International Journal of Mathematics and Statistics Invention (IJMSI)
E-ISSN: 2321 – 4767 P-ISSN: 2321 - 4759
www.ijmsi.org Volume 2 Issue 6 || June. 2014 || PP-14-16

I_{rwg} –Regular and I_{rwg} –Normal Spaces

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ABSTRACT: *In this paper, we characterize and discuss the properties of I_{rwg}- normal spaces and I_{rwg}-regular spaces.*

KEY WORDS: *Irwg- closed set , Irwg-regular space, Irwg-normal space.*

I. INTRODUCTION

In 1990, T.R.Hamlett and D.Jankovic [1], introduced the concept of ideals in topological spaces and after that [2, 3, 4, 5, 6] several authors turned their attention towards generalizations of various concepts of topology by considering ideal topological spaces. A nonempty collection I of subsets of a topological space (X, τ) is called a topological ideal if it satisfies the following two conditions:

(i) If $A\in I$ and $B\subset A$ implies $B\in I$ (heredity)

(ii) If $A \in I$ and $B \in I$, then $A\cup B \in I$ (finite additivity)

By a space (X, τ), we always mean a topological space (X, τ). If $A\subset X$, cl(A) and int(A) will, respectively denote the closure and interior of A in (X, τ). Let (X, τ ,I) be an ideal topological space and $A\subset X$. $A^*(I, \tau) = \{x \in X / U \cap A\notin I$ for every $U \in \tau(x)\}$ where $\tau(x) = \{ U \in \tau : x \in U\}$, is called the local function of A with respect to I [7] . For every topological space (X, τ, I) there exists a topology τ^* finer than τ defined as $\tau^*= \{U \subseteq X : cl^*(X - U)= X - U\}$. A Kuratowski closure operator $cl^*(.)$ for topology $\tau^*(I, \tau)$, $cl^*(A) = A \cup A^*$. Clearly, if $I = \{\emptyset\}$, then $cl^*(A) = cl(A)$ for every subset A of X. In this paper, we define I_{rwg}- normal space and I_{rwg}- regular space in ideal topological spaces and discuss their properties and characterizations.

II. PRELIMINARIES

Definition 2.1: [12] An ideal space (X, τ ,I) is said to be I-normal if for every pair of disjoint closed sets A and B of X, there exist disjoint open sets U and V such that $A-U \in I$ and $B-V \in I$. Clearly, if $I = \{\emptyset\}$, then normality and I-normality coincide.

Definition 2.2: [13] A subset A of an ideal space (X, τ, I) is said to be a regular weakly generalized closed set with respect to ideal I [I_{rwg}- closed] if $A^* \subset U$ whenever $A \subset U$ and U is regular open.

Lemma 2.3: [11,Theorem 5]: Let (X, τ ,I) be an ideal space and A be a subset of X. If $A \subset A^*$, then $A^* = cl(A^*) = cl(A) = cl^*(A)$.

III. REGULAR WEAKLY GENERALIZED NORMAL SPACES IN IDEAL TOPOLOGICAL SPACES

Definition 3.1: An ideal space (X, τ, I) is said to be an I_{rwg} -normal space if for every pair of disjoint closed sets A and B, there exist disjoint I_{rwg} -open sets U and V such that $A \subset U$ and $B \subset V$.

Remark 3.2: Every normal space is I_{rwg} -normal but the converse need not be true as seen from the following example.

Example 3.3: Consider the ideal space (X, τ, I) where X = {a,b,c}, τ = {∅, X,{b},{a,b},{b,c}} and I = {∅, {b}}. Since every I_{rwg} -open set is * - closed, every subset of X is I_{rwg} -closed and hence every subset of X is I_{rwg} -open. This implies that (X, τ, I) is I_{rwg} -normal. Now {a} and {c} are disjoint closed subsets of X which not separated by disjoint open sets and so (X, τ) is not normal.

Theorem 3.4: Let (X, τ, I) be an ideal topological space. Then if X is I_{rwg} -normal iff for every closed set A and an open set V containing A, there exists an I_{rwg} -open set U such that $A \subset U \subset cl^*(U) \subset V$.

