13}}{2}$.

2.3GMLS5 with alphabet $A = \{0, 1, 2, 3, 4\}$:

We take the alphabet as$A = \{0,1,2,3,4\}$ and the corresponding forbidden block be taken as $F = \{11,22,33,44,12,21,13,31,23,32,14,41,24,42,34,43\}$. Then the resulting shift is a golden mean lookalike shift (GMLS5) with alphabet $A = \{0,1,2,3,4\}$. The adjacency Matrix is given by

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } A^2 = \begin{bmatrix} 5 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$ therefore A is irreducible.

The characteristic equation$|A - \lambda I| = 0$, becomes $-\lambda^3(-\lambda^2 + \lambda + 4) = 0$. The roots are $\lambda = 0,0,0, \frac{1-\sqrt{17}}{2}, \frac{1+\sqrt{17}}{2}$. The entropy is $entropy_{GMLS5} = log\,\frac{1+\sqrt{17}}{2}$.

2.4.GMLS6 with alphabet $A = \{0, 1, 2, 3, 4, 5\}$:

In this case the forbidden block is taken as $F = \{11,22,33,44,55,12,21,13,31,23,32,14,41,24,42,34,43,15,51,25,52,35,53,45,54\}$. Then the resulting shift is a golden mean lookalike shift (GMLS5) with alphabet $A = \{0,1,2,3,4,5\}$.

The adjacency Matrix is $A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$. It can be shown as above that it is also irreducible matrix.

The characteristic equation is $\lambda^4(\lambda^2 - \lambda - 3) = 0$, and roots are $\lambda = 0,0,0, \frac{1-\sqrt{13}}{2}, \frac{1+\sqrt{13}}{2}$. Therefore the entropy is $entropy_{GMLS6} = log\,\frac{1+\sqrt{13}}{2}$.

2.5. GMLS7 with alphabet $A = \{0, 1, 2, 3, 4, 5, 6\}$:

In this case the forbidden block is taken as
$F =$
$\{11,22,33,44,55,66,12,21,13,31,23,32,14,41,24,42,34,43,15,51,25,52,35,53,45,54,16,61,26,62,36,63,46,63,$
. Then the resulting shift is a golden mean lookalike shift (GMLS6) with alphabet $A = \{0,1,2,3,4,5,6\}$.

The adjacency Matrix is $A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$. It is also irreducible matrix. The characteristic equation is $-\lambda^5(\lambda^2 - \lambda - 2) = 0$ and the roots are $\lambda = 0,0,0,0,0,-1,2$. Therefore entropy is $entropy_{GMLS6} = log2$

2.6. GMLS8 with alphabet $A = \{0, 1, 2, 3, 4, 5, 6, 7\}$:

The forbidden block is
$F =$
$\{11,22,33,44,55,66,77,12,21,13,31,23,32,14,41,24,42,34,43,15,51,25,52,35,53,45,54,16,61,26,62,36,63,46,$

$46,64,56,65,17,71,27,72,37,73,47,74,57,75,67,76\}$. The adjacency matrix is

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$, which is an irreducible matrix.

The characteristic equation is $\lambda^6(-\lambda^2 + \lambda + 3) = 0$ and the Eigen values are given by $\lambda = 0,0,0,0,0,0, \frac{1-\sqrt{13}}{2}, \frac{1+\sqrt{13}}{2}$. Therefore the entropy is $entropy_{GMLS4} = log\,\frac{1+\sqrt{13}}{2}$

2.7. GMLS9 with alphabet $A = \{0,1,2,3,4,5,6,7,8\}$:

The forbidden block is
$F =$
$\{11,22,33,44,55,66,77,88,12,21,13,31,23,32,14,41,24,42,34,43,15,51,25,52,35,53,45,54,16,61,26,62,36,63,$

$46,64,56,65,17,71,27,72,37,73,47,74,57,75,67,76,18,81,28,82,38,83,48,84,58,85,68,86,78,87\}$. In this case also, the adjacency matrix A is irreducible and the characteristic equation is $-\lambda^7(-\lambda^2 + \lambda + 4) = 0$. The roots are $\lambda = 0,0,0,0,0,0,0, \frac{1-\sqrt{17}}{2}, \frac{1+\sqrt{17}}{2}$. Therefore the entropy is $entropy_{GMLS8} = log\,\frac{1+\sqrt{17}}{2}$

Table

Alphabet	Characteristic Equation	Greatest positive Eigen value	Entropy	Remarks
$A = \{0,1,2\}$	$\lambda(-\lambda^2 + \lambda + 2) = 0$	2	$log2$	No. of alphabet 3, odd
$A = \{0,1,2,3\}$	$\lambda^2(-\lambda^2 + \lambda + 3) = 0$	$\frac{1+\sqrt{13}}{2}$	$log\,\frac{1+\sqrt{13}}{2}$	No. of alphabet 4, even
$A = \{0,1,2,3,4\}$	$-\lambda^3(-\lambda^2 + \lambda + 4) = 0$	$\frac{1+\sqrt{17}}{2}$	$log\,\frac{1+\sqrt{17}}{2}$.	No. of alphabet 5, odd
$A = \{0,1,2,3,4,5\}$	$\lambda^4(\lambda^2 - \lambda - 3) = 0$	$\frac{1+\sqrt{13}}{2}$	$log\,\frac{1+\sqrt{13}}{2}$.	No. of alphabet 6, even
$A = \{0,1,2,3,4,5,6\}$	$-\lambda^5(\lambda^2 - \lambda - 2) = 0$	2	$log2$	No. of alphabet 7, odd
$A = \{0,1,2,3,4,5,6,7\}$	$\lambda^6(-\lambda^2 + \lambda + 3) = 0$	$\frac{1+\sqrt{13}}{2}$	$log\,\frac{1+\sqrt{13}}{2}$.	No. of alphabet 8, even
$A = \{0,1,2,3,4,5,6,7,8\}$	$-\lambda^7(-\lambda^2 + \lambda + 4) = 0$	$\frac{1+\sqrt{17}}{2}$	$log\,\frac{1+\sqrt{17}}{2}$.	No. of alphabet 9, odd

III. CONCLUSIONS

Baring the case of alphabet $A = \{0,1,2\}$, the characteristic equation for GMLSn is $(-1)^{n-2}\lambda^{n-2} + (-1)^{n-2}\lambda C_{n-1} = 0$, for all $n \geq 4$, where C_{n-1} denotes the right hand side expression of the characteristic equation of GMLS$n-1$. If n is even then $(-\lambda^2 + \lambda + 3)$ is the only non-trivial factor. If n is odd and it is of the form $3 + m4$, where m is a non-negative integer, then the only non-trivial factor is $(\lambda^2 - \lambda - 2)$whereas if n is of the form $5 + m4$, then $(-\lambda^2 + \lambda + 4)$ is the only non-trivial factor. We can conclude that GMLS with alphabet having even number of symbols have entropy $log\,\frac{1+\sqrt{13}}{2}$. There are two groups for alphabets having odd number of symbols. The first group has alphabets having $3 + m4$ numbers of symbols, where m is a non-negative integer. Each member of this group has entropy $log2$. The second odd group has alphabet in which there are $5 + m4$ numbers of symbols, where m is a non-negative integer, and each member of this group has entropy $log\frac{1+\sqrt{17}}{2}$.

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