

On The Use Of Exponentially Weighted Moving Average (Ewma) Control Chart In Monitoring Road Traffic Crashes.

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ABSTRACT: *This study is aimed at designing Exponentially Weighted Moving average (EWMA) control chart in monitoring road crashes. The data used in this study is a secondary data from the accident record office of the Federal Road Safety Commission (FRSC), Zone RS8 Command Headquarter, Offa Road, GRA, Ilorin, Kwara State, Nigeria for the period of January, 2000 to December, 2010. Cumulative Exponentially Weighted Moving Average (EWMA) control chart scheme was designed and used to study the rate of road crashes. Finally, the performance of the control chart schemes was compared by varying the weighted parameter ($w=0.15, 0.20$ and 0.30). It was observed that EWMA control chart scheme is sensitive in detecting small shift in the process but more sensitive in detecting shift in the process mean with higher weighted parameter (W). Therefore, the higher the weighted parameter, the faster it detects an out of control signal. Thus providing validation of the methodology employed to set up Minitab using Neubauer table in determining the weighted parameter, limit q of the EWMA chart and the nominal ARL.*

I. INTRODUCTION

The mandate of Federal Road Safety Commission (FRSC) which is reducing road accidents on our roads can be viewed as rendering services to the citizens of Nigeria and the more the road accidents are curtailed, the better the costumers (citizens) are satisfied. Shewhart chart has been widely used in determining shift in a process, however the main disadvantage of any of Shewhart charts is that they use only the last information of the combined process and ignore any other process given by the sequence of all points. This characteristic makes Shewhart control charts relatively insensitive to a small change in a process, because the cumulative or weight of the previous observation are disregarded. Small changes could be detected through CUSUM, EWMA and Moving Average control charts. These charts could be used as a complementary tool for X-bar, through which, small and large changes could be detected. Some authors, such as Dugans (1974), Lucas (1976), stated that the cumulative sum control chart is much more efficient than the usual **X – bar** control chart for detecting small variations in the average. Other authors including Crowders (1987) presented the exponentially weighted moving average control chart, as good choice to detect small change in process average. Various modification and supplemental criteria have also suggested that control charts based on moving average is also an effective chart in detecting small process change. In this study, road crashes from the Accidents Record Office of Federal Road Safety Commission (FRSC) which comprises of Kwara, Kogi and Ekiti state is studied using cumulative-sum (CUSUM), exponentially weighted moving average (EWMA) and moving average control charts.

II. MATERIALS AND METHODS

THE EXPONENTIAL WEIGHTED MOVING AVERAGE (EWMA)

The Exponential Weighted Moving Average (EWMA) chart is used for monitoring process by averaging the data in a way that give less weight to old data as samples are taken and give a high weight to most recent data. It is also effective in detecting small shifts. The EWMA chart methodology was developed by Roberts (1959) who purposed it for use by some organizations, particularly in the process industries, financial and management control as the basis of new control/performance charts system. The EWMA charting procedure is sometime used to monitor the rate of occurrence of rare events where the time between two successive occurrences is exponentially distributed. This procedure can also be used extensively in time series modeling and forecasting. The EWMA for individual value may be defined as:

$$Z_i = wX_i + (1 - w)Z_{i-1}, \text{ where } 0 < w \leq 1, i = 1, 2, \dots, n, n + 1, \dots \dots \dots 1$$

The above equation is due to Roberts (1959).

The use of this control chart was first published in the article “Control chart tests based on geometric moving averages” (Roberts, 1959). While Shewhart charts only consider the most recent data point in testing to determine if statistical limits have been exceeded, EWMA charts consider all previous points using a weighing factor that makes the outcome more influenced by recent points. In statistical quality control, the EWMA chart (or exponentially-weighted moving average chart) is a type of control chart used to monitor either variables or attributes-type data. While other control charts treat rational subgroups of samples individually, the EWMA chart tracks the exponentially-weighted moving average of all prior samples. EWMA weights samples in geometrically decreasing order so that the most recent samples are weighted most highly while the most distant samples contribute very little. Although EWMA chart assumes normal distribution, it is relatively robust in the face of non-normality. The chart monitors only the process mean. EWMA Charts are generally used for detecting small shifts in the process mean and they detect shifts of 0.5sigma to 2sigma much faster than Shewhart charts with the same sample size. In brief, after multiplication by a weighting factor w , the current measurement is added to the sum of all former measurements, which is weighted with $(1 - w)$. Thus, at each time $t (t = 1, 2, \dots)$, the test statistic Z_t .

