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Steady State Analysis of an optimal designof N-Policybatch arrival queueing system with server's single vacation, setup time, closed down time, second optional fast slow service and breakdown

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ABSTRACT :*This paper deals with a M[X]/(GF,GS)/1 queueing system in which all the customers undergo first essential service (FES) and only some of them receive second optional service(SOS) by the same server. In addition to this the server is unreliable and hence subjected to random breakdowns while in service and the server leaves for single vacation when the system is empty. After returning from vacation if there are N or more customers in the system then the server does setup work before it starts the service. Explicit analytical expressions for various performance measures are derived. A cost model for the optimal operating N- Policy that minimizes the total expected cost per unit time is determined.*

KEYWORDS: *$M^{[X]}$/ (G_F, GS)/1 queueingsystem, singlevacation, N-Policy, setup time, breakdown, optimalcostmodel*

I. INTRODUCTION

In day-to-day life, one encounters numerous examples of queuing situations, where all arriving customers require the main service and only some may require the subsidiary service provided by the server. K.C.Madhan [1] has done some initial work on the steady state behavior of M/G/1 queue with second optional service and later Choudhry and Paul [2] extended the results of Madhan [1] to a batch arrival queue under N-policy. The authors mentioned above have focused on reliable servers. Queueing models with second optional service and breakdowns accommodate real world situations more closely. Therefore, it would be practical to consider the [3]N-policy for the batch arrival $M^{[X]}$/G/1 queueing system in which the service is unreliable and all the arriving customers demand the first essential service (FES) [5]where as only some of them demand the second optional service (SOS).There are several vacation policies and this paper deals with single vacation policy[7]. Also the server needs a random amount of time for preparatory work which is termed as server's setup time (or) startup time[8]. In this model, it is assumed that after returning from vacation if the server finds N (or) more customers in the system then the server is turned on for the start up work of random length D and as soon as the server finishes the setup work, the busy period initiates. The steady state behavior of queue size distribution is analyzed for this model, using the supplementary variable technique. Various performance measures such as expected system size and expected length of the cycle are also calculated. The PGF of the system size distribution at an arbitrary epoch is derived.

II MODELDESCRIPTION

The $M^{[X]}/(G_F,G_S)/1$ Queueing System under consideration has the following specification.

Compound Arrival Process and closed downtime

The customers are assumed to arrive in batches according to compound Poisson process with arrival rate l. The number of units arrive at an arbitrary instant is a random variable X whose probability distribution is given by Pr(X = k) = g_k, k = 1, 2, 3 …

N-Policy Setup Time and Single Vacation

A cycle begins right after the system becomes empty and the server leaves the system for vacation. After returning from vacation, if the server finds N (or) more customers present in the system then the server takes random amount of setup time for preparatory work before starting the service. The setup time is a random variable with finite moments which has the general distribution D(t) and density function d(t). The customers arriving during the vacation period and setup period will join the queue and wait for their turns. If the server returning from vacation finds less than N customers in the system then he stays idle in the system (i.e.) the server takes only single vacation. The period during which the server remains idle in the system to start the preparatory work after returning from vacation is called build up period. The vacation time follows the general distribution V(t) with finite moments and density function v(t).The server may requires some amount of time for doing the service after the service is completed.C(t) be cumulative distribution of closed down time,c(t) be probability density function.

Busy Period and Server's Breakdown

Immediately after the setup time the busy period starts and customers are served one by one according to FCFS queue discipline. During busy period the server provides each unit two types of heterogeneous services of which, one is optional. (i.e.) the server provides first essential service (FES) to all the arriving customers and after the completion of FES the customers may leave the system with probability (1 – r) (or) may opt for the SOS with probability r (0 £ r £ 1). During the services (FES (or) SOS) the server may undergo breakdowns according to the Poisson process with rates a_i, i = 1, 2. Whenever the breakdowns occur the server is sent immediately for repair and the repair times follow the general distributions $U_i(t)$, i = 1, 2 with finite mean and variance. Once the server gets repaired, he is sent back to the service facility to resume the service. Thus the vacation period, setup period, busy period and break down periods constitute a cycle. It is also assumed that the arrival processes, vacation time, service time and setup time are independent of each other.

