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Difference Labeling of Some Graph Families

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ABSTRACT: *A graph with vertex V and edge E is said to have difference labeling if for an injection f from V to the non-negative integers together with weight function f^* on E given by*
$f^*(uv) = |f(u) - f(v)|$ for each edge $uv \in E$.
A graph with difference labeling defined on it is called a labeled graph.
In this paper we investigate difference labeling on a gear graph G_n, Ladder L_n , Fan graph F_n, Friendship graph T_n, Helm graph H_n, wheel graph w_n.

KEYWORDS: *Difference labeling, common weight decomposition.*

I. INTRODUCTION:

In this paper, we consider only finite simple undirected graph. The graph G has vertex set $V = V(G)$ and edge set $E = E(G)$. The set of vertices adjacent to vertex u of G is denoted by $N(u)$, for notation and terminology we refer to Bondy and Muthy[2].A difference labeling of a graph G is realized by assigning distinct integer values to its vertices and then associating with each edge uv the absolute difference of those values assigned to its end vertices.The concept of difference Labelings was introduced by G.S.Bloom and S.Ruiz [1] and was further investigated by Arumugam and Meena[6]. Vaithilingam and Meena [7] further investigated difference labeling of crown graph $C_n^*$and grid graph$P_m * P_n^*$, pyramid graph fire cracker, banana trees.

Definition: 1.1: Let $G = (V, E)$ be a graph. A difference labeling of G is an injection f from V to the set of non-negative integer with weight function f^* on E given by $f^*(uv) = |f(u) - f(v)|$ for every edge uv in G.A graph with a difference labeling defined on it is called a labeled graph.

Definition: 1.2: A decomposition of labeled graph into parts, each part containing the edge having a common-weight is called a common – weight decomposition.

Definition: 1.3: A common weight decomposition of G in which each part contains m edges is called m-equitable.

Definition: 1.4: A gear graph is obtained from the wheel by adding a vertex between every pair of adjacent vertices of the cycle. The gear graph G_n has $2n + 1$ vertices and $3n$ edges.

Definition: 1.5: The Ladder graph L_n is the Cartesian product $P_n \times P_2$ and the Fan graph F_n is the graph obtained by joining all the vertices of a path P_n to a further vertex called the Centre.

Definition: 1.6: A fan graph obtained by joining all vertices of a path P_n to a further vertex, called the centre. Thus F_n contains n+1 vertices say C, $v_1, v_2, v_3 \dots v_n$ and (2n-1) edges, say cv_i , $1 \le i \le n$ and $v_i v_i + 1$, $1 \le i \le n - 1$.

Definition: 1.7: The friendship graph T_n is a set of n triangles having a common central vertex. For the i^{th} triangles let x_i and y_j denote the open vertices.

Definition: 1.8: The helm H_n is a graph obtained from a wheel by attaching a Pendant edge at each vertex of the n-cycle.

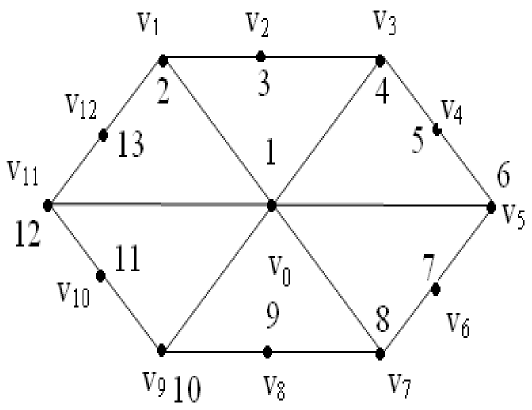
Definition: 1.9: A wheel W_n, $n \ge 3$ is a graph obtained by joining all vertices of cycle C_n to a further vertex c called the centre.

$$V(W_n) = \{c, v_1, v_2, \dots v_n\}$$
$$E(W_n) = \{cv_i / 1 < i < n\} \cup \{v_i v_{i+1} / 1 < i < n - 1\} \cup \{v_n v_1\}$$

II. MAIN RESULTS:

Theorem 2.1:
The gear graph G_n is a labeled graph with common weight decomposition.