Basically, the method is to form a new moving average at each sample point by calculating a weighted average of new value and previous moving-average. Therefore, in exponentially weighted moving average chart, all of the past data has some effect on the current value, but it rapidly loses influence.

Consider the following notations:

Z_t = Exponentially Weighted Moving Average

w = EWMA weighted parameter ($0 < w \leq 1$)

μ_0 = Process mean

X_{ij} = j^{th} measure of i^{th} subgroup, with

σ = Process standard deviation

n_i = Sample size of i^{th} group

q = sigma limit

\bar{X}_i = Mean of measurement in i^{th} subgroup. If $n_{i=1}$, then subgroup mean reduce to the observation in the group

Each point on the chart indicates the value of exponentially weighted moving average (EWMA) for the subgroup. The EWMA for the i^{th} subgroup (Z_i) is defined recursively as:

$$Z_t = wX_i + (1 - w)Z_{i-1}, 1 \leq t, \text{ where } w \text{ is the weighted parameter } (0 < w \leq 1).$$

III. COMPUTATION OF CONTROL LIMIT

By default, the central line on an EWMA chart indicates an estimate for μ , which is computed as:

$$\hat{\mu} = \bar{\bar{X}} = \frac{n_1 X_1 + \dots + n_N X_N}{n_1 + \dots + n_N} \dots \dots \dots 2$$

The default limits are computed as $q = 2.30$ (referred to as 2.30σ limits).

The upper and lower control limits are

$$UCL = \bar{\bar{X}} + q\sigma \sqrt{\frac{w}{n(2-w)}} \dots \dots \dots 3$$

$$LCL = \bar{\bar{X}} - q\sigma \sqrt{\frac{w}{n(2-w)}} \dots \dots \dots 4$$

where ‘n’ is the subgroup size (1).

CHOOSING THE VALUE OF WEIGHTED PARAMETER (w)

The following shows how we might set up an EWMA chart by the five-step procedure:

- Select an in-control Run Length (RL) of 45. This is also the average run length (ARL) of the Westgard algorithm.
- Define the optimum shift to be detected as 1.0 as used in this paper.
- Using this shift sizes and the above ARL, gives a weighting factor w of 0.15 (Fig. 2).
- With these selected values derived from Fig. 2, EWMA chart limit q of 2.30 is selected (Fig. 3). For one control sample per month ($n = 1$), upper control limit (UCL) and lower control limit (LCL) are calculated as equation 3 and 4 above.

- Enter the weight parameter (w), historical mean, sigma (standard deviation) and the sigma limit (S limit) to the MINITAB 14 EWMA chart dialog box as displayed in Fig. 1 below.

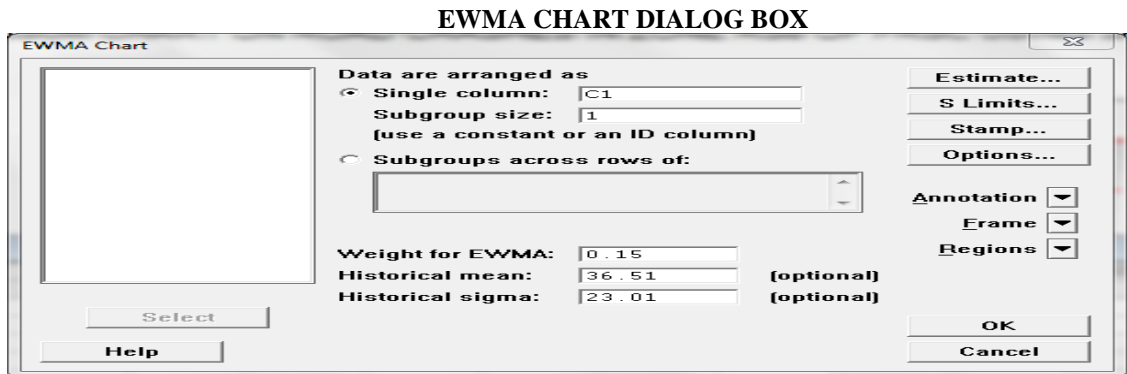


Figure 1

In order to accurately compare EWMA control chart techniques, the same ARL value must be used for the design.

