*International Journal of Mathematics and Statistics Invention (IJMSI)*
*E-ISSN: 2321 – 4767 P-ISSN: 2321 - 4759*
*www.ijmsi.org || Volume 2 || Issue 3 || March 2014 || PP-103-110*

# Unsteady MHD Couette Flow between Two Infinite Parallel Porous Plates in an Inclined Magnetic Field with Heat Transfer

Joseph K.M[1], Daniel S[1] and Joseph G.M[2]
[1]*Department of Mathematics, Kaduna State University – Nigeria*
[2]*Department of Electrical Engineering, Ahmadu Bello University – Nigeria*

**ABSTRACT:** *Unsteady MHD couette flow between two infinite parallel porous plates in an inclined magnetic field with heat transfer has been studied. The lower plate is considered porous. The governing equations of the flow field are solved by variable separable technique and the expression for the velocity* $u$, *temperature* $\theta$, *skin frictions* $(\tau_1 \text{ and } \tau_2)$ *and Nusselt numbers*$(Nu_1 \text{ and } Nu_2)$ *are obtained. The effects of various parameters such as Hartman number* $Ha$ *and Prandtl number* $Pr$ *on the flow field have been studied, the results are presented both graphically, in tabular form and are discussed quantitatively.*
***Keywords**:* MHD, Couette Flow, Porous Plate, Heat Transfer, Unsteady.

## I. INTRODUCTION

Unsteady MHD couette flow between two infinite parallel porous plates in an inclined magnetic field with heat transfer has many applications in different field of engineering and technology. The interaction between the conducting fluid and the magnetic field radically modifies the flow, with effects on such important flow properties as heat transfer, the detail nature of which is strongly dependent on the orientation of the magnetic field. When fluid moves through a magnetic field, an electric field and consequently a current may be induced, and in turn the current interacts with the magnetic field to produce a body force on fluid. The production of this current has led to MHD power generators, MHD devices, nuclear engineering and the possibility of thermonuclear power has created a great practical need for understanding the dynamics of conducting fluids. The influence of a magnetic field in viscous incompressible flow of electrically conducting fluid is of use in extrusion of plastics in the manufacture of rayon, nylon etc.

Hannes Alfven (1942), a Swedish electrical engineer first initiated the study of MHD. Shercliff (1956) considered the steady motion of an electrically conducting fluid in pipes under transverse magnetic fields. Sparrow and Cess (1961) observed that the free convection heat transfer to liquid metals may be significantly affected by the presence of magnetic field. Drake (1965) considered flow in a channel due to periodic pressure gradient and solved the resulting equation by separation of variables methods. Singh and Ram (1978) studied Laminar flow of an electrically conducting fluid through a channel in the presence of a transverse magnetic field under the influence of a periodic pressure gradient and solved the resulting differential equation by the method of Laplace transform. More to this, Ram et al (1984) have analyzed Hall effects on heat and mass transfer flow through porous media. Soundelgekar and Abdulla Ali (1986) studied the flow of viscous incompressible electrically conducting fluid past an impulsively started infinite vertical isothermal plate. Singh (1993) considered steady MHD fluid flow between two parallel plates. John Mooney and Nick Stokes (1997) considered the numerical requirements for MHD flows with free surfaces. Raptis and Perdikis (1999) considered the effects of thermal radiation and free convection flow past a moving vertical plate. Al-Hadhrami (2003) discussed flow through horizontal channels of porous material and obtained velocity expressions in terms of the Reynolds number. Ganesh (2007) studied unsteady MHD Stokes flow of a viscous fluid between two parallel porous plates. Stamenkovic et al (2010) investigates MHD flow of two immiscible and electrically conducting fluids between isothermal, insulated moving plates in the presence of applied electric and magnetic fields. He matched the solution at the interface and it was found that decrease in magnetic field inclination angle flattens out the velocity and temperature profiles. Rajput and Sahu (2011) studied the effect of a uniform transverse magnetic field on the unsteady transient free convection flow of an incompressible viscous electrically conducting fluid between two infinite vertical parallel porous plates with constant temperature and variable mass diffusion. Manyonge et al (2012) studied steady MHD Poiseuille flow between two infinite parallel porous plates in an inclined magnetic field and discover that high magnetic field strength decreases the velocity. Sandeep and Sugunamma (2013) analysed the effect of an inclined magnetic field on unsteady free convection flow of a dusty viscous fluid between two infinite flat plates filled by a porous medium. Heat transfer effects on rotating MHD coquette flow in a channel partially field by a porous medium with hall current has been discussed by Singh and Rastogi (2012).