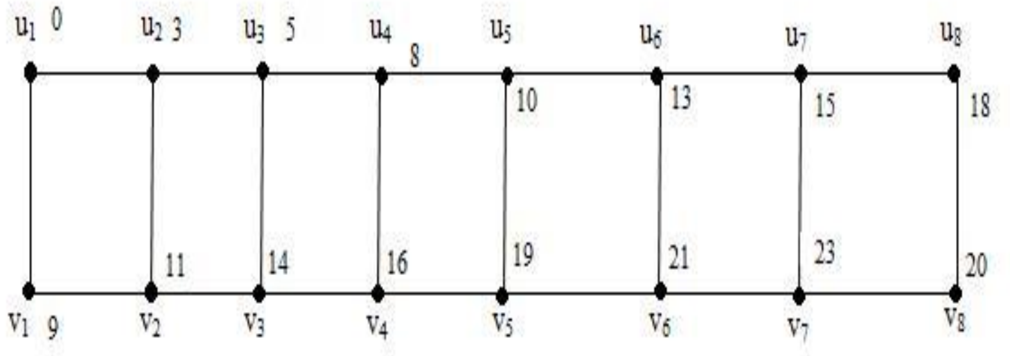
Proof:
Let $G = G_n$ be a gear graph. Let $V(G) = \{v_0, v_1, v_2, \dots v_{2n}\}$
Let $E(G) = \{ v_0v_{2i-1}, 1 \le i \le n \} U \{v_iv_{i+1}, 1 \le i \le n-1 \} U \{v_{2n}v_1\}$
$|V(G)| = 2n+1$
$|E(G)| = 3n$
$f: V(G) \to \{1,2,3 \dots 2n+1\}$ as follows
Let $f(v_0) = 1$
$f(v_i) = i+1 \qquad 1 \le i \le 2n$
Define the weight function f^* on E as $f^*(v_iv_{i+1}) = |f(v_i) - f(v_{i+1})|$ for $1 \le i \le 2n$
Then f^* has a value 1 on each edge on the circumference.
For the edge connecting the centre and the vertex on the circumference
$f^*(v_0v_i) = |f(v_0) - f(v_i)|$
There exist weights in the relation $1,3,5, \dots ie(2n-1)$
The gear graph is decomposed as $2nP_1P_{(2n-1)}$ graph.
Therefore the gear graph G_n is a labeled graph.
Example G_6



Theorem 2.2:
A labeling exists for every ladder of n vertices with common weight decomposition.

Proof:
Let $G = L_n = P_n \times p_2$ be a ladder graph. Let $V(L_n) = \{u_i, v_i, 1 \le i \le n\}$
Let $E(L_n) = \{ u_iu_{i+1}, v_iv_{i+1}, 1 \le i \le n-1 \} U \{u_iv_i, 1 \le i \le n \}$
Case i:
When n is even
Define a function $f: V(L_n) \to \{u_iv_i\}i = 1,2,3 \dots n$ to the set of positive integers as follows
Let $f(u_{2i-1}) = 5j - 5 \qquad i,j = 1,2, \dots \dots \frac{n}{2}$
$f(u_{2i}) = 5j - 2 \qquad i,j = 1,2, \dots \dots \frac{n}{2}$
$f(v_{2i-1}) = 5j + 4 \qquad 1 \le i,j \le \frac{n}{2} - 1$
$f(v_{2i}) = 5j + 6 \qquad 1 \le i,j \le \frac{n}{2} - 1$
$f(v_{n-1}) = 3n - 1$
$f(v_n) = 3n - 4$
Define the weight function f^* on G as $f^* = f(uv) = |f(u) - f(v)|$ then f^* decomposed the ladder L_n as
$nP_2 + (n-1)P_3 + (n-5)P_7 + (n-4)P_8$
Therefore when n is even, the ladder graph decomposes into parts as shown above.
The ladder is a labeled graph.

Example L_8



Case ii:

When n is odd.