III STEADYSTATESYSTEMSIZEEQUATIONS

To obtain the steady state system size equations of the model using supplementary variable technique, we employ the remaining service time, the remaining setup time and the remaining vacation time of the server as the supplementary variables. The following notations and probabilities are used to derive the steady state equations of the model.

$P_{n1}(x, t)\,dt = \Pr(N(t) = n, x\ S_1^0(t) \leq x + dt, Y(t) = 1) \qquad n \geq 1$

$P_{n2}(x, t)\,dt = \Pr(N(t) = n, x\ S_2^0(t) \leq x + dt, Y(t) = 3) \qquad n \geq 1$

$B_{n1}(x, y, t)\,dt = \Pr(N(t) = n, S_1^0(t) = x, y \leq U_1(t) \leq y + dt, Y(t) = 2), \qquad n \geq 1$

$B_{n2}(x, y, t)\,dt = \Pr(N(t) = n, S_2^0(t) = x, y \leq U_2(t) \leq y + dt, Y(t) = 4), \qquad n \geq 1$

$D_n(x, t)\,dt = \Pr(N(t) = n, x \leq D^0(t) \leq x + dt, Y(t) = 5), \qquad n \geq N$

$C_n(x, t)\,dt = \Pr(N_q(t) = n, x \leq C^0(t) \leq x + dt, Y(t) = 1), \qquad n \geq 0$

IV STEADY STATE EQUATIONS

Under the steady state, the system size probabilities are assumed to be independent of time and the steady state equations are given by,

$$l R_0 = Q_0(0) \tag{1}$$

$$l R_n = Q_n(0) + l \sum_{k=1}^{n} R_{n-k}\ gk, \qquad 1 £ n £ N – 1 \tag{2}$$

$$\frac{-d}{dx} P_{11}(x) = - (l + a_1) P_{11}(x) + (1 – r) P_{21}(0) s_1(x) + B_{11}(x, 0) + P_{22}(0) s_1(x) \tag{3}$$

$$\frac{-d}{dx} P_{n1}(x) = - (l + a_1) P_{n1}(x) + (1 – r) P_{n+11}(0) s_1(x)$$

$$+ B_{n1}(x, 0) + P_{n+12}(0) \quad s_1(x) + l \sum_{k=1}^{n-1} P_{n-k1}(x)\, g_k, \ 2 \leq n \leq N\text{-}1 \tag{4}$$

$$\frac{-d}{dx} P_{n1}(x) = - (l + a_1) P_{n1}(x) + (1 – r) P_{n+11}(0) s_1(x) + B_{n1}(x, 0)$$

$$+ D_n(0) s_1(x)+ P_{n+12}(0) s_1(x) + l \sum_{k=1}^{n-1} P_{n-k1}(x)\,(x)gk, \quad n \geq N \tag{5}$$

$$\frac{-d}{dx} P_{12}(x) = - (l + a_2) P_{12}(x) + r P_{11}(0) s_2(x) + B_{12}(x, 0) \tag{6}$$

$$\frac{-d}{dx} Pn2(x) = - (l+ a2) Pn2(x) + r Pn1(0) s2(x) + l\sum_{k=1}^{n-1} P_{n-k2}(x)\ gk + B_{n2}(x, 0), \quad n \geq 2 \tag{7}$$

$$\frac{-d}{dx} Q_0(x) = - lQ_0(x) + (P_{11}(0) (1 – r) + P_{12}(0)) v(x) \tag{8}$$

$$\frac{-d}{dx} Qn(x) = - l Qn(x) + l \sum_{k=1}^{n-1} Qn - k\ (x)g^k, \quad n \geq 1 \tag{9}$$

$$\frac{-d}{dx} D_N(x) = -l D_N(x) + Q_N(0) d(x) + l \sum_{k=1}^{N} R_{N-k}\ g_{k\,d(x)},$$