In this paper, we considered one dimensional couette flow of an electrically conducting fluid between two infinite parallel porous plates under the influence of inclined magnetic field with heat transfer.

## II. PROBLEM FORMULATION

A magnetic field of field strength represented by the vector $B$ at right angle to the flow of an electrically conducting fluid moving with velocity $V$ was introduced. Here, an electric field vector denoted by $E$ is induced at right angle to both $V$ and $B$ because of their interaction.

We assume that the conducting fluid exhibits adiabatic flow in spite of magnetic field, then we denote the electrical conductivity of the fluid by a scalar σ.

Lorentz force comes in place because the conducting fluid cuts the lines of the magnetic field in electric generator. This vector $F$ is parallel to $V$ but in opposite direction but is perpendicular to the plane of both $J$ and $B$.

Laminar flow through a channel under uniform transverse magnetic field is important because of the use of MHD generator, MHD pump and electromagnetic flow meter.

Here, we consider an electrically conducting, viscous, unsteady, incompressible fluid moving between two infinite parallel plates both kept at a constant distance $2h$.

The equations of motion are the continuity equation

$$\nabla . V = 0 \qquad (2.1)$$

And the Navier-Stokes equation

$$\rho\left[\left(\frac{\partial}{\partial t} + V.\nabla\right)\right] V = f_B - \nabla P + \mu \nabla^2 V \qquad (2.2)$$

Where $\rho$ is the fluid density, $f_B$ is the body force per unit mass of the fluid, μ is the fluid viscosity and $P$ is the pressure acting on the fluid. If one dimensional flow is assumed, so that we choose the axis of the channel formed by the two plates as the $x - axis$ and assume that flow is in this direction. Observed that $\bar{u}, \bar{v}$ $and$ $\bar{w}$ are the velocity components in $\bar{x}$, $\bar{y}$ and $\bar{z}$ directions respectively. Then this implies $\bar{v} = \bar{w} = 0$ and $\bar{u} \neq 0$, then the continuity equation is satisfied.

From this we infer that $\bar{u}$ is independent of $\bar{x}$ and this will make [(V.∇)V] in the Navier-stokes equation to vanish. The body force $f_B$ is neglected and replace with Lorentz force and from the assumption that the flow is one dimensional, it means that the governing equation for this flow is

$$\frac{\partial \bar{u}}{\partial \bar{t}} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{x}} + \upsilon \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{F_x}{\rho} \qquad (2.3)$$

Where $\upsilon = \frac{\mu}{\rho}$ is the kinematics viscosity and $F_x$ is the component of the magnetic force in the direction of x-axis. Assuming unidirectional flow so that $\bar{v} = \bar{w} = 0$ and $B_x = B_z = 0$ since magnetic field is along y-direction so that $V = i\bar{u}$ and $B = B_{0j}$ where $B_0$ is the magnetic field strength component. Now,

$$F_x = \sigma[(i\bar{u} \times jB_0)] \times jB_0 \qquad (2.4)$$

So that we have

$$\frac{F_x}{\rho} = -\frac{\sigma}{\rho} B_0^2 \bar{u} \qquad (2.5)$$

Then (2.3) becomes

$$\frac{\partial \bar{u}}{\partial \bar{t}} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{x}} + \upsilon \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\sigma}{\rho} B_0^2 \bar{u} \qquad (2.6)$$

From (2.6), when angle of inclination is introduced, we have

$$\frac{\partial \bar{u}}{\partial \bar{t}} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{x}} + \upsilon \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\sigma}{\rho} B_0^2 \bar{u} \sin^2(\alpha) \qquad (2.7)$$

Where α is the angle between V and B. Equation (2.7) is general in the sense that both field can be assessed at any angle α for 0 ≤ α ≤ π.

Because of the porosity of the lower plate, the characteristic velocity $\upsilon_0$ is taken as a constant so as to maintain the same pattern of flow against suction and injection of the fluid in which it is moving perpendicular to the fluid flow. The origin is taken at the centre of the channel and $\bar{x}, \bar{y}$ coordinate axes are parallel and perpendicular to the channel walls respectively. The governing equation, that is, the momentum equation is as follows

$$\rho \frac{\partial \bar{u}}{\partial t} = -\upsilon_0 \frac{\partial \bar{u}}{\partial \bar{y}} - \frac{\partial \bar{p}}{\partial \bar{x}} + \mu \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\sigma}{\rho} B_0^2 \bar{u} \sin^2(\alpha) \qquad (2.8)$$