Define a function $f: V(G)$ to the set of positive integers as follows

Let $f(u_{2i-1}) = 5j - 5 \quad i, j = 1,2, \ldots \ldots n - 4$

$f(u_{2i}) = 5j - 2 \quad i, j = 1,2, \ldots \ldots n - 5$

$f(v_{2i-1}) = 5j + 2 \quad i, j = 1,2,3 \ldots n - 5$

$f(v_{2i}) = 5j + 4 \quad i, j = 1,2 \ldots n - 6$

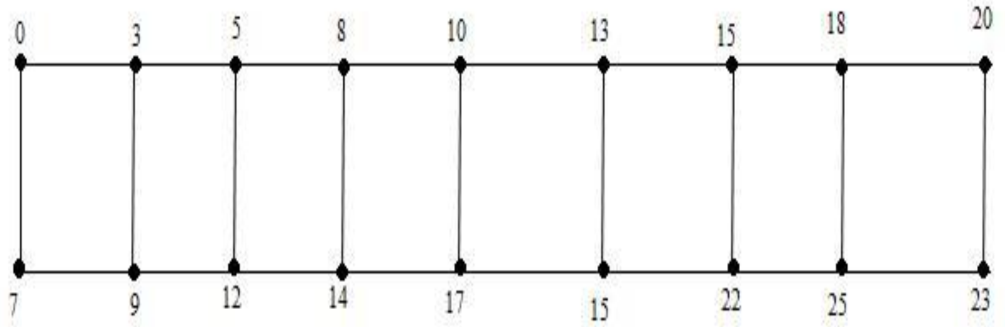
$f(v_{n-1}) = 3n - 2 \quad ; f(v_n) = 3n - 4$

Define the weight function f^* on G as $f(uv) = |f(u) - f(v)|$ then f^* decomposed the ladder L_n as $nP_3 U(n-1)P_2 U(n-6)P_6 U(n-4)P_7$

Therefore when n is odd, the ladder graph L_n becomes a labeled graph.

The ladder is a labeled graph.

Example when $n = 9$



Theorem 2.3:

The fan graph F_n is a labeled graph with common weight decomposition.

Proof:

Let $G = F_n$ be a fan graph. Let $V(G) = \{v_0, v_1, v_2 \ldots v_n\}$

Let $E(E) = \{v_0, v_i, 1 \le i \le n\} U \{v_i v_{i+1}, 1 \le i \le n - 1\}$

$|V(G)| = n + 1$

Now define a function f for its vertices to the set of integer as follows

Let $f(v_o) = 5j + 6 \quad i = 4$

$f(v_{2i-1}) = 5j - 5 \quad i, j = 1,2, \ldots \ldots \frac{n}{2}$

$f(v_{2i}) = 5j - 2 \quad i, j = 1,2, \ldots \frac{n}{2}$

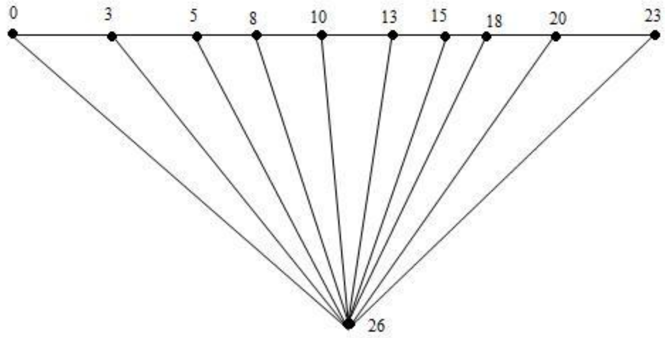
Define the weight function f^* on the edge of G as $f^* = f(uv) = |f(u) - f(v)|$ then f^* decomposed the edge G as

$\frac{n}{2} P_3 U \left(\frac{n}{2} - 1\right) P_2 U P_{5n+1} U P_{5n+3}$

Therefore when n is even, the fan graph decomposes by its weight function.

Therefore it is a labeled graph.

Example F_{10}



Case ii:

When n is odd, here $n = 5$

Define f as $f(V(G)) \to$ set of integers as follows.

Let $f(u_{2i-1}) = 5j - 2 \quad i, j = 1,2, \dots \dots n-2$

$f(u_{2i}) = 5j + 1 \quad i, j = 1,2, \dots n-3$

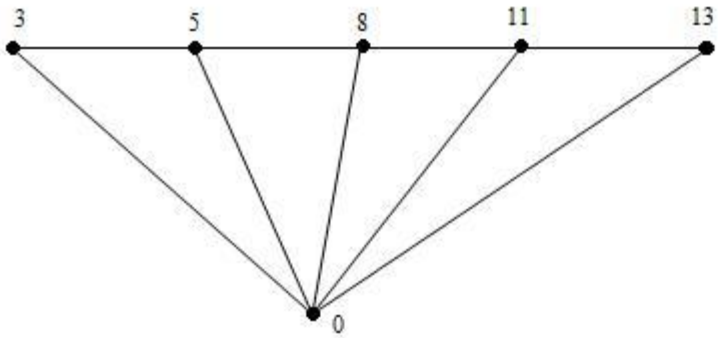
$f(u_o) = 0$

Define the weight function f^* on the edge of G as $f^* = f(uv) = |f(u) - f(v)|$ then f^* decomposed the edge G as

$$(n-2)P_3\, U (n-3)P_2\, U \sum_{i=1}^{2} P_{5i+1}\, U \sum_{i=1}^{2} P_{5i+3}$$

Therefore G is a labeled graph when n is odd.