(10)

$$\frac{-d}{dx} D_n(x) = -l D_n(x) + Q_n(0) d(x) + l \sum_{k=n-N+1}^{n} R_{N-k}\ g_{k\,d(x)} + l\sum_{k=1}^{n-N} D_{n-k}\, g_k, \ n \geq N+1 \tag{11}$$

$$\frac{-d}{dy} B_{11}(x, y) = -l B_{11}(x, y) + a_1 P_{11}(x) u_1(y) \tag{12}$$

$$\frac{-d}{dy} B_{n1}(x, y) = -l B_{n1}(x, y) + a_1 P_{n1}(x) u_1(y) + l \sum_{k=1}^{n-1} B_{n-k1}(x, y)\, g_k, \quad n \geq 2 \tag{13}$$

$$\frac{-d}{dy} B_{12}(x, y) = -l B_{12}(x, y) + a_2 P_{12}(x) u_2(y) \tag{14}$$

$$\frac{-d}{dy} B_{n2}(x, y) = -l B_{n2}(x, y) + a_2 P_{n2}(x) u_2(y) + l \sum_{k=1}^{n-1} B_{n-k2}(x, y)\, g_k, \quad n \geq 2 \tag{15}$$

$\frac{-d}{dx}$ Cn(x) = -l Cn(x) + $\sum_{m=a}^{b} P_{mn}\ C(x) + \sum_{k=1}^{n} C_{n-k}\ (x) g_k$
(16)

$\frac{-d}{dx}$Cn(x) = -l Cn(x) + $\sum_{k=1}^{n-a} C_{n-k}\ (x)$ lg_k n ³ a

V LISTDEFINITION

The following Laplace Stieltjes Transform (LST) are defined to derive the PGF of the system size

$P_{ni}^{*}(\theta) = \int_0^{\theta} e^{-\theta x} P_{ni}(x)dx$; $S_i^{*}(\theta) = \int_0^{\theta} e^{-\theta x} dS_i(x)$, i=1,2

$B_{ni}^{*}(\theta, y) = \int_0^{\theta} e^{-\theta x} B_{ni}(x, y)dx$,i=1,2

$D_n^{*}(\theta) = \int_0^{\theta} e^{-\theta x} \mathrm{D}_n(x)dx$ $D^{*}(\theta) = \int_0^{\theta} e^{-\theta x}\,\mathrm{d\,D(x)}$

$Q_n^{*}(\theta) = \int_0^{\theta} e^{-\theta x} \mathrm{Q}_n(x)dx$ $V^{*}(\theta) = \int_0^{\theta} e^{-\theta x}\,\mathrm{d\,V(x)}$

$D_n^{*}(\theta) = \int_0^{\theta} e^{-\theta x} \mathrm{D}_n(x)dx$; $D^{*}(\theta) = \int_0^{\theta} e^{-\theta x}\,\mathrm{d\,D(x)}$

$\theta\, P^*_{11}(\theta) - P_{11}(0) = (\lambda + a_1)\, P^*_{11}(\theta) - (1 - r)\, P_{21}(0)\, S_1(\theta) - B^*_{11}(\theta, 0) - P_{22}(0) S^*_1(\theta)$ (18)

$\theta\, P^*_{n1}(\theta) - P_{n1}(0) = (\lambda + a_1)\, P^*_{n1}(\theta) - (1 - r)\, P_{n+11}(0)\, S^*_1(\theta) - B^*_{n1}(\theta, 0)$
$- P_{n+1}(0) S^*_1(\theta) - \lambda\ \sum_{k=1}^{n-1} P^*_{n-k1}(\theta) g_k,\ 2 \le n \le$ N-1 (19)

$\theta\, P^*_{n1}(\theta) - P_{n1}(0) = (\lambda + a_1)\, P_{n1}(\theta) - (1 - r)\, P_{n+11}(0)\, S^*_1(\theta) - B^*_{n1}(\theta, 0)$

