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Exchange rate volatility of Nigerian Naira against some major currencies in the world: An application of multivariate GARCH models

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ABSTRACT : *Exchange rates are important financial problem that is receiving attention globally. This study uses daily data over the period January, 1999 to February, 2014 consisting of 3950 observations. The study is aimed to determine the volatility spillover of the nine selected countries against the U.S. Dollar simultaneously via Multivariate GARCH models. And hence, to observe some stylize facts/common features of good volatility modeling on financial time series. To achieve the stated objectives, we employed three multivariate volatility models: DVECH, BEKK and CCC and the result indicated that the restricted BEKK, DVECH and CCC results exhibit rather similar behavior for each considering countries. The result also indicated that the skewness is greater than zero, that is to say the distribution is positively skewed which is an indication of a non symmetric series, meaning that there is an asymmetric effects in these models. The kurtosis is also greater than 3; relatively large kurtosis suggests that the distribution of the exchange rate return series is leptokurtic which is another stylize fact. Conclusively, base on information criteria, the DVECH model is found to be the best model. But according to parsimonious principle, the BEKK model is considered to be the best because it has least number of parameters.*

KEYWORDS:MULTIVARIATE *GARCH, Exchange rate, Volatility*

I. INTRODUCTION

The volatility of exchange rate returns is one of the central variables in mainstream financial economics. In fact, much empirical work has been done in this area. Many authors are convinced that exchange rate returns are in large part predictable, but only over the long term (in less than a five-year period, the predictions seem less reliable). Fama and French (1988, 1993, 1996), among others prominent and well-known researchers, demonstrated empirically that a few economic factors can explain the variability of returns. Consequently, with this knowledge, we can forecast the expected exchange rate returns quite well. Understanding the volatility of the exchange rate means having indirect knowledge of the distribution of the returns if they are normally distributed (Gaussian) because for this distribution, we need only the two first moments. If we know the distribution or the variability of returns, then we can forecast with higher accuracy the returns themselves. But more than one question arises here: (1) Do we really know the volatility of exchange rates? (2) Is it this constant over time or stochastic? (3) Are the daily or monthly exchange rates really normally distributed? But if we are unable to answer these questions with precision, can we postulate that the exchange rate returns are predictable? In fact, if you find a model that explains something accurately, then you can be certain that this will forecast, with small probability of failure, the true future (expected) return. Volatilities and correlations are the two most important elements in asset pricing, portfolio management and risk assessment. Since the seminal 1982 paper of Engel's autoregressive conditional heteroskedasticity (ARCH) model, lots of efforts have been spent on univariate volatility modeling. Most famous one among them is the Bollerslov's generalized ARCH (GARCH) model. As time goes by and computing power improves, researchers find it more and more important and necessary to generalize the univariate ARCH/GARCH models to their multivariate versions. This will continue to be the trend thereafter. (Xiaojun Song, 2009).

One of the central aspects in financial econometrics is the modeling, measuring and forecasting of second and possible higher moments, because the volatility for instance is not directly observable. One of the most important models for volatility is the class of multivariate generalized autoregressive conditional heteroscedasticity (MGARCH) models. They allow us to specify a dynamic process for the whole time varying variance–covariance matrix of the time series thus jointly modeling the first and second moments. The main applications of MGARCH models are in portfolio management, hedging, analysis of volatility spillovers across markets, option pricing and Value–at–Risk (VaR) of portfolios.

Since correlations between asset returns and markets are important in many financial applications, multivariate volatility models have also been extended to describe the time–varying feature of the correlations in recent years.The univariate GARCH framework was developed by Bollerslev (1986), based on the ARCH models by Engle (1982). Engle proposed a function for the conditional variance of the time series that depends on the realized error of the period before. (Xiaojun Song, 2009).Analogous to the expansion from the AR models to ARMA models, Bollerslev developed the GARCH model by taking the own history of the volatility into account. But in this framework the restrictions to univariate time series doesn't take the volatility spillover into account. The possibility of interaction between one or more time series is completely excluded. Therefore Bollerslev, Engle and Wooldridge (1988) proposed the basic framework for the multivariate GARCH model by including additional parameters in order to capture this effect. Many expansions to the basic MGARCH models have therefore been developed. Based on the recent theoretical and empirical developments and discoveries in MGARCH models, this paper focuses on the investigation of volatilities and correlations of some selected currencies in the world, these are Nigerian Naira, Euro, Great Britain Pound (GBP), Singapore Dollars, South African Rand, Malaysian Ringgit, Japanese Yen, Chinese Yuan and Canadian Dollars. Now, let's give some shorts overviews on the economic standard of the countries of eight selected currencies plus the Euro for simple comparison.

Nigeria is a middle income, mixed economy and emerging market, with expanding financial, service, communications and technology and entertainment sectors. It is ranked 26th in the world in terms of GDP (nominal: 30th in 2013 before rebasing, 40th in 2005, 52nd in 2000), and is the largest economy in Africa (based on rebased figures announced in April 2014). It is also on track to become one of the 20 largest economies in the world by 2020. Its re-emergent, though currently underperforming, manufacturing sector is the third-largest on the continent, and produces a large proportion of goods and services for the West African region. Previously hindered by years of mismanagement, economic reforms of the past decade have put Nigeria back on track towards achieving its full economic potential. Nigerian GDP at purchasing power parity (PPP) has almost tripled from $170 billion in 2000 to $451 billion in 2012, although estimates of the size of the informal sector (which is not included in official figures) put the actual numbers closer to $630 billion. Correspondingly, the GDP per capita doubled from $1400 per person in 2000 to an estimated $2,800 per person in 2012 (again, with the inclusion of the informal sector, it is estimated that GDP per capita hovers around $3,900 per person). (Population increased from 120 million in 2000 to 160 million in 2010). These figures are to be revised upwards by as much as 80% when metrics are recalculated subsequent to the rebasing of its economy in April 2014. (Anon., 2014).