A graphical method for determining the ARL for an EWMA control chart is seen in Fig. 2 and 3 below, Neubauer (1997). One can obtain the weighting factor " w " as a function of the desired shift sensitivity of the chart. For a shift of 1.0 standard deviation, for example, a weighting factor of 0.15 can be obtained from the chart as shown in figure 2 and we set ARL at 45, using Fig. 3, the sigma limit (q) was obtained to be 2.30.

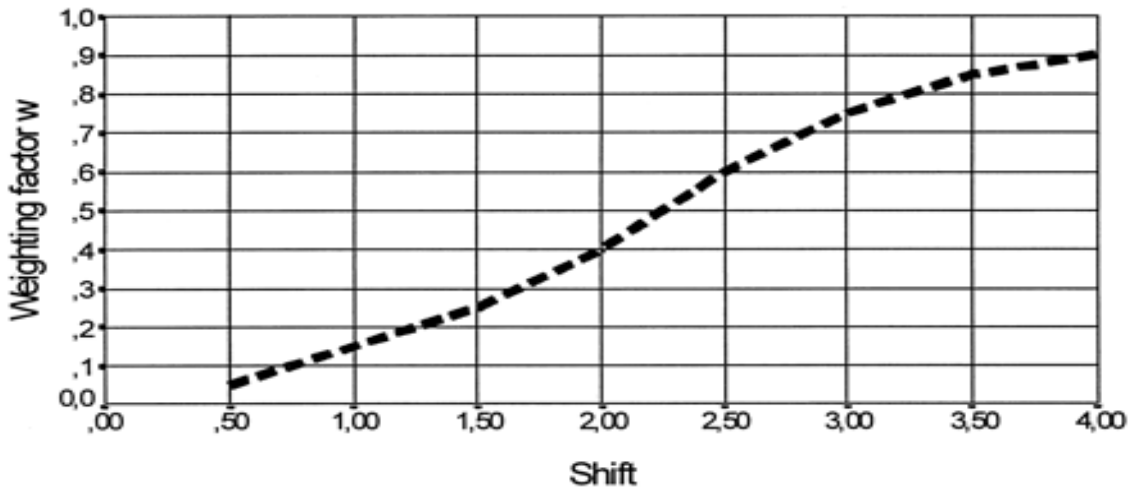


Fig.2: Optimal w for EWMA charts according to the shift d from Neubauer (1997)

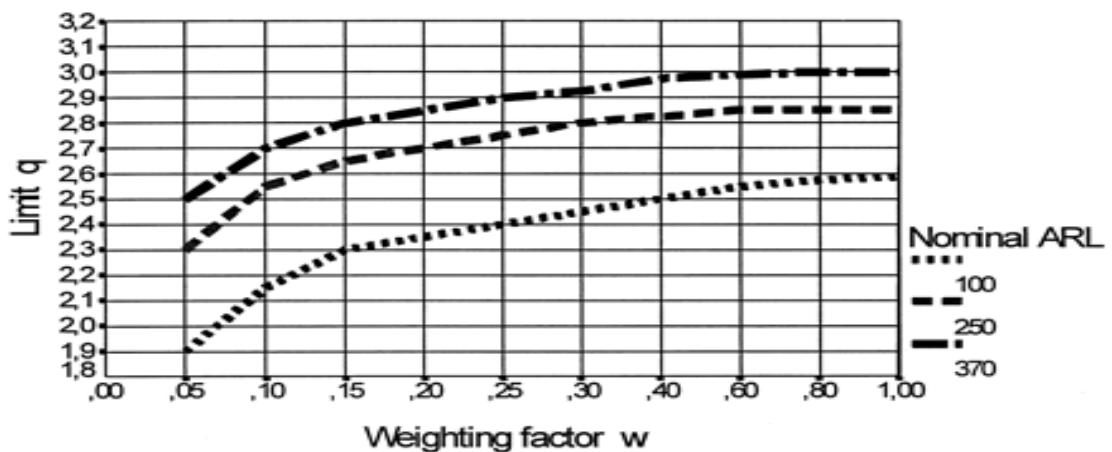


FIG. 3: Determining the limit q of the EWMA chart after selection of w and the nominal ARL from Neubauer 1997.

ASSESSMENT OF PERFORMANCE OF EWMA CHART

A number of authors have studied the design of EWMA control scheme based on Average Run Length (ARL) computation. Ideally, the ARL should be short when a shift occurs and should be long when there is no shift. The effect of “w” (weight parameter) on the ARL was described by Robert (1959), who used simulation method. The ARL function was approximated by Robinson and Ho (1978).