$- D_n(0)\, S^*_1(\theta) - P_{n+12}(0)\, S^*_1(\theta) - \lambda\ \sum_{k=1}^{n-1} P^*_{n-k1}(\theta) g_k,\ n \ge N$ (20)

$\theta\, P^*_{12}(\theta) - P_{12}(0) = (\lambda + a_2)\, P^*_{12}(\theta) - r\, P_{11}(0)\, S^*_2(\theta) - B^*_{12}(\theta, 0)$ (21)

$\theta\ P^*_{n2}(\theta) - P_{n2}(0) = (\lambda + a2)\ P^*_{n2}(\theta) - r\ P_{n1}(0)\ S^*_2(\theta) - \lambda\ \sum_{k=1}^{n-1} P^*_{n-k2}(\theta) g_k - B * n2(\theta, 0)$
(22)

$$\theta Q^*_0(\theta) - Q_0(0) = \lambda Q^*_0(\theta) - (P_{11}(0)(1-r) + P_{12}(0)) V^*(\theta) \quad (23)$$

$$\theta Q^*_n(\theta) - Q_n(0) = \lambda Q^*_n(\theta) - \lambda \sum_{k=1}^{n} Q^*_{n-k}(\theta)\, g_k, \; n \geq 1 \quad (24)$$

$$\theta D^*_N(\theta) - D_N(0) = \lambda D^*_N(\theta) - Q_n(0) D^*(\theta) - \lambda \sum_{k=1}^{N} R_{n-k}\, g_k\, D^*(\theta), \quad (25)$$

$$\theta D^*_n(\theta) - D_n(0) = \lambda D^*_n(\theta) - Q_n(0) D^*(\theta) - \lambda \sum_{k=n-N+1}^{n} R_{n-k}\, g_k\, D^*(\theta) - \lambda \sum_{k=1}^{n-N} D^*_{n-k}(\theta)\, g_k, \quad n \geq N+1$$
(26)

$$\theta C^*_n(\theta) - C_n(0) = \lambda C^*_n(\theta) - \sum_{m=a}^{b} P_{mn}(0)\, C^*(\theta) - \sum_{k=1}^{n} C^*_{n-k}(\theta)\, \lambda\, g_k, \; n \leq a \quad (27)$$

$$\theta C^*_n(\theta) - C_n(0) = \lambda C^*_n(\theta) - \sum_{k=1}^{n-a} C^*_{n-k}, \; n \geq a \quad (28)$$

$$\frac{-d}{dy} B^*_{11}(\theta, y) = -\lambda B^*_{11}(\theta, y) + a_1 P^*_{11}(\theta)\, u_1(y) \quad (29)$$

$$\frac{-d}{dy} B^*_{n1}(\theta, y) = -\lambda B^*_{n1}(\theta, y) + a_1 P^*_{n1}(\theta)\, u_1(y) + \lambda \sum_{k=1}^{n-1} B_{n-k1}(\theta, y) \quad (30)$$

$$\frac{-d}{dy} B^*_{12}(\theta, y) = -\lambda B^*_{12}(\theta, y) + a_2 P^*_{12}(\theta)\, u_2(y) \quad (31)$$

$$\frac{-d}{dy} B^*_{n2}(\theta, y) = -\lambda B^*_{n2}(\theta, y) + a_2 P^*_{n2}(\theta)\, u_2(y) + \lambda \sum_{k=1}^{n-1} B^*_{n-k2}(\theta, y)\, g_k, \; n \geq 2$$
(32)