Although much has been made of its status as a major exporter of oil, Nigeria produces only about 2.7% of the world's supply (Saudi Arabia: 12.9%, Russia: 12.7%, USA:8.6%). To put oil revenues in perspective: at an estimated export rate of 1.9 Mbbl/d (300,000 m^3/d), with a projected sales price of $65 per barrel in 2011, Nigeria's anticipated revenue from petroleum is about $52.2 billion (2012 GDP: $451 billion). This accounts about 11% of official GDP figures (and drops to 8% when the informal economy is included in these calculations). Therefore, though the petroleum sector is important, it remains in fact a small part of the country's overall vibrant and diversified economy. (Anon.,2014).

In 1986, Nigeria adopted the structural adjustment programme (SAP) of the IMF/World Bank. With the adoption of SAP in 1986, there was a radical shift from inward-oriented trade policies to out ward –oriented trade policies in Nigeria. These are policy measures that emphasize production and trade along the lines dictated by a country's comparative advantage such as export promotion and export diversification, reduction or elimination of import tariffs, and the adoption of market-determined exchange rates. Some of the aims of the structural adjustment programme adopted in 1986 were diversification of the structure of exports, diversification of the structure of production, reduction in the over-dependence on imports, and reduction in the over-dependence on petroleum exports. The major policy measures of the SAP were:

[1] Deregulation of the exchange rate
[2] Trade liberalization
[3] Deregulation of the financial sector
[4] Adoption of appropriate pricing policies especially for petroleum products
[5] Rationalization and privatization of public sector enterprises and
[6] Abolition of commodity marketing boards. (Onasanya *et al*, 2013)

The **economy of South Africa** is the second largest in Africa behind Nigeria, it accounts for 24% of its gross domestic product in terms of purchasing power parity, and is ranked as an upper-middle income economy by the World Bank; this makes the country one of only four countries in Africa in this category (the others being

Botswana, Gabon and Mauritius). Since 1996, at the end of over twelve years of international sanctions, South Africa's Gross Domestic Product has since almost tripled to $400 billion, and foreign exchange reserves have increased from $3 billion to nearly $50 billion; creating a growing and sizable African middle class, within two decades of establishing democracy and ending apartheid. According to official estimates, a quarter of the population is unemployed, According to a 2013 Goldman Sachs report, that number increases to 35% when including people who have given up looking for work. A quarter of South Africans live on less than US $1.25 a day. South Africa has a comparative advantage in the production of agriculture, mining and manufacturing products relating to these sectors. South Africa has shifted from a primary and secondary economy in the mid-twentieth century to an economy driven primarily by the tertiary sector in the present day which accounts for an estimated 65% of GDP or $230 billion in nominal GDP terms. The country's economy is reasonably diversified with key economic sectors including mining, agriculture and fisheries, vehicle manufacturing and assembly, food processing, clothing and textiles, telecommunication, energy, financial and business services, real estate, tourism, transportation, and wholesale and retail trade.

The unemployment rate is very high, at more than 25%, and the poor have limited access to economic opportunities and basic services. Poverty also remains a major problem. In 2002, according to one estimate, 62% of Black Africans, 29% of Coloureds, 11% of Asians, and 4% of Whites lived in poverty. The high levels of unemployment and inequality are considered by the government and most South Africans to be the most salient economic problems facing the country. These issues, and others linked to them such as crime, have in turn hurt investment and growth, consequently having a negative feedback effect on employment. Crime is considered a major or very severe constraint on investment by 30% of enterprises in South Africa, putting crime among the four most frequently mentioned constraints. (Anon.,2014).

The socialist market **economy of China** is the world's second largest economy by nominal GDP and by purchasing power parity after the United States. It is the world's fastest-growing major economy, with growth rates averaging 10% over the past 30 years. China is also the largest exporter and second largest importer of goods in the world. China is the largest manufacturing economy in the world, outpacing its world rival in this category, the service-driven economy of the United States of America. ASEAN–China Free Trade Area came into effect on 1 January 2010. China-Switzerland FTA is China's first FTA with a major European economy, while China–Pakistan Free Trade Agreement came in effect in 2007 is the first FTA signed with a South Asian state. The

economy of China is the fastest growing consumer market in the world. On a per capita income basis, China ranked 87th by nominal GDP and 92nd by GDP (PPP) in 2012, according to the International Monetary Fund (IMF). The provinces in the coastal regions of China tend to be more industrialized, while regions in the hinterland are less developed. As China's economic importance has grown, so has attention to the structure and health of the economy. Xi Jinping’s Chinese Dream is described as achieving the “Two 100s”: the material goal of China becoming a “moderately well-off society” by 2021, the 100th anniversary of the Chinese Communist Party, and the modernization goal of China becoming a fully developed nation by 2049, the 100th anniversary of the founding of the People’s Republic.

The internationalization of the Chinese economy continues to affect the standardized economic forecast officially launched in China by the Purchasing Managers Index in 2005. At the start of the 2010s, China remained the sole Asian nation to have an economy above the $10-trillion mark (along with the United States and the European Union). Most of China's economic growth is created from Special Economic Zones of the People's Republic of China that spread successful economic experiences to other areas. The development progress of China's infrastructure is documented in a 2009 report by KPMG. (Anon.,2014).

The United Kingdom has the 6th-largest national economy in the world (and 3rd-largest in Europe) measured by nominal GDP and 8th-largest in the world (and 2nd-largest in Europe) measured by purchasing power parity (PPP). The UK's GDP per capita is the 22nd-highest in the world in nominal terms and 22nd-highest measured by PPP. In 2012, the UK was the 10th-largest exporter in the world and the 6th-largest importer. In 2012, the UK had the 3rd-largest stock of inward foreign direct investment and the 2nd-largest stock of outward foreign direct investment. The British economy comprises (in descending order of size) the economies of England, Scotland, Wales and Northern Ireland. The UK has one of the world's most globalised economies. One-sixth of the tax revenue comes from VAT (value added tax) from the consumer market of the British Economy.The service sector dominates the UK economy, contributing around 78% of GDP, with the financial services industry particularly important. London is the world's largest financial centre and has the largest city GDP in Europe.