IV. DATA ANALYSIS

The analysis of the data (number of road crashes) obtained from the Federal Road Safety Commission (FRSC) were considered using EWMA control chart scheme

The overall mean per month is approximately $\bar{X} = 36.51$, the standard deviation from the historical data is $\sigma = 23.01$ and the subgroup size (n) is 1.

EWMA CHART METHOD

The EWMA statistic was calculated using three different values of w (weighted parameters), 0.15, 0.20 and 0.30 so as to come out with the most appropriate one.

The following shows how we set up an EWMA-chart by the three-step procedure:

- ❖ Select an in-control Run Length (RL) of 45. This is also the average run length (ARL) of the Westgard algorithm.
- ❖ Define the optimum shift to be detected as 1.0, 1.2 and 1.5. Using this and the above ARL, gives a weighting factor w of 0.15, 0.20 and 0.30 (Fig. 2).
- ❖ With these selected values derived from Fig. 2, EWMA chart limit (q) were obtained to be 2.30, 2.35 and 2.42 from Fig. 3. For one control sample per month (n = 1), the central line, upper control limit (UCL) and lower control limit (LCL) are calculated as follows.

$$CENTRAL\ LINE = \frac{(n_1x_1 + \dots + n_Nx_N)}{n_1 + \dots + n_N} = \bar{X} = 36.51s$$

CONTROL LIMITS

$$UCL = \mu_0 + q\sigma \sqrt{\frac{w}{n(2-w)}} = 36.51 + 2.30(23.01) \sqrt{\frac{0.15}{1(2-0.15)}} = 36.51 + 15.07 = 51.58$$

$$LCL = \mu_0 - q\sigma \sqrt{\frac{w}{n(2-w)}} = 36.51 - 2.30(23.01) \sqrt{\frac{0.15}{1(2-0.15)}} = 36.51 - 15.07 = 21.44$$

Using the same procedure, the control limits calculation for w = 0.1, 0.2 and 0.3 is shown in table 1 below.

TABLE OF EWMA CONTROL LIMITS

WEIGHTED/SIGMA LIMIT VALUE	CONTROL LIMITS		
	LCL	CL	UCL
w = 0.15 q = 2.30	21.44	35.51	51.58
w = 0.20 q = 2.35	18.49	36.51	54.53
w = 0.30 q = 2.42	13.12	36.51	59.90

Table 1.

COMPUTATION OF EWMA STATISTIC

The EWMA statistic is calculated using three (3) different values of w (weighted parameter) so as to come out of the appropriate value.

The statistic is

$$Z_i = wx_i + (1-w) Z_{i-1}, 0 < w \leq 1$$

For w = 0.15,

$$Z_1 = 0.15(15) + (1 - 0.15)36.51 = 33.28$$

$$Z_2 = 0.15(31) + (1 - 0.15)33.28 = 32.948$$

Using the same procedure, the EWMA statistic for w = 0.15, 0.20 and 0.30 is shown in table 2.

Fig. 4,5 and 6 below represents the EWMA charts for $w = 0.15, 0.20$ and 0.30 respectively when the values of Z_i where plotted against sample number.

EWMA CHARTS ON ROAD CRASHES IN ZONE RS8

The figures below display the EWMA charts with different weight parameters (w).