The LST with respect to repair time are defined by,

$$B^{**1}_{ni}(\theta, \theta_1) = \int_0^{\infty} e^{-\theta_1 y} B^*_{ni}(\theta, y)\, dy$$

$$U^{*1}_i(\theta_1) = \int_0^{\infty} e^{-\theta_1 y}\, u_i(y)\, dy$$

Taking the LST on both sides of equations (29) to (32) we have,

$$\theta_1 B^{**1}_{11}(\theta, \theta_1) - B^{*1}_{11}(\theta, 0) = \lambda B^{**}_{11}(\theta, \theta_1) - a_1 P^*_{11}(\theta) U^{*1}_1(\theta_1) \quad (33)$$

$$\theta_1 B^{**1}_{n1}(\theta, \theta_1) - B^{*1}_{n1}(\theta, 0) = \lambda B^{**}_{n1}(\theta, \theta_1) - a_1 P^*_{11}(\theta) U^{*1}_1(\theta_1) - \lambda \sum_{k=1}^{n-1} B^{**1}_{n-k1}(\theta, \theta_1)\, g_k, \; n \geq 2 \quad (34)$$

$$\theta_1 B^{**1}_{12}(\theta, \theta_1) - B^{*1}_{12}(\theta, 0) = \lambda B^{**}_{12}(\theta, \theta_1) - a_2 P^*_{12}(\theta) U^{*1}_2(\theta_1) \quad (35)$$

$$\theta_1 B^{**1}_{n2}(\theta, \theta_1) - B^{*1}_{n2}(\theta, 0) = \lambda B^{**}_{n2}(\theta, \theta_1) - a_2 P^*_{n2}(\theta) U^{*1}_2(\theta_1) - \lambda \sum_{k=1}^{n-1} B^{**1}_{n-k2}(\theta, \theta_1)\, g_k, \; n \geq 2 \quad (36)$$

VI PGF OF THE SYSTEM SIZE PROBABILITIES

The following partial PGFs for $|z| \leq 1$ are defined to determine the system size distribution.

$$R(z) = \sum_{n=0}^{N-1} R_n\, z^n$$

$$P^*_i(z,\theta) = \sum_{n=1}^{\infty} P^*_i(\theta) z^n, \qquad P_i(z,0) = \sum_{n=0}^{N-1} P_{ni}(0)\, z^n, \; i=1,2$$

$$D^*(z,\theta) = \sum_{n=N}^{\infty} D^*_n(\theta) z^n, \qquad D(z,0) = \sum_{n=N}^{N-1} D_n(0)\, z^n$$

$C^*(z,\theta) = \sum_{n=N}^{\infty} C_n^*(\theta) z^n$, $C(z,0) = \sum_{n=N}^{N-1} C_n(0) z^n$

$Q^*(z,\theta) = \sum_{n=0}^{\infty} Q_n^*(\theta) z^n$, $Q(z,0) = \sum_{n=0}^{\infty} Q_n(0) z^n$

$B_i^{**1}(z,\theta,\theta_1) = \sum_{n=1}^{\infty} B_{ni}^{**1}(\theta,\theta_1) z^n$

$B_i^{**1}(z,\theta,0) = \sum_{n=1}^{\infty} B_{ni}^{**1}(\theta,0) z^n , i = 1,2$

VII. IDENTITIES

Here some important identities used in this paper are listed out

* $\sum_{n=2}^{\infty} z^n \left(\sum_{k=1}^{n-1} P_{n-k1}^*(\theta)\, g_k\right) = \left(\sum_{n=1}^{\infty} P_{n1}^*(\theta)\, z^n\right)\left(\sum_{k=1}^{\infty} g_k z^k\right)$
$= P_2^*(z,\theta) X(z)$
* $\sum_{n=2}^{\infty} z^n \left(\sum_{k=1}^{n-1} P_{n-k1}^*(\theta)\, g_k\right) = \left(\sum_{n=1}^{\infty} P_{n1}^*(\theta)\, z^n\right)\left(\sum_{k=1}^{\infty} g_k z^k\right)$
$= P_2^*(z,\theta) X(z)$
* $\sum_{n=2}^{\infty} z^n \left(\sum_{k=1}^{n-1} B_{n-k1}^{**1}(\theta,\theta 1)\, g_k\right) = \left(\sum_{n=1}^{\infty} B_{n1}^{**1}(\theta,\theta 1)\, z^n\right)\left(\sum_{k=1}^{\infty} g_k z^k\right)$
$= B_{n1}^{**1}(\theta,\theta 1) X(z)$
* $\sum_{n=N}^{\infty} z^n \left(\sum_{k=n-N+1}^{n} R_{n-k}\, g_k\right) + \sum_{n=1}^{N-1} z^n \left(\sum_{k=1}^{n} R_{n-k}\, g_k\right)$
$= \left(\sum_{n=0}^{N-1} R_n z^n\right)\left(\sum_{k=1}^{\infty} g_k z^k\right)$
$= R(z) X(z)$