Proof: Let A be a closed set and V be an open set containing A. Then A and X−V are disjoint closed set and so there exist disjoint I_{rwg} -open sets U and W such that $A \subset U$ and $X - V \subset W$. Thus, $U \cap W = \emptyset$ implies that $U \cap int^*(W) = \emptyset$ which implies that $U \subset X - int^*(W)$ and so $cl^*(U) \subset X - int^*(W)$. Again, $X - V \subset W$ implies that $X - W \subset V$ where V is open implies $cl^*(X - W) \subset V$ and so $X - int^*(W) \subset V$. Thus, $A \subset U \subset cl^*(U) \subset X - int^*(W) \subset V$. Therefore, $A \subset U \subset cl^*(U) \subset V$ where U is I_{rwg} -open.

Conversely, Let A and B be two disjoint closed subsets of X. By hypothesis, there exists an I_{rwg} -open set such that $A \subset U \subset cl^*(U) \subset X - B$. Now, $cl^*(U) \subset X - B$ implies that $B \subset X - cl^*(U)$. If $X - cl^*(U) = W$, then W is an I_{rwg} -open, since every * - closed set is I_{rwg} -closed.

Hence U and W are the required disjoint I_{rwg} -open sets containing A and B respectively. Therefore X is I_{rwg} -normal.

Theorem 3.5: Let (X, τ, I) be an ideal topological space where I is completely codense. Then (X, τ, I) is an I_{rwg} -normal space iff (X, τ) is a normal space.

Proof: X is I_{rwg} -normal if and only if for each pair of disjoint closed sets A and B, there exist disjoint I_{rwg} -open sets U and V such that $A \subset U$ and $B \subset V$. Since (X, τ, I) is an ideal topological space where I is completely codense, every I_{rwg} -open sets is open . Therefore X is normal.

Theorem 3.6: Let (X, τ, I) be an ideal topological space which is I_{rwg} - normal, then the following hold.

[1] If F is closed and A is a g -closed set such that $A \cap F = \emptyset$, then there exist disjoint I_{rwg} -open sets U and V such that $cl(A) \subset U$ and $F \subset V$.

[2] For every closed set A and every g-open set B containing A, there exists an I_{rwg} -open set U such that $A \subset int^*(U) \subset U \subset B$.

[3] For every g -closed set A and every open set B containing A, there exists an I_{rwg}- closed set U such that $A \subset U \subset cl^*(U) \subset B$.

Proof: a) Let X be an I_{rwg} -normal space. Since $A \cap F = \emptyset$, $A \subset X - F$ where X – F is open. Therefore, by hypothesis $cl(A) \cap F = \emptyset$. Since X is I_{rwg} -normal, there exist disjoint I_{rwg} -open sets U and V such that $cl(A) \subset U$ and $F \subset V$.

b) Let A be a closed set and B be a g-open set containing A. Then X – B is g-closed set such that $A \cap (X - B) = \emptyset$. By (a), there exist disjoint I_{rwg} -open sets U and V such that $A \subset U$ and $X - B \subset cl(X - B) \subset V$. $A \subset U$ implies that $A \subset int^*(U)$, by [13], $A \subset int^*(U) \subset U \subset X - V \subset B$.

c) Let A be a g -closed set and B be an open set containing A. Then X – B is a closed set contained in the g-open set X – A. By (b), there exists an I_{rwg} – open set V such that $X - B \subset int^*(V) \subset V \subset X - A$. Therefore, $A \subset X - V \subset cl^*(X - V) \subset B$. If U = X – V, then $A \subset U \subset cl^*(U) \subset B$ and so U is the required I_{rwg} – closed set.

Theorem 3.7: Let (X, τ, I) be an ideal topological space . Then every closed subspace of an I_{rwg} -normal space is I_{rwg} -normal.

Proof: Let Y be a closed subspace of a I_{rwg} -normal space (X, τ, I). Let τ_1 be the relative topology for Y. Let E_1 and F_1 be any two disjoint τ_1- closed subsets of Y. Then there exist τ- closed subsets E and F such that $E_1 = Y \cap E$ and $F_1 = Y \cap F$. Since Y and E are τ- closed, E_1 is also τ- closed and F_1 is also τ- closed. Thus E_1 and F_1 are the disjoint subsets of an I_{rwg} -normal space (X, τ, I). Therefore, there exist disjoint I_{rwg} -open sets U and V such that $E_1 \subset U$ and $F_1 \subset V$. Hence for every disjoint closed sets E_1 and F_1 in Y , we can find disjoint I_{rwg} -open sets U and V such that $E_1 \subset U$ and $F_1 \subset V$. Therefore, $E_1 \subset U \cap Y$ and $F_1 \subset V \cap Y$ where $U \cap Y$ and $V \cap Y$ are disjoint I_{rwg} – open sets in Y. Hence (Y, τ_1 , I) is I_{rwg} -normal.