The UK aerospace industry is the second- or third-largest national aerospace industry depending on the method of measurement. The pharmaceutical industry plays an important role in the economy and the UK has the third-highest share of global pharmaceutical R&D. The automotive industry is also a major employer and exporter. The British economy is boosted by North Sea oil and gas production; its reserves were valued at an estimated £250 billion in 2007. There are significant regional variations in prosperity, with the South East of England and southern Scotland the richest areas per capita. (Anon.,2014).

The **economy of Japan** is the third largest in the world by nominal GDP, the fourth largest by purchasing power parity and is the world's second largest developed economy. According to the International Monetary Fund, the country's per capita GDP (PPP) was at $35,855 or the 22nd highest in 2012. Japan is a member of Group of Eight. The Japanese economy is forecasted by the Quarterly Tankan survey of business sentiment conducted by the Bank of Japan.Japan is the world's third largest automobile manufacturing country, has the largest electronics goods industry, and is often ranked among the world's most innovative countries leading several measures of global patent filings. Facing increasing competition from China and South Korea, manufacturing in Japan today now focuses primarily on high-tech and precision goods, such as optical instruments, Hybrid vehicles, and robotics. Beside the Kantō region, the Kansai region is one the leading industrial clusters and the manufacturing center for the Japanese economy. Japan is the world's largest creditor nation, generally running an annual trade surplus and having a considerable net international investment surplus. As of 2010, Japan possesses 13.7% of the world's private financial assets (the 2nd largest in the world) at an estimated $14.6 trillion. As of 2013, 62 of the Fortune Global 500 companies are based in Japan. (Anon.,2014).

Canada has the eleventh or 14th-largest economy in the world (measured in US dollars at market exchange rates), is one of the world's wealthiest nations, and is a member of the Organization for Economic Co-operation and Development (OECD) and Group of Seven (G7). As with other developed nations, the Canadian economy is dominated by the service industry, which employs about three quarters of Canadians. Canada is unusual among developed countries in the importance of the primary sector, with the logging and oil industries being two of Canada's most important. Canada also has a sizable manufacturing sector, centred in Central Canada, with the automobile industry and aircraft industry especially important. With a long coastal line, Canada has the 8th largest commercial fishing and seafood industry in the world. Canada is one of the global leaders of the entertainment software industry. (Anon.,2014).

Singapore is a highly developed trade-oriented market economy. Singapore's economy has been ranked as the most open in the world, least corrupt, most pro-business, with low tax rates (14.2% of Gross Domestic Product, GDP) and has the third highest per-capita GDP in the world; in terms of Purchasing Power Parity (PPP). Government-linked companies play a substantial role in Singapore's economy, which are owned through the sovereign wealth fund Temasek Holdings, which holds majority stakes in several of the nation's largest companies, such as Singapore Airlines, SingTel, ST Engineering and Media Corp. The economy of Singapore is a major Foreign Direct Investment (FDI) outflow financier in the world. Singapore has also benefited from the inward flow of FDI from global investors and institutions due to her highly attractive investment climate and a stable political environment.Exports, particularly in electronics, chemicals and services including the posture that Singapore is the regional hub for wealth management provide the main source of revenue for the economy, which allows it to purchase natural resources and raw goods which she lacks. Moreover, water is scarce in Singapore therefore water is defined as a precious resource in Singapore along with the scarcity of land to be treated with land fill of Pulau Semakau. Singapore has limited arable land that Singapore has to rely on the agrotechnology park for agricultural production and consumption. Human Resource is another vital issue for the health of Singaporean economy. Singapore could thus be said to rely on an extended concept of intermediary trade to Entrepôt trade, by purchasing raw goods and refining them for re-export, such as in the wafer fabrication industry and oil refining. Singapore also has a strategic port which makes it more competitive than many of its neighbours in carrying out such entrepot activities. Singapore has the highest trade to GDP ratio in the world, averaging around 400% during 2008–11.The Port of Singapore is the second-busiest in the world by cargo tonnage. In addition, Singapore's port infrastructure and skilled workforce, which is due to the success of the country's education policy in producing skilled workers, is also fundamental in this aspect as they provide easier access to markets for both importing and exporting, and also provide the skill(s) needed to refine imports into exports. (Anon.,2014).

Malaysia has a newly industrialised market economy, which is relatively open and state-oriented. The state plays a significant, but declining role in guiding economic activity through macroeconomic plans. In 2012, the economy of Malaysia was the third largest economy in South East Asia behind more populous Indonesia and Thailand and 29th largest economy in the world by purchasing power parity with gross domestic product stands at US$492.4 billion and per capita US$16,922. In 2010, GDP per capita (PPP) of Malaysia stood at US$14,700. In 2009, the PPP GDP was US$383.6 billion, and the PPP per capita GDP was US$8,100.The Southeast Asian country experienced an economic boom and underwent rapid development during the late 20th century and has GDP per capita of $17,200 today, to be considered a newly industrialized country. On the income distribution, there are 5.8 million households in 2007. Of that, 8.6% have a monthly income below RM1,000, 29.4% had between RM1,000 and RM2,000, while 19.8% earned between RM2,001 and RM3,000; 12.9% of the households earned between RM3,001 and RM4,000 and 8.6% between RM4,001 and RM5,000. Finally, around 15.8% of the households have an income of between RM5,001 and RM10,000 and 4.9% have an income of RM10,000 and above. As one of three countries that control the Strait of Malacca, international trade plays a large role in its economy. At one time, it was the largest producer of tin, rubber and palm oil in the world. Manufacturing has a large influence in the country's economy. Malaysia is the world's largest Islamic banking and financial centre. (Anon.,2014).