* $\frac{d}{dZ}\left(\frac{1-D^*(W_X(z))V^*(W_X(z))}{1-X(z)}\right)_{z=1} = \frac{E(X)[(E(D^2)+2E(D)E(V)+E(V^2)]}{2}$

* $\frac{d}{dZ}\left(\frac{D^*(W_X(z))(1-V^*(W_X(z))}{W_X(z)}\right)_{z=1} = E(X)\left(\frac{E(V^2)}{2} + E(V)E(D)\right)$

* $\frac{d}{dZ}\left(\frac{1-D^*(W_X(z))}{W_X(z)}\right)_{z=1} = \frac{E(X)(E(D^2)}{2}$

VIII. STEADY STATE CONDITION

The closed form expressions after extensive simplification for $D^*(z,\theta)$ and $Q^*(z,\theta)$ are as follows

$$Q^*(z,\theta) = \frac{P_1(0)(V^*(W_X(z))V^*(\theta)}{\theta - W_X(z)} \quad (37)$$

$$D^*(z,\theta) = \frac{(D^*(W_X(z)) - (D^*(\theta)}{\theta - W_X(z)} P_1(0)(V^*(W_X(z)) - R(z)(W_X(z)) \quad (38)$$

Using the equations(19) and (20) to (32),we get

$$B_i^{**1}(z,\theta,0) = a_i P_i^*(z,\theta) U_i^{*1}(W_X(z)) \quad (39)$$

$$B_i^{**1}(z,\theta,\theta_1) = \frac{a_i P_i^*(z,\theta) U_i^{*1}(W_X(z)) - U_i^{*1}(\theta_1))}{\theta_1 - W_X(z)} \quad (40)$$

$$P_1^*(z,\theta) = \frac{z(P_1(0)(D^*(w_X(z))V^*(w_X(z)) - 1) - D^*(w_X(z))R_z(w_X(z))H_{a1}^*(w_X(z)) - S_1^*(\theta)}{(Z - H^*(z))(\theta - ha_1(w_X(z)))} \quad (41)$$

$$P_2^*(z,\theta) = \frac{rzH_{a1}^*(w_X(z))(P_1(0)(D^*(w_X(z))V^*(w_X(z)) - 1) - D^*(w_X(z))R_z(w_X(z))H_{a2}^*(w_X(z)) - S_2^*(\theta)}{(Z - H^*(z))(\theta - ha_2(w_X(z)))} \quad (42)$$

Where $P_1(0) = P_{11}(0)(1-r) + P_{12}(0)$, $W_X(z) = (1 - X(z))$,

$H^*(z) = H_{a1}^*(W_X(z))\left((1-r) + rH_{a2}^*(W_X(z))\right)$, $H_{a1}^*(W_X(z)) = S_i^*(ha_i(W_X(z))$ and

$ha_i(W_X(z)) = W_X(z) + a_i(1 - U_i^* W_X(z))$, $i=1,2$

The equations (37),(38),(40),(41) and (42) at $\theta = \theta_1 = 0$ respectively give,

$$Q^*(z,0) = \frac{(1 - v^*(w_X(z)))(P_1(0))}{(w_X(z))}$$

$$D^*(z,0) = \frac{\left(1 - D^*(W_X(z))\right)\left(P_1(0)V^*(W_X(z))\right) - R(z)(W_X(z))}{(W_X(z))}$$