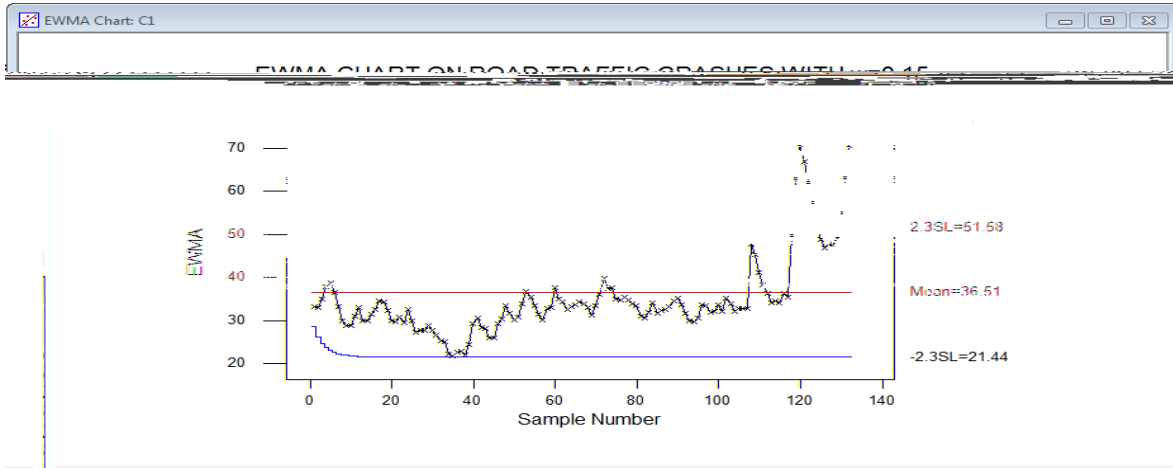


Fig. 4.

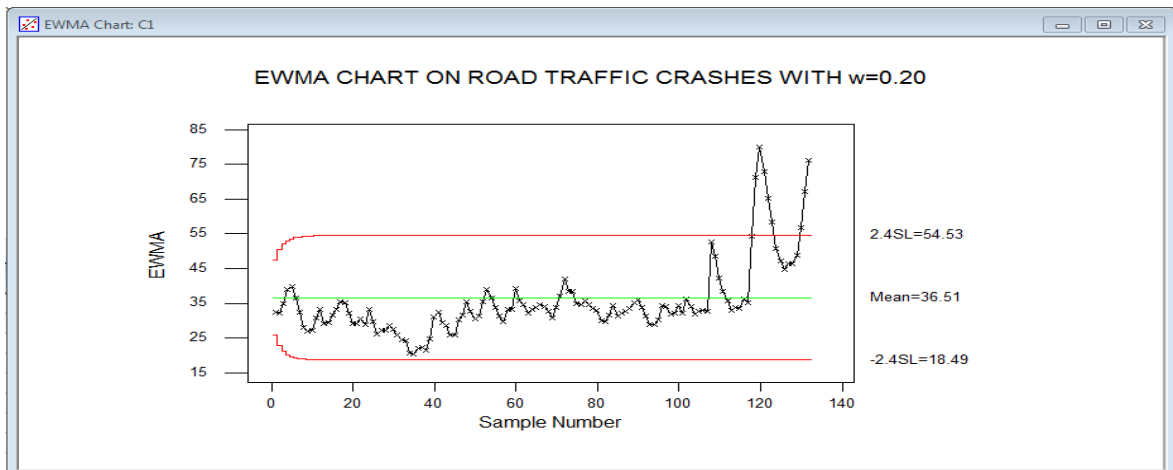


Fig. 5.

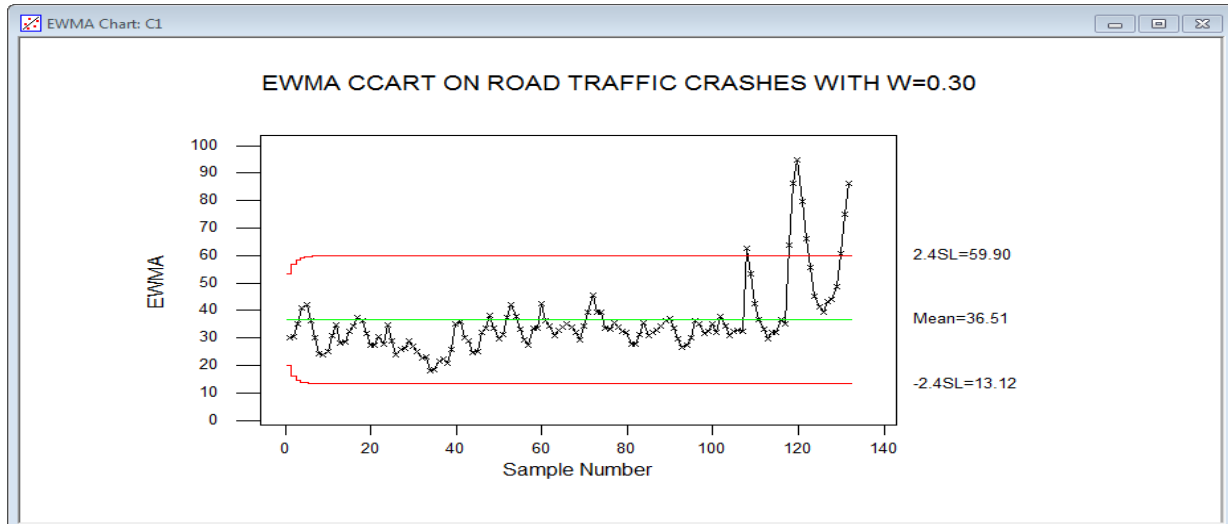


Fig. 6.