IV. REGULAR WEAKLY GENERALIZED REGULAR SPACES IN IDEAL TOPOLOGICAL SPACES

Definition 4.1: An ideal space (X, τ, I) is said to be an I_{rwg} – regular space if for each pair consisting of a point x and a closed set B not containing x, there exist disjoint non empty I_{rwg} - open sets U and V such that $x \in U$ and $B \subset V$.

Theorem 4.2: In an ideal topological space (X, τ, I), the following are equivalent.

a) X is I_{rwg} -regular.

b) For every open set V containing $x \in X$, there exists an I_{rwg} -open set U of X such that $x \in U \subset cl^*(U) \subset V$.

Proof: a) ⇒ b) Let V be an open subset such that $x \in V$. Then $X-V$ is a closed set not containing x. Therefore, there exist disjoint I_{rwg} -open sets U and W such that $x \in U$ and $X-V \subset W$. Now, $X-V \subset W$ implies that $X-V \subset int^*(W)$ which implies that $X - int^*(W) \subset V$. Again, $U \cap W = \emptyset$ implies that $U \cap int^*(W) = \emptyset$ and so $cl^*(U) \subset X - int^*(W)$. Hence $x \in U \subset cl^*(U) \subset X - int^*(W) \subset V$ and so $x \in U \subset cl^*(U) \subset V$.

b) ⇒ a) Let B be a closed set not containing x. By hypothesis, there exists and I_{rwg} -open set U such that $x \in U \subset cl^*(U) \subset X - B$. Now, $cl^*(U) \subset X - B$ implies that $B \subset X - cl^*(U)$. If $W = X - cl^*(U)$, then U and W are disjoint I_{rwg} – open sets such that $x \in U$ and $B \subset W$.

Theorem 4.3: If (X, τ, I) is an I_{rwg} -regular, T_1 -space where I is completely codense, then X is regular.

Proof: Let B be a closed set not containing $x \in X$. Then there exists an I_{rwg} – open set U of x such that $x \in U \subset cl^*(U) \subset X - B$. Since X is a T_1 -space , {x} is closed and so $\{x\} \subset int^*(U)$. Since I is completely codense, so $x \in int^*(U) \subset int(cl(int(int^*(U))) = G$ and $B \subset X - cl^*(U) \subset int(cl(int(X - cl^*(U))) = H$. Therefore, G and H are the required disjoint open sets containing x and B, respectively. Hence X is regular.

Theorem 4.4: Let (X, τ, I) be an ideal space. Then the following are equivalent

[1] X is I_{rwg} -regular.
[2] For each $x \in X$ and open set U containing x, there is an open set V containing x such that $cl(V) - U \in I_{rwg}$.
[3] For each $x \in X$ and closed set A not containing x, there is an open set V containing x such that $cl(V) \cap A \in I_{rwg}$.

Proof: a) ⇒ b) Let $x \in X$ and U be an open set containing x. Then there exist disjoint open sets V and W such that $x \in V$ and $(X-U)-W \in I_{rwg}$. If $(X-U)-W = I \in I_{rwg}$, then $(X-U) \subset W \cup I$. Now $V \cap W = \emptyset$ implies that $V \subset X - W$ and so $cl(V) \subset X - W$. Now $cl(V) - U \subset (X - W) \cap (W \cup I) = (X-W) \cap I \subset I \in I_{rwg}$.

b) ⇒ c) Let A be closed in X such that $x \notin A$. Then, there exists an open set V containing x such that $cl(V) - (X - A) \in I_{rwg}$ which implies that $cl(V) \cap A \in I_{rwg}$.

c) ⇒ a) Let A be closed in X such that $x \notin A$. Then there exists an open set V containing x such that $cl(V) \cap A \in I_{rwg}$. If $cl(V) \cap A = I \in I_{rwg}$, then $A - (X - cl(V)) = I \in I_{rwg}$. V and $(X-cl(V))$ are the required disjoint open sets such that $x \in V$ and $A - (X - cl(V)) \in I_{rwg}$. Hence X is I_{rwg} -regular.

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