$$B_i^{**1}(z,0,0) = \frac{a_i P_i^*(z,0) U_i^{*1}(W_X(z))}{(W_X(z))}, \quad i=1,2$$

$$P_1^*(z,0) = \frac{z(P_1(0)(D^*(w_X(z))V^*(w_X(z)) - 1) - D^*(w_X(z))R_z(w_X(z))\left(1 - H_{a1}^*(w_X(z))\right)}{(Z - H^*(z))(ha_1(w_X(z)))}$$

$$P_2^*(z,0)=\frac{rzH_{a1}^*(w_x(z))(P_1(0)(D^*(w_x(z))V^*(w_x(z))-1)-D^*(w_x(z))R_z(w_x(z))(1-H_{a2}^*(w}{(Z-H^*(z))(ha_2(w_x(z)))}$$

Let $P_B(z)=\sum_{i=1}^{2}(P_i^*(z,0)B_i^{**1}(z,0,0))$

Then $P_B(z)=\dfrac{-z\emptyset(z)(1-H^*(z))}{(W_X(z))(Z-H^*(z))}$

Where $\emptyset(z)=(P_1(0)((1-(D^*(W_X(z))V^*(W_X(z))+D^*(W_X(z))R_z(W_X(z))$

Let $P_1(z)$ gives the PGF of the system size probabilities when the server is idle.

Then $P_1(z)=Q^*(z,0)+D^*(z,0)+R(z)$

$$P_1(z)=\frac{(1-V^*(W_X(z)))(P_1(0)}{W_X(z)}+\frac{((1-(D^*(W_X(z)))(P_1(0)V^*(W_X(z))-R_z(W_X(z))}{W_X(z)}+R_z$$

To calculate R_z ,for $0\le n\le$ N-1, let $\prod_0=1$ and $\prod_n=\sum_{i=1}^{n}g_i\prod_{n-i}$; $\Psi_0=\alpha_0$ and $\Psi_n=\sum_{i=0}^{n}\alpha_i\prod_{n-i}$

Using equation (1) and (2) we get

$R(z)=\frac{P_1(0)}{\lambda}\sum_{n=0}^{N-1}\Psi_n Z^n=P_1(0)\Psi_z$ where $\Psi_z=\frac{1}{\lambda}\sum_{n=0}^{N-1}\Psi_n$

Then $P_1(z)=\dfrac{\Phi(z)}{W_X(z)}$

If p(z) denotes the total probability generating function of the number of customers in the system in steady state ,then,

$P(z)\quad=P_B(z)+P_1(z)$

$=\frac{\Phi(z)(z-1)H^*(z)}{(W_X(z))(z-H^*(z))}$ where $\Phi(z)$ involves the unknown $P_1(0)$ and this can be calculated using

the normalizing condition P(1)=1

And $P_1(0)=\dfrac{1-\rho_c}{E(D)+E(V)+\sum_{n=0}^{N-1}\frac{\Psi_n}{\lambda}}$

Where $\rho_c=\lambda E(X)E(H_c)$, $E(H_c)=E(S_1)(1+a_1E(U_1))+rE(S_2)(1+a_2E(U_2))$

Substituting for $P_1(0)$ in $P(z)$ we have,

$$P(z)=\frac{(1-\rho_c)(z-1)H^*(z)}{z-H^*(z)}\left[\frac{\frac{1-D^*(W_X(z))V^*(W_X(z))}{W_X(z)}+D^*(W_X(z))\sum_{n=0}^{N-1}\frac{\Psi_n}{\lambda}z^n}{E(D)+E(V)+\sum_{n=0}^{N-1}\frac{\Psi_n}{\lambda}}\right]$$

IX. PERFORMANCEMEASURES

In this section, the probability that the server is on vacation (P_V), in busy period (P_{Busy}), in breakdown state (P_{Br}) and in setup state (P_D) are calculated.

i. P_V = the probability that the server is on vacation
= $E(V)\, P_1(0)$, where $P_1(0) = P_{11}(0)\,(1-r) + P_{12}(0)$

ii. P_{Busy} = the probability that the server is busy
= $\lambda\, E(X)\, (E(S_1) + r\, E(S_2)) = P_{Busy}$.