From the EWMA charts above, the process is out of control for the three values of weighted parameter (w) and they indicate out of control signals at different points. For w =0.15 and 0.20, both charts detected the first out of control signal at point 119 which correspond to November, 2009 while for w= 0.30, the first out of control signal was at sample point 118 which corresponds to October, 2009. Therefore, the higher the weighted parameter, the faster it detects an out of control signal. Thus providing validation of the methodology employed to set up Minitab using Neubauer table in determining the weighted parameter, limit q of the EWMA chart and the nominal ARL. Therefore, the higher the weighted parameter, the faster it detects an out of control signal.

COMPUTATION OF EWMA STATISTIC ON ROAD TRAFFIC CRASHES IN ZONE RS8 FOR THE YEAR 2001 to 2010

Sample No. (i)	No of Road Crashes (X _i)	$z_t = [wX_t + (1 - w)z_{t-1}]$		
		w=0.15	w=0.20	w=0.3
1	15	33.28	32.21	30.06
2	31	32.94	31.97	30.34
3	46	34.90	34.77	35.04
4	55	37.92	38.82	41.03
5	44	38.83	42.06	41.93
6	23	36.46	38.25	36.24
7	15	33.24	33.60	29.87
8	11	29.90	29.08	24.21
9	23	28.87	27.86	23.85
10	28	28.74	27.89	25.09
11	44	31.03	31.11	30.76
12	44	32.98	33.69	34.74
13	12	29.83	29.35	27.91
14	30	29.86	29.48	28.54
15	41	31.53	31.79	32.28
16	39	32.65	33.23	34.29
17	45	34.50	35.58	37.51
18	33	34.28	35.07	36.15
19	21	32.29	32.25	31.61
20	17	30.00	29.20	27.23
21	28	29.70	28.96	27.46
22	37	30.80	29.97	30.32
23	22	29.50	28.38	27.83
24	50	32.58	32.70	20.98
25	15	29.94	29.16	19.18
26	12	27.25	25.73	17.03
27	31	27.81	26.78	21.22

28	27	27.69	26.83	22.95
29	35	28.79	28.46	26.57
30	22	27.77	27.17	25.20
31	20	26.60	25.73	23.64
32	18	25.31	24.19	21.95
33	23	24.96	23.95	22.26
34	6	22.12	20.36	17.38
35	20	21.80	20.29	18.17
36	28	22.73	21.83	21.12
37	24	22.92	22.26	21.98
38	17	22.03	21.21	20.47
39	38	24.43	24.57	25.74
40	57	29.32	31.06	35.12
41	38	30.62	32.44	35.98
42	16	28.43	29.16	29.99
43	26	28.07	28.52	28.79
44	14	25.96	25.62	24.36
45	26	25.97	25.70	24.85
46	48	29.27	30.17	31.79
47	37	30.43	31.53	33.36
48	50	33.37	35.22	38.35
49	22	31.66	32.58	33.44
50	21	30.06	30.26	29.71
51	35	30.80	31.21	31.30
52	52	33.98	35.37	37.51
53	53	36.83	38.90	42.16
54	27	35.36	36.52	37.61
55	22	33.36	33.61	32.93
56	21	31.51	31.09	29.35
57	23	30.23	29.47	27.44
58	48	32.90	33.18	33.61
59	33	32.92	33.14	33.43
60	64	37.58	39.31	42.60
61	21	35.09	35.65	36.12
62	30	34.33	34.52	34.28
63	22	32.48	32.02	30.60
64	38	33.31	33.21	32.83
65	36	33.71	33.77	33.77
66	38	34.35	36.62	35.04
67	31	33.85	33.89	33.83
68	28	32.97	32.71	32.08
69	22	31.32	30.57	29.06
70	46	33.52	33.65	34.14
71	51	36.12	37.13	39.20
72	61	39.85	41.90	45.75
73	24	37.47	38.32	39.22
74	39	37.70	38.45	39.15
75	20	35.05	34.77	33.41
76	33	34.74	34.41	33.28
77	40	35.53	35.53	35.30
78	30	34.70	34.42	33.71
79	29	33.85	33.34	32.30
80	31	33.42	32.87	31.91
81	18	31.12	29.90	27.73
82	28	30.65	29.52	27.81
83	39	31.90	31.41	31.17
84	46	34.02	34.33	35.62
85	19	31.77	31.26	30.63
86	35	32.25	32.01	31.94
87	35	32.66	32.61	32.86
88	37	33.31	33.49	34.10
89	41	34.46	34.99	36.17
90	39	35.14	35.79	37.02
91	25	33.62	33.63	33.41
92	21	31.73	31.11	29.69
93	19	29.82	28.69	26.48
94	29	29.70	28.75	27.24
95	36	30.65	30.20	29.87
96	51	33.70	34.36	36.21
97	32	33.45	33.89	34.94