Since the flow is isentropic, the energy equation is given as

$$\frac{\partial \bar{T}}{\partial \bar{t}} = \frac{k}{\rho c_p}\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} \quad (2.9)$$

Where $k$ the thermal conductivity of the fluid, ρ is is the density, $c_p$ is the specific heat constant pressure and $\bar{T}$ the temperature.
The boundary conditions are

$$\bar{u}(y,t) = 0, \bar{T} = \bar{T}_\infty \quad at\ \bar{t} = 0,$$
$$\bar{u}(-L,\bar{t}) = 0, \bar{u}(L,\bar{t}) = \frac{v}{L}, \bar{T} = \bar{T}_w \quad at\ \bar{t} > 0 \quad (2.10)$$

In other to solve equations (2.8) and (2.9) subject to the boundary conditions (2.10), we introduce the following dimensionless parameters:

$$\bar{x} = xL,\ \bar{y} = yL,\ \bar{p} = p\rho\frac{v^2}{L^2},\ \bar{u} = \frac{uv}{L},\ \bar{t} = \frac{tL^2}{v}, P_r = \frac{\mu c_p}{k}, \theta = \frac{\bar{T}-\bar{T}_\infty}{\bar{T}_w-\bar{T}_\infty},\ Ha^2 = \frac{\sigma L^2 B_0^2}{\mu} \quad (2.11)$$

Equations (2.8) and (2.9) now become

$$\frac{\partial u}{\partial t} = -\frac{R_g}{\rho}\frac{\partial u}{\partial y} + \frac{\partial^2 u}{\partial y^2} - M^2 u \quad (2.12)$$

Where $M = M^* sin\alpha$ and $M^*$=L$B_0\sqrt{\frac{\sigma}{\mu}} = Ha$. We assume that the rate of $\frac{\partial p}{\partial x} = 0$ (since it is couette flow)

$$P_r\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} \quad (2.13)$$

The boundary conditions in dimensionless form are

$$u(y,t) = 0, \theta(-1,t) = 0 \quad at\ t = 0$$
$$u(-1,t) = 0,\ u(1,t) = 1,\ \theta(1,t) = 1 at\ t > 0 \quad (2.14)$$

## III. SOLUTION OF THE PROBLEM

In this section we used separation of variable technique to solve equations (2.12) and (2.13) subject to the boundary condition (2.14). We assumed the solutions of equations (2.12) and (2.13) to be respectively in the form

$$u(y,t) = u(y)u(t) \quad (3.1)$$
$$\theta(y,t) = \theta(y)\theta(t) \quad (3.2)$$

As a consequence of equations (3.1) and (3.2), the solutions of the velocity profile and temperature distribution are

$$u(y,t) = e^{-\lambda^2 t}(c_1 e^{m_1 y} + c_2 e^{m_2 y}) \quad (3.3)$$
$$\theta(y,t) = e^{-\frac{\lambda^2}{Pr}t}(c_3 cos\lambda y + c_4 sin\lambda y) \quad (3.4)$$

Where,

$$B = M^2 + \lambda^2$$
$$m_1 = \frac{A+\sqrt{A^2+4B}}{2}$$
$$m_2 = \frac{A-\sqrt{A^2+4B}}{2}$$
$$c_1 = -c_2 e^{m_1-m_2}$$
$$c_2 = \frac{-e^{\lambda^2 t - m_1}}{e^{m_1-m_2} - e^{m_2-m_1}}$$
$$c_3 = \frac{e^{-\frac{\lambda^2}{Pr}t}}{2sin\lambda}$$
$$c_4 = c_3 tan\lambda$$

**SKIN FRICTION $\tau$**

We considered the skin frictions $\tau_1$ *and* $\tau_2$ at $y = -1$ *and* $y = 1$ respectively.

$$\tau_1 = \frac{du}{dy}\Big|_{y=-1} = e^{-\lambda^2 t}(c_1 m_1 e^{-m_1} + c_2 m_2 e^{-m_2}) \quad (3.5)$$

$$\tau_2 = \frac{du}{dy}\Big|_{y=1} = e^{-\lambda^2 t}(c_1 m_1 e^{m_1} + c_2 m_2 e^{m_2}) \quad (3.6)$$

**NUSSELT NUMBER $Nu$**

We also considered the Nusselt numbers $Nu_1$ *and* $Nu_2$ at $y = -1$ *and* $y = 1$ respectively.

$$Nu_1 = \lambda e^{-\frac{\lambda^2}{Pr}t}(c_3 siny + c_4 cos\lambda) \quad (3.7)$$

$$Nu_2 = \lambda e^{-\frac{\lambda^2}{Pr}t}(-c_3 siny + c_4 cos\lambda) \quad (3.8)$$

## IV. DISCUSSION OF RESULTS

To study the effect of inclined Hartmann in an unsteady MHD couette flow between two infinite parallel porous plates with heat transfer. The velocity profile and the temperature distribution are shown graphically against $y$ for different values of the Hartmann number $Ha$ and Prandlt number $Pr$.