iii. P_{Br} = the probability that the server is in break down state
P_{Br} = $\lambda\, E(X)\, (E(S_1)\, a_1\, E(U_1) + r\, E(S_2)\, a_2\, E(U_2)) = p_{Br}$

iv. P_D = the probability that the server is doing his setup work
= $E(D)\, P_1(0)$

Mean system size

Let L_N denote the expected system size of the unreliable $M^{[X]}/G/1$ queue with N-policy, single vacation and setup time.

Then $L_N = \left(\frac{d}{dz}P(z)\right) z = 1$

By calculating,
$$L_N = \frac{(\lambda\, E(X))^2\, E(H_C^2) + \lambda\, E(X(X-1))\, E(H_C)}{2(1-\rho_c)}\rho_c$$
$$+ \frac{\left(\lambda E(X) E(D) \sum_{n=0}^{N-1}\Psi_n + \sum_{n=0}^{N-1} n\Psi_n\right) + \frac{\lambda^{*} E(X)}{2} E(D^2) + 2E(D)E(V) + E(V^2))}{\lambda(E(D)E(V)) + \sum_{n=0}^{N-1}\Psi_n}$$

Expected Cycle Length

Let $E(T_N)$,E(B), $E(T_c)$,E(Br), E(D),E(C) represent the expected idle period ,expected busy period,expected cycle,expected breakdown period,expected setup period and the expected completion period respectively.Then the long –run fraction of time the server is idle and busy are given by,

$$\frac{E(T_N)}{E(T_{cy})} = P_1 = E(V)\ P_1(0)$$

$$\frac{E(B)}{E(T_{cy})} = P_{Busy} = \rho_{Busy}$$

$$\frac{E(Br)}{E(T_{cy})} = P_{Br} = \rho_{Br}$$

$$\frac{E(D)}{E(T_{cy})} = P_D = E(D)\ P_1(0)$$

From the above calculations $E(T_{cy}) = \frac{1}{P_1(0)} = \frac{E(D) + E(V) + \sum_{n=0}^{N-1}\frac{\Psi_n}{\lambda}}{1-\rho_c}$

Then E(C) = Expected completion period

= $E(B) + E(Br)$

$= (\rho_{Busy} + \rho_{Br})$

$= \frac{\rho_c}{1-\rho_c}\left(E(D) + E(V) + \sum_{n=0}^{N-1}\frac{\Psi_n}{\lambda}\right)$

Proof

By calculation, $\square\, T_C^S(k+1) - \square\, T_C^S(k) = \frac{\Psi_k}{C_k\, C_{k+1}}\,(h(k))$,

where $h(k) = -A + l\,(E(D) + E(V))\,(C_{Busy}\,(1 - r_C) + l\,E(X)\,E(D)\,C_h + k\,C_h) + C_h \sum_{n=0}^{k}(k-n)\Psi_n$

The sign of $h(k)$ determines whether $T_C(k)$ increases (or) decreases.

If k be the first integer such that $h(k) > 0$, then

$h(k+1) = h(k) + C_h(l\,(E(D) + E(V))) + C_h\, Y_n$

$h(k+1) > 0$

This implies $h(k+1) > 0$ whenever $h(k) > 0$

Therefore $N^* =$ first k, for which $h(k) > 0$

(i.e.) $N^* = \min\{k \geq 1 / h(k) > 0\}$

X. CONCLUSION

This is an extension of the work on Non- Markovian queueing system combining N - Policy with setup time and vacation, carried out by several researchers including Medhi and Templeton [5], Minh [6], Lee and Park [4], Lee et al. [7], Hur and Paik [8]. But these authors have focused only on reliable servers. In this paper, for the Non- Markovian unreliable queueing system with N- Policy,second optional service,setup time and vacation,the PGF of the system size is presented in closed form.Further, various performance measures are derived.

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