98	23	31.88	31.71	31.36
99	34	32.20	32.17	32.15
100	42	33.67	34.13	35.11
101	24	32.22	32.11	31.77
102	52	35.19	36.09	37.84
103	26	33.81	34.07	34.29
104	23	32.19	31.85	30.90
105	36	32.76	32.68	32.42
106	33	32.80	32.75	32.60
107	32	32.68	32.60	32.42
108	133	47.73	52.68	62.60
109	32	45.37	48.54	53.42
110	17	41.11	42.23	42.49
111	23	38.39	38.39	36.64
112	25	36.38	35.71	33.15
113	21	34.07	32.77	29.50
114	37	34.51	33.61	31.75
115	32	34.13	33.29	31.83
116	48	36.21	27.43	36.68
117	31	35.43	28.14	34.98
118	131	49.77	48.72	63.54
119	139	63.15	66.77	86.35
120	116	71.08	76.62	95.24
121	44	67.02	70.09	79.87
122	35	62.22	63.08	66.41
123	31	57.54	56.66	55.79
124	20	51.91	49.33	45.05
125	32	48.92	45.86	41.14
126	35	46.83	43.69	39.29
127	53	47.76	45.55	43.41
128	46	47.50	45.65	44.19
129	59	49.23	48.31	48.63
130	89	55.20	45.64	60.74
131	109	63.27	60.65	75.22
132	113	70.73	50.56	86.55

Table 2

V. RESULT

EWMA was designed and applied using three (3) different values of weighted parameters/smoothing constant (w) which are 0.15, 0.20 and 0.30. Table 2 shows the EWMA statistic tabulation. The values of control limit for the three values of w are show in table 1. The values of Z_i (EWMA) were then plotted against the sample number for the three values of w and the corresponding EWMA control charts are shown in Fig. 4 to 6 respectively. It can be seen from the charts that the process is out of control for the three values of w . For $w=0.15$ and 0.20, both charts detected the first out of control signal at point 119 which correspond to November, 2009 while for $w=0.30$, the first out of control signal was at sample point 118 which corresponds to October, 2009. Therefore, the higher the weighted parameter, the faster it detects an out of control signal.

VI. CONCLUSION

The effectiveness of EWMA chart relative to determining shifts in underlying pattern and the efficiency of the charts relative to the time and expertise required to generate the chart was taken into consideration. EWMA chart was very capable at detecting changes in the Road Traffic Crashes recurrent (persistent) pattern when properly configured within Minitab. That is, it is very effective at detecting small change like mean shift of 1.0 sigma as used in this paper work. In view of our findings, we strongly observed that EWMA generally provided a slightly faster alarm Since the EWMA methodology detects shift quickly for this type of dataset, it may be an efficient choice for monitoring it. Conclusively, for such applications within this organization EWMA can be an efficient tool.

VII. RECOMMENDATION

From Fig. 4 and 5, the first out of control signal at point 119 corresponding to November, 2009 and continues to February while from Fig. 6 first out of control signal was at sample point 118 which corresponds to October, 2009 and also continues to February. These months correspond to the Christmas and New-year celebration periods in Nigeria when most people use the highways to travel to their home towns. It is then suggested that the Federal Road Safety Commission (FRSC) and other Agencies who are engaged in road safety

activities should increase their earnest effort by deploying more Road Safety officers to strategic locations on the high ways and making sure that road users adhere to traffic rules so as to ensure that Nigeria roads are under proper check and safe for all traffic roads users in order to reduce the high rate of road traffic crashes probably leading to loss of lives.

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