Figure 1, 2, 3 and 4 depicts decrease in velocity $u$ as $Ha$ increases with effect of increase in the angle of inclination $\alpha$ on velocity.

Figure 7 shows that temperature $\theta$ increases with an increase Prandlt number $Pr$.

Tables 1, 2, 3 and 4 show decrease in skin friction $\tau_1$ and increase in skin friction $\tau_2$ with increase in Hartman number $Ha$ and angle of inclination $\alpha$.

Tables 5 and 6 show the variation of Nusselt numbers $Nu_1$ *and* $Nu_2$ with different values of Prandtl number $Pr$ and time $t$. Both tables show that $Nu_1$ *and* $Nu_2$ decreases with increase in Prandtl number $Pr$ and time $t$.

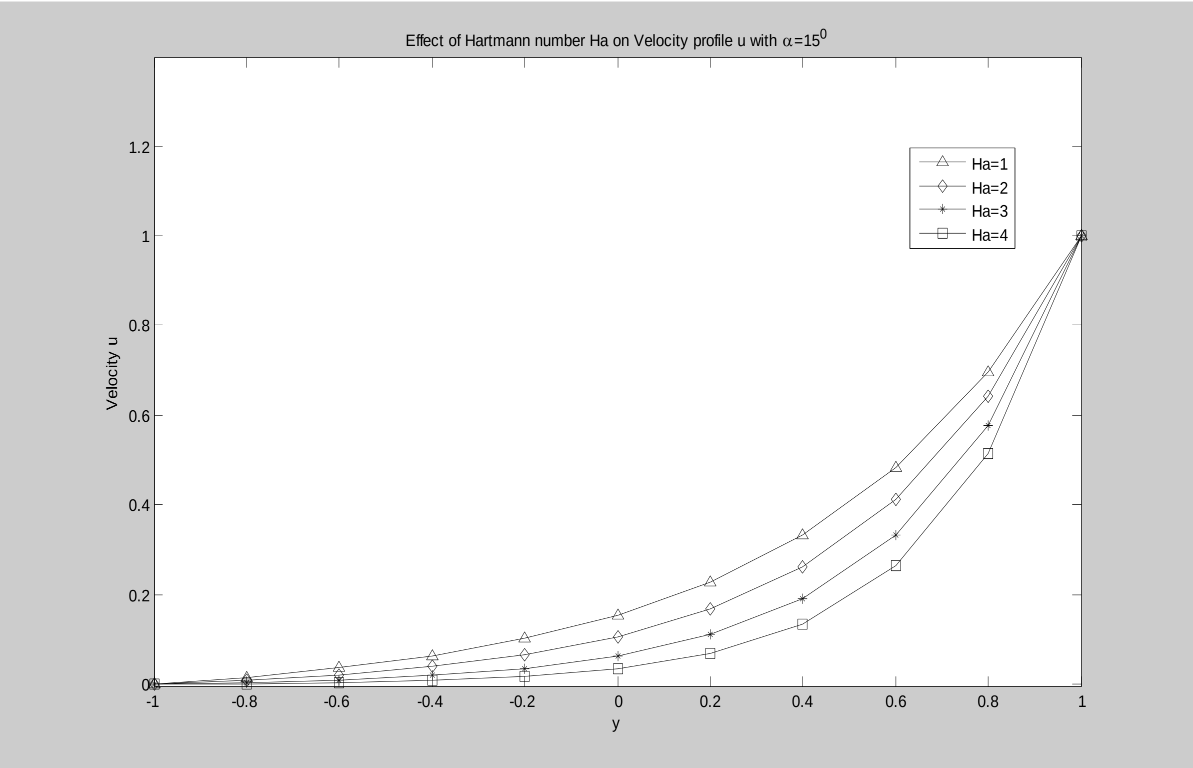


Figure 1: Effect of Hartmann number $Ha$ on velocity profile $u$ with $\alpha = 15^0$ , $A = 1, t = 0.5, and\ \lambda = 1.$