The **euro** is the official currency of Germany, which is a member of the European Union. The Euro Area refers to a currency union among the European Union member states that have adopted the euro as their sole currency. In Germany, interest rate decisions are taken by the Governing Council of the European Central Bank (ECB).On January 1, 1999 Euro became the currency for 11 member states of the European Union (the countries were Belgium, Germany, Spain, France, Ireland, Italy, Luxembourg, the Netherlands, Austria, Portugal and Finland). Since then the Dollar-Euro exchange rate has completed a full turning. Euro depreciated since its introduction steadily and without any major interruption until February 2002. Then it began to rise against Dollar smoothly and reached a height of 0.74 Euros to 1 US$ in December 2004. Three years of depreciation of the Euro followed by three years of appreciation without wild fluctuations asks for an explanation which would adequately account for the position of the Euro as an emerging international currency. Theories of the exchange rate which are based on interest rate and price differentials or revisions of expectations causing short term capital movements cannot provide for such an explanation. Since both, Dollar and Euro are international currencies we apply the theory of world money to explain the steep downward and upward movement of Euro against Dollar in the period 1999 to 2014 together with eight major currencies in the world (i. e. Nigerian Naira, Great Britain Pound (GBP), Singapore Dollars, South African Rand, Malaysian Ringgit, Japanese Yen, Chinese Yuan and Canadian Dollars). (Rasul S., 2004).

II. LITERATURE REVIEW

Many studies provide evidence that correlation is evolving through time. Longin and Solnik (1995) showed that correlation in international equity returns across 1960–1990 is highly volatile. Engle (2002) verified the important evidence of time–varying correlation of many classes of assets. Tse and Tsui (2002) applied time–varying correlation model to exchange rate data, national stock market data and the sectoral price data and provided the time–varying correlation evidence for the three real datasets. Solnik, Boucrelle, and Le Fur (1996) found that correlation is increasing in periods of high market volatility for the industrialized countries when risk diversification is needed most. Campbell, Koedijk and Kofman (2002) showed that market correlations increase in the bear market. Volatility changes not only due to the dynamic evolution of own market volatility but also changes of interdependence across markets. Hamao, Masulis, and Ng (1990) examined the combination of correlations in price changes and volatility across international stock markets. Engle and Susmel (1993) found that there is common volatility in international equity markets. Bollerslev and Engle (1993) checked the common persistence effect in the conditional variances, that is, the volatility. Bae and Karolyi (1994) found that the spillover of stock volatility between Japan and the United States is closely related to goods news or bad news. Karolyi (1995) used a bivariate GARCH model to investigate the transmission of stock returns and volatility between the United States and Canada, finding that volatility is transferred from U.S. to Canada most of the time. See King, Sentana and Wadhwani (1994), Lin, Engle, and Ito (1994) and Ng (2000) for more evidences of volatility transmission and linkage. Lanza, Manera, and McAleer (2006) and Manera, McAleer, and Grasso (2006) examined correlation and volatility in the oil forward and future markets. Edwards and Susmel (2001) and Edwards and Susmel (2003) investigated the volatility dependence and contagion in equity and interest rate respectively in emerging markets. Balasubramanyan and Premaratne (2003) and Balasubramanyan (2004) provided the evidence of volatility comovement and spillover from Asian markets. Yang (2005) used a DCC anaylsis to examine the role of Japan on the Asian Four Tigers, finding that stock market correlations fluctuate widely over time and volatilities are contagious across markets. Kuper and Lestano (2007) analyzed the financial market interdependence of Thailand and Indonesia. See Andersen, Bollerslev, Christoffersen, and Diebold (2005) for a review of volatility and correlation modeling for financial markets.

Little or no work has been done on dollar-Naira exchange rate together with the exchange rates of the major currencies in the world particularly using Multivariate GARCH family models. The exchange rate volatility has implications for many issues in the area of finance and economics. Such issues include impact of foreign exchange rate volatility on derivative pricing, global trade patterns, countries balance of payments position, government policy making decisions and international capital budgeting.

III. MATERIALS AND METHODS

Data: We use the daily exchange rates returns of Nigerian Naira together with eight measure currencies in the world (i. e. Euro, Great Britain Pound (GBP), Singapore Dollars, South African Rand, Malaysian Ringgit, Japanese Yen, Chinese Yuan and Canadian Dollars). The data covers the period from January 4, 1999 to February 21, 2014 which consists of 3950 observations obtained from the Federal Reserve Bank of Louis, U.S.A. The return on exchange rate is defined as:

$$r_t = \log\left(\frac{e_t}{e_{t-1}}\right) \qquad 1$$

Where e_t is the exchange rate at time t and e_{t-1} represent exchange rate at time t-1. The r_t of equation (1) will be used in observing the volatility of the exchange rate between the selected currencies over the period 1999-2014.

Multivariate GARCH Representations

The Diagonal VECH Representation : Let's have a quick glance at the VECH representation before diagonal VECH for crystal clear. Applying the VECH operator to a symmetric matrix stacks the lower triangular elements into a column. Since $\boldsymbol{H}_t$ is a symmetric matrix, in specifying the multivariate GARCH model we can employ the VECH transformation of $\boldsymbol{H}_t$. consider the following specification:

$$vech(H_t) = vech(A_0) + \sum_{i=1}^{q} A_i vech\left(\varepsilon_{t=i}\varepsilon_{t=i}^{/}\right) + \sum_{i=1}^{p} B_i vech(H_{t-i})$$

2

where $\varepsilon_t = \left(\varepsilon_{1t}, \varepsilon_{2t}, ..., \varepsilon_{Nt}\right)^{/}$ are the error terms associated with the conditional mean equations for y_{1t} to y_{Nt}, A_0 is an $(N \times N)$ positive definite matrix of parameters and A_i and B_i are $\left[N(N+1)/2 \times N(N+1)/2\right]$ matrices of parameters. In the case two variables $(N = 2)$ and $p = q = 1$, the multivariate GARCH representation given by (2) can be written out in full as:

$$\begin{bmatrix} h_{11.t} \\ h_{12.t} \\ h_{22.t} \end{bmatrix} = \begin{bmatrix} a_{11}^0 \\ a_{12}^0 \\ a_{22}^0 \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_{1.t-1}^2 \\ \varepsilon_{1.t-1}\varepsilon_{2.t-1} \\ \varepsilon_{2.t-1}^2 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} h_{11.t-1} \\ h_{12.t-1} \\ h_{22.t-1} \end{bmatrix}$$

3

where $h_{11.t}$ is the conditional variance of the error associated with y_{1t}, $h_{22.t}$ is the conditional variance of the error associated with y_{2t} and $h_{12.t}$ is the conditional covariance between the errors.