Example F_5



Theorem 2.4:

The friendship graph T_n is a labeled graph with common weight decomposition.

Proof:

Let $G = T_n$ be a friendship graph. Let $V(G) = \{v_0, v_1, v_2 \dots v_{2n}\}$ where v_0 the centre vertex

Let $E(G) = \{ v_0v_i, 1 \le i \le 2n \}\, U \{v_iv_{i+1}, i = 1,2, \dots n\}$

$|V(G)| = n + 1$

$|E(G)| = 3n$

Now define a function f for its vertices to the set of integer as follows

Let $f(v_o) = 0$

$f(v_{2i-1}) = 5j - 2 \quad 1 \le i, j \le n$

$f(v_{2i}) = 5j + 1 \quad 1 \le i, j \le n$

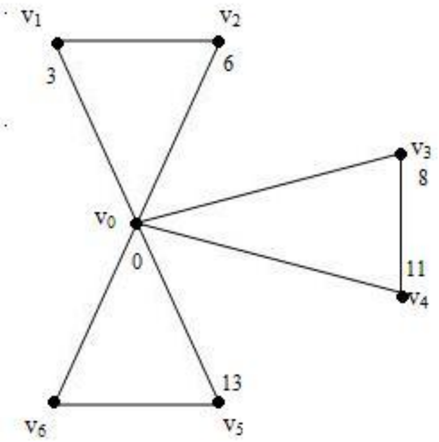
Define the weight function f^* on the edge of G as $f(uv) = |f(u) - f(v)|$ then f^* decomposed the edge G as

When $n = 3$ the total edge is 9, the decomposition

$$(n+1)P_3\, U \sum_{i=1}^{3} P_{5i+1}\, U \sum_{i=1}^{2} P_{5i+3}$$

Therefore G is a labeled graph.

Example T_3

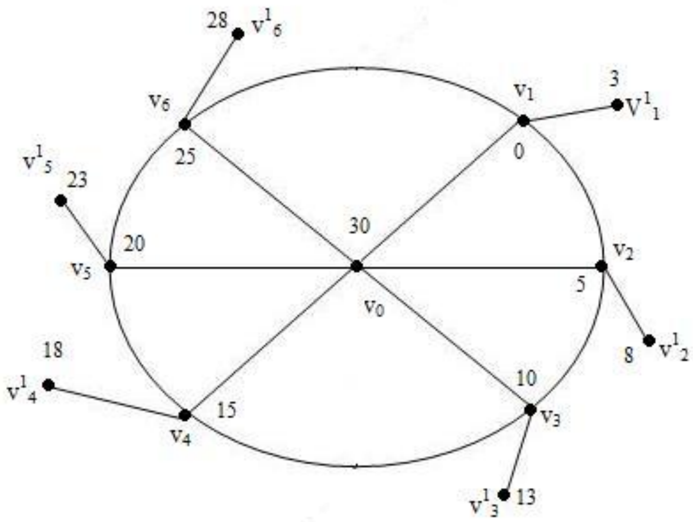


Theorem 2.5:
The helm graph H_n is a labeled graph with common weight decomposition.

Proof:
Let $G = H_n$ be a helm graph. Let $V(G) = \{v_0, v_1, v_2 \dots v_n, v_1^{'}, v_2^{'}, \dots v_n^{'}\}$ where v_0 the centre vertex $v_i, 1 \le i \le n$ is the vertex on the circumference and $v_i^{'}, 1 \le i \le n$ is a vertex attached at each $v_i^{'}$.
Let $V(G) = \{ v_0 v_i, 1 \le i \le n, v_i v_i^{'}, 1 \le i \le n\}$
$|V(G)| = n + 1$
$E(G) = \{ v_0 v_i, 1 \le i \le n \} U \{v_i v_i^{'}, 1 \le i \le n\} U \ v_0 v_n$
$|E(G)| = 3n$
Now define a function f on the vertices of G to the set of integer as follows
Let $f(v_i) = 5j$ where $1 \le i \le n, 0 \le i \le n - 1$
$f(v_i^{'}) = 5j - 2 \quad 1 \le i, j \le n$
$f(v_0) = 5n$
Define the weight function f^* on the edge of G as $f^* = f(uv) = |f(u) - f(v)|$ then f^* decomposed the edge G as