In the diagonal representation (due to Bollerslev, Engle and Woodridge. 1988) A_i and B_i in (2) are diagonal matrices. This assumption forces the individual conditional variances to have GARCH (p, q) form and the covariances to have a GARCH (p, q) form. As an example, consider the diagonal representation of Vech ($\mathbf{H_t}$) in the case of two variables (N = 2) and p = q = 1:

$$\begin{bmatrix} h_{11.t} \\ h_{12.t} \\ h_{22.t} \end{bmatrix} = \begin{bmatrix} a_{11}^0 \\ a_{12}^0 \\ a_{22}^0 \end{bmatrix} + \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_{1.t-1}^2 \\ \varepsilon_{1.t-1}\varepsilon_{2.t-1} \\ \varepsilon_{2.t-1}^2 \end{bmatrix} + \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{bmatrix} \begin{bmatrix} h_{11.t-1} \\ h_{12.t-1} \\ h_{22.t-1} \end{bmatrix}$$

4

The BEKK Representation

The BEKK representation (due to Baba et al. 1990) assumes the following model for $\mathbf{H}_t$:

$$H_t = A_0 + \sum_{i=1}^{q} A_i^* \varepsilon_{t-i}\varepsilon_{t-i}^{/} A_i^{*/} + \sum_{i=1}^{p} B_i^* H_{t-i} B_i^{*/}$$

5

Where A_i^* and B_i^* are $(N \times N)$ matrices of parameters and A_0 is defined as before, writing out (5) for $N = 2$ and $p = q = 1$ gives:

$$\begin{bmatrix} h_{11.t} & h_{12.t} \\ h_{21.t} & h_{22.t} \end{bmatrix} = \begin{bmatrix} a_{11}^0 & a_{12}^0 \\ a_{12}^0 & a_{22}^0 \end{bmatrix} + \begin{bmatrix} a_{11}^* & a_{12}^* \\ a_{21}^* & a_{22}^* \end{bmatrix} \begin{bmatrix} \varepsilon_{1.t-1}^2 & \varepsilon_{1.t-1}\varepsilon_{2.t-1} \\ \varepsilon_{1.t-1}\varepsilon_{2.t-1} & \varepsilon_{2.t-1}^2 \end{bmatrix} \begin{bmatrix} a_{11}^* & a_{21}^* \\ a_{12}^* & a_{22}^* \end{bmatrix}$$
$$+ \begin{bmatrix} b_{11}^* & b_{12}^* \\ b_{21}^* & b_{22}^* \end{bmatrix} \begin{bmatrix} h_{11.t-1} & h_{12.t-1} \\ h_{21.t-1} & h_{22.t-1} \end{bmatrix} \begin{bmatrix} b_{11}^* & b_{21}^* \\ b_{12}^* & b_{22}^* \end{bmatrix} \qquad 6$$

The Constant Conditional Correlation Representation

In the past years, a new class of multivariate GARCH models has been developed.

They focus on the parameterization of the conditional correlation matrix. Such models have the flexibility of univariate GARCH models with respect to the conditional variances. They need simple conditions to ensure the positive definiteness of Ht and the estimation is much easier than the usual MARCH models. The constant conditional correlation (CCC) model of Bollerslev (1990) is a fruitful endeavor to explore the MGARCH model indirectly in the correlation direction instead of modeling the variance covariance matrix Ht directly. CCC model has several advantages mentioned above. Now we define the structure of the constant conditional correlation matrix R and the variance covariance matrix H_t as follows:

$$R = \begin{bmatrix} 1 & \cdots & \rho_{1N} \\ \vdots & \cdots & \vdots \\ \rho_{N1} & \cdots & 1 \end{bmatrix}$$

7

where ρ_{ij} is the correlation coefficient measuring the correlation of variable *i* with variable *j*. He then defines the conditional variance matrix $\mathbf{H}_t$ as:

$$H_t = D_t R D_t^{/}$$

8

Where

$$D_t = diag(\sigma_{1t}, \sigma_{2t}, ..., \sigma_{Nt}).$$

The basic idea is that every variance–covariance matrix can be decomposed in the above way. Therefore, we can characterize the dynamics in the following way.

$$H_t = \begin{pmatrix} \sigma_{1t}^2 & \sigma_{12,t} \cdots & \sigma_{1N,t} \\ \sigma_{12,t} & \sigma_{2t}^2 \cdots & \sigma_{2N,t} \\ \vdots & & \vdots \\ \sigma_{1N,t} & \sigma_{2N,t} \cdots & \sigma_{Nt}^2 \end{pmatrix}$$

9

$$\sigma_{it}^2 = w_i + \sum_{j=1}^{q} \alpha_{i,j}\varepsilon_{i,t-j}^2 + \sum_{j=1}^{p} \beta_{i,j}\sigma_{i,t-j}^2 \qquad i-1,...,n$$

10

$$\sigma_{ij,t} = \rho_{ij}\sigma_{it}\sigma_{jt} \qquad\qquad i, j = 1,\ldots,n, i \neq j$$

11

The usual conditions to ensure the positivity of the variances and the stationarity hold: $w_i = 0, \alpha_{i,j} > 0, \beta_{i,j} > 0$ and $\sum_{j=1}^{q} \alpha_{i,j} + \sum_{j=1}^{p} \beta_{i,j} < 1$. The total number of parameters is $(p+q+1)N + \frac{N(N-1)}{2}$, when $p = q = 1, N = 2$ 7 parameters need to be estimated, which is not so many but still lack parsimony. Positive definiteness of the variance covariance matrix is controlled by the correlation matrix, while only the usual requirements of positivity constraints for GARCH model suffice. In order to obtain the parameters, maximum likelihood estimation method can be used. (Xiaojun Song, 2009)

IV. DATA ANALYSIS:

In this chapter, we shall focus on the analysis of data and econometric interpretation which is typically the case in financial application. The aim is to examine the volatility spillover (own or cross) of the local currencies of the selected countries against the U.S dollars simultaneously via MGARCH models. Note that "Own–volatility spillovers" is used to indicate a one–way causal relationship between past volatility shocks and current volatility in the same market. "Cross–volatility spillovers" is used to indicate a one–way causal relationship between past volatility shocks in one market and current volatility in another market. The argument is straightforward and easily understood as these nine countries have been historically closely interlinked due to their economic advancements, their popularity in their sub-continents (region) and more importantly the globalization. The researcher(s) have seldom investigated the nine markets together. We applied log-difference transformation to convert the data into continuously compounded returns, because the return (log values) of both currencies is not stationary (see figure 4.1) and are stationary when they are first differenced (see figure 4.2).

Fig 4.1: Graphical Representation of Exchange Rates of the nine selected currencies:

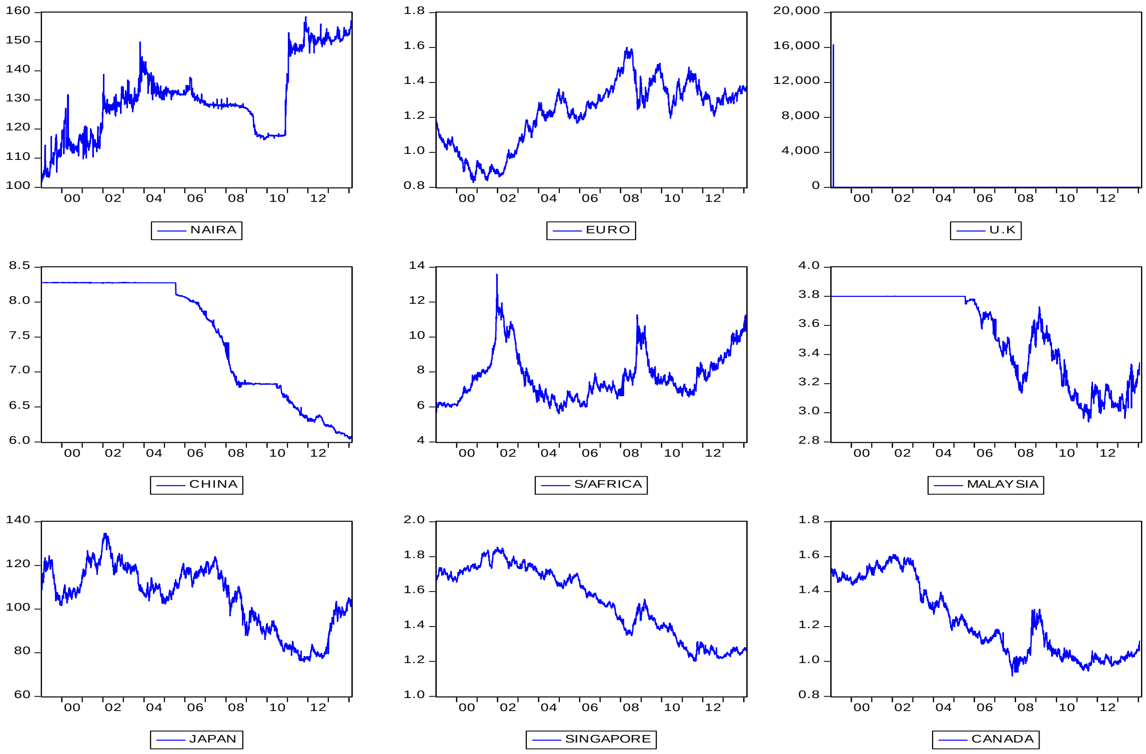

Figure 4.1 indicates that all the series are not stationary as they contain a trend components which should be remove before modeling. These trend components have been taken care of, as explained above which can be seen in figure 4.2.

Fig 4.2: Graphical Representation of the First Difference of the Logarithms of Daily Exchange rates of the nine selected currencies:

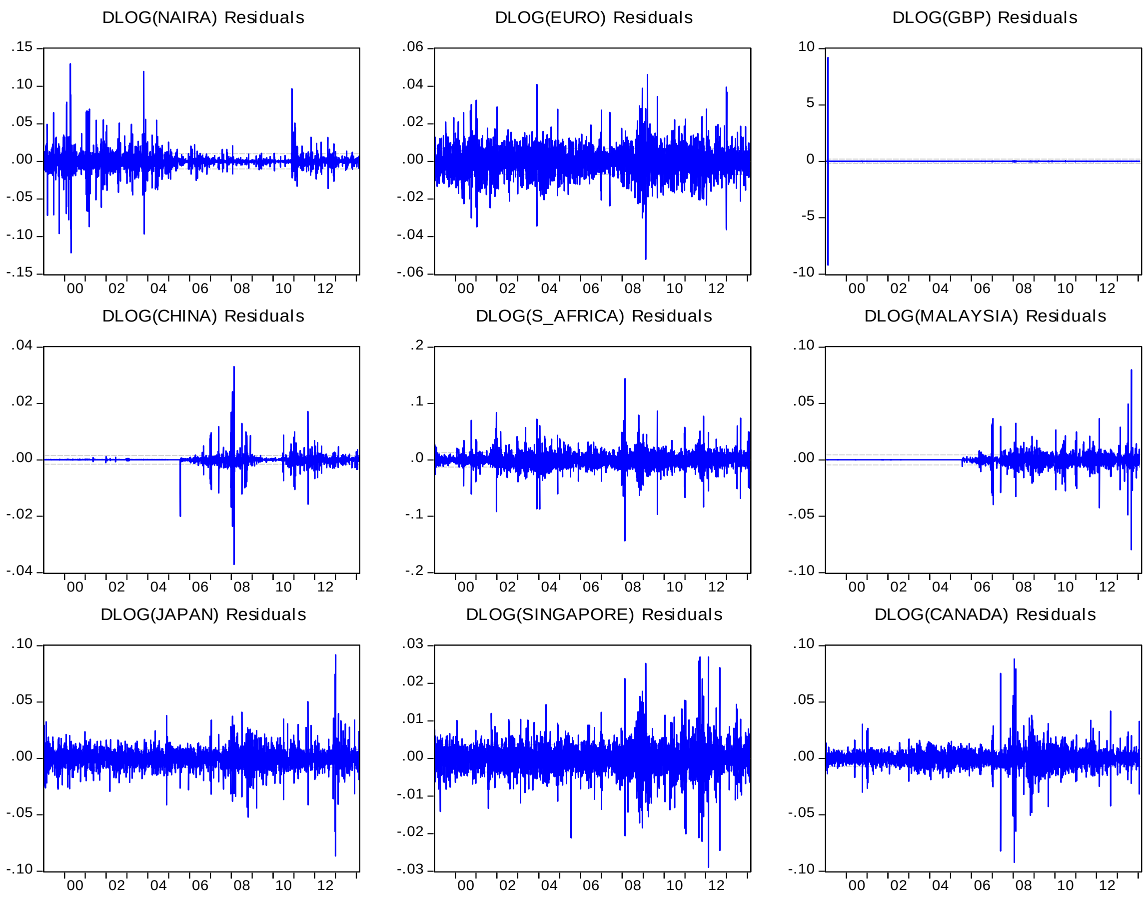

Figure 4.2 shows that some periods are riskier than the others. Also, the risky periods are scattered randomly and there is some degree of autocorrelation in the riskiness of financial returns (i.e. large changes (of either sign) tend to be followed by large changes and small changes (of either sign) tend to be followed by small changes, this is termed as volatility clustering (Mendelbret, 1963) and is one f the stylized facts of volatility of financial time series. We also observed that the clustering of periods of volatility that is large movements being followed by further large movements; the variance of exchange rate returns of these countries (with the exception of the Great Britain) is not constant over time. This is an indication of shock persistence. (International journal of academic research, 2011) which is another stylized fact of volatility f financial time series.

Autocorrelation Function (ACF):

Having discovered that the exchange rates series could be modeled as MGARCH, the next is to examine the ACF to see the degree of correlation in the data points of the series. The one with higher degree of correlation will be the right candidate to model with. We formally confirm the presence of the autocorrelation in the exchange rates series by Portmanteau test.

Table 4.1 System Residual Portmanteau Tests for Autocorrelations
Null Hypothesis: no residual autocorrelations up to lag h
Included observations: 3949

Lags	Q-Stat	Prob.	Adj Q-Stat	Prob.	df
1	2817.018	0.0000	2817.731	0.0000	81
2	2916.977	0.0000	2917.741	0.0000	162
3	3008.902	0.0000	3009.736	0.0000	243
4	3119.072	0.0000	3120.018	0.0000	324
5	3267.282	0.0000	3268.416	0.0000	405
6	3347.610	0.0000	3348.866	0.0000	486
7	3412.667	0.0000	3414.039	0.0000	567
8	3489.148	0.0000	3490.675	0.0000	648
9	3578.795	0.0000	3580.526	0.0000	729

df is degrees of freedom for (approximate) chi-square distribution

Table 4.1 reveals that the null hypothesis of no autocorrelation can be rejected in all cases. Hence, we concluded that there are strong autocorrelations in the residuals of the returns.

Jarque Bera Test for Normality: To achieve the overall objective of the research, we examine the characteristics of the unconditional distribution of the exchange rate. This will enable us to explore and explain some stylized facts embedded in the financial time series. Jarque Bera normality test is used to demonstrate this and the results are given in the table below:Note that the Jarque-Bera test is a goodness–of–fit measure of departure from normality, based on the sample kurtosis and skewness. Under the null hypothesis of normality, the statistic JB has an asymptotic chi-square distribution with two degrees of freedom.

Table 4.2 System Residual Normality Tests for the Nine Selected Currencies

Null Hypothesis: residuals are multivariate normal
Sample: 1/05/1999 2/21/2014
Included observations: 3949

Component	Skewness	Chi-sq	df
1	0.879085	508.6246	1
2	0.026677	0.468376	1
3	44.88030	1325706.	1
4	7.105127	33226.11	1
5	0.398252	104.3885	1
6	9.862163	64014.78	1
7	0.283621	52.94355	1
8	0.026361	0.457350	1
9	0.429407	121.3596	1
Joint		1423735.	9

Component	Kurtosis	Chi-sq	df
1	17.12798	32842.49	1
2	5.754510	1248.431	1
3	2521.141	1.04E+09	1
4	290.0217	13555179	1
5	8.287359	4599.953	1

6	309.1061	15417714	1
7	9.123025	6168.904	1
8	9.596338	7159.483	1
9	12.89901	16123.49	1
Joint		1.07E+09	9

Component	Jarque-Bera	df	Prob.
1	33351.11	2	0.0000
2	1248.899	2	0.0000
3	1.04E+09	2	0.0000
4	13588405	2	0.0000
5	4704.342	2	0.0000
6	15481729	2	0.0000
7	6221.848	2	0.0000
8	7159.941	2	0.0000
9	16244.85	2	0.0000
Joint	1.07E+09	18	0.0000

Table 4.2 indicated that the skewness is greater than zero (for the normal distribution), that is to say the distribution is positively skewed which is an indication of a non symmetric series, meaning that there is an asymmetric effects in these models (i.e. volatility is higher in a falling market than in a rising market) which is another stylize fact of financial time series. The kurtosis is also greater than 3 (the kurtosis of a normal distribution). Recall that; relatively large kurtosis suggests that the distribution of the exchange rate return series is leptokurtic (i.e. exhibit fat tail) which is another stylize fact. Thereafter, Jarque Bera normality test statistic indicates that, neither returns series has a normal distribution.

Modeling of restricted BEKK, DVEC and CCC models in Multivariate version

Table 4.3, contains the number coefficients, log-likelihood and information criteria for multivariate BEKK, DVECH and CCC models. And the table is given below:

Table 4.3: Summary of the Estimated Parameters of Multivariate BEKK, DVEC and CCC models

	BEKK	DVECH	CCC
No. of	36	144	72
	134658.4	134779.9	134572.7
	3.788819	3.792238	3.786406
	-68.18051	-68.18735	-68.11884
	-68.12325	-67.95831	-68.00432
	-68.16020	-68.10611	-68.07822
Log likelihood Avg. log			

AIC **SIC** **HIC**			

We can observe (from table 4.3) that base on information criteria, the DVECH model despite its estimation complexity due to large number of parameters needed to be estimated is found to be the best model because it has maximum likelihood, lower AIC and BIC. Following the DVECH model is the BEKK model with only 36 parameters. But according to parsimonious principle (Robert E. (1984)) the BEKK model is regarded as the best model because it has least number of parameters.
Now the next thing to do is to examine the correlations between the residuals (variances) of these models. These correlations have been explored in the table below:

Table 4.4 conditional correlation between the selected currencies

	DLOG(NAIRA)	DLOG(EURO)	DLOG(GBP)	DLOG(CHINA)	DLOG(S_AFRICA)	DLOG(SINGAPORE)	DLOG(JAPAN)	DLOG(MALAYSIA)	DLOG(CANADA)
DLOG(NAIRA)	1.000000	-0.021869	-0.000939	-0.009028	0.019182	0.033311	0.040382	-0.006979	0.029895
DLOG(EURO)	-0.021869	1.000000	0.035218	-0.058176	-0.395642	-0.489539	-0.132060	-0.163410	-0.398919
DLOG(GBP)	-0.000939	0.035218	1.000000	0.004946	-0.017672	-0.022081	-0.048124	-0.004954	-0.024244
DLOG(CHINA)	-0.009028	-0.058176	0.004946	1.000000	-0.159501	0.254555	0.048867	0.354794	-0.223182
DLOG(S_AFRICA)	0.019182	-0.395642	-0.017672	-0.159501	1.000000	0.375183	-0.037636	0.222131	0.501856
DLOG(SINGAPORE)	0.033311	-0.489539	-0.022081	0.254555	0.375183	1.000000	0.266694	0.508793	0.370951
DLOG(JAPAN)	0.040382	-0.132060	-0.048124	0.048867	-0.037636	0.266694	1.000000	-0.042282	-0.039131
DLOG(MALAYSIA)	-0.006979	-0.163410	-0.004954	0.354794	0.222131	0.508793	-0.042282	1.000000	0.206021
DLOG(CANADA)	0.029895	-0.398919	-0.024244	-0.223182	0.501856	0.370951	-0.039131	0.206021	1.000000

We can see that the nature of the correlation between the selected currencies is not identical; some of the markets are positively correlated while others are negatively correlated. For example, Naira, Rand, Singapore dollars, Yen and Canadian dollars are positively correlated. Also most of the Asian markets are positively correlated, e.g. Yuan, Singapore dollars, Yen and Ringgit. This is due to their closeness; their cultural and geographical similarities among other factors which have been conform to the arguments in most of the literatures.

V. SUMMARY AND CONCLUSION:

As stated earlier, an exchange rate is the current market price for which one currency can be exchanged for another. Currency exchange rates are among the most analyzed and forecasted indicators in the world. However, the exchange rate is determined by the level of supply and demand on the international markets. Similarly, changes in foreign exchange market rate are often difficult to understand and predict because the market is very large and volatile. So this research is aimed to establish the volatility model i.e. the Multivariate Generalized Autoregressive Conditional Heteroskedasticity (MGARCH) time series models to analyze the daily dollar-Naira exchange rates together with eight major currencies in the world between the periods of 4th January, 1999 and 21st February, 2014 inclusive. And hence, to observe some stylize facts/common features of good volatility modeling on financial time series (such as fat tails, volatility clustering, and etc.). To achieve the stated objectives, we employed three multivariate volatility models (i.e. DVECH, BEKK and CCC) and the result indicated that the restricted BEKK, DVECH and CCC results exhibit rather similar behavior for each considering countries. We further observe that in all three models, the time of the greatest peak match with time when the United States experienced serious financial meltdown. This shows that there is a serious interdependence of measure world currencies on the United States Dollar. Similarly, the indicated that the skewness is greater than zero (for the normal distribution), that is to say the distribution is positively skewed which is an indication of a non symmetric series, meaning that there is an asymmetric effects in these models (i.e. volatility is higher in a falling market than in a rising market) which is another stylize fact of financial time series. The kurtosis is also greater than 3 (the kurtosis of a normal distribution). Recall that; relatively large kurtosis suggests that the distribution of the exchange rate return series is leptokurtic (i.e. exhibit fat tail) which is another stylize fact. Thereafter, Jarque Bera normality test statistic indicates that, neither returns series has a normal distribution.The results also indicated that the nature of the correlation between the selected currencies is not identical; some of the markets are positively correlated while others are negatively correlated. For example, Naira, Rand, Singapore dollars, Yen and Canadian dollars are positively correlated. Also most of the Asian markets are positively correlated, e.g. Yuan, Singapore dollars, Yen and Ringgit. This is due to their closeness; their cultural and geographical similarities among other factors which have been conform to the arguments in most of the literatures. Conclusively, base on information criteria, the DVECH model is found to be the best model because it has maximum likelihood and lower Akaike information criteria (AIC) as well as lower Schwarz information criteria (SIC). But according to parsimonious principle, the BEKK model is considered to be the best because it has least number of parameters.

RECOMMENDATIONS: The results of the analysis reveals that, the DVECH and BEKK models are recommended to be the best models respectively because most of their variances/covariances are statistically significant and they have maximum likelihood, lower Akaike information criteria (AIC) and lower Schwarz information criteria (SIC). The research can serve as a step to observe the volatility modeling of the Naira exchange rates with other currencies so as to re-identify some silence phenomena of the exchange rate. Data of some specified periods can be tested by a feature researcher(s) by developing new and more models to capture the effect and predictions of the volatility behavior of the Naira exchange rate.

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