$$nP_3 \, U nP_5 \, U \sum_{i=1}^{5} P_{5i+5} \, U P_{25}$$

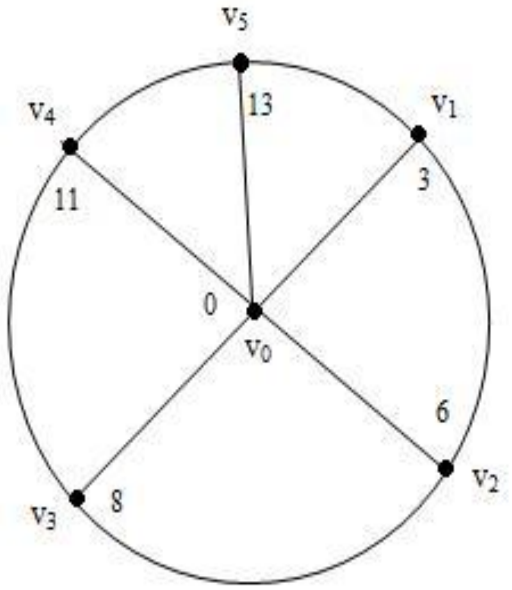
Therefore G is a labeled graph.
Example common weight decomposition of Helm graph H_6



Theorem 2.6:
The wheel graph W_n is a labeled graph with common weight decomposition.
Proof:
Let $G = W_n$ be a wheel graph. Let $V(G) = \{v_0, v_1, v_2 \dots\ v_n\}$
$|V(G)| = n + 1$
$E(G) = \{\ v_0 v_i, 1 \le i \le n\ \} U\ \{v_0 v_n\}$
Now define a function f on the vertices of G to the set of integer as follows
When n is odd,
Let $f(v_0) = 0$
$f(v_{2i-1}) = 5j - 2 \quad 1 \le i, j \le n - 3$
$f(v_{2i}) = 5j + 1 \quad 1 \le i, j \le n - 4$
Define the weight function f^* on the edge of G as $f^* = f(uv) = |f(u) - f(v)|$ then f^* decomposed the edge G as

$$(n-2)P_3 U (n-3)P_2 U \sum_{i=1}^{2} P_{5i+1} U \sum_{i=1}^{2} P_{5i+3} U P_{10}$$

By the common weight decomposition the wheel graph becomes a labeled graph when n is odd.
Therefore G is a labeled graph.
Example w_5

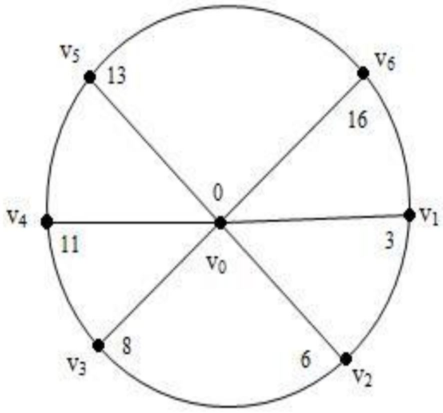


When n is even
Let $f(v_0) = 0$
$f(v_{2i-1}) = 5j - 2 \quad 1 \le i, j \le \frac{n}{2}$
$f(v_{2i}) = 5j + 1 \quad 1 \le i, j \le \frac{n}{2}$
Define the weight function f^* on the edge of G as $f^* = f(uv) = |f(u) - f(v)|$ then f^* decomposed the edge G as

$$(n-2)P_3 U (n-4)P_2 U \sum_{i=1}^{3} P_{5i+1} U \sum_{i=1}^{2} P_{5i+3} U P_{13}$$

By the common weight decomposition the wheel graph becomes a labeled graph when n is even.
Therefore G is a labeled graph.

Example w_6



III. CONCLUDING REMARKS:

Labeled graph is the topics of current interest due to its diversified applications. Here we investigate six results corresponding to labeled graphs similar work can be carried out for other families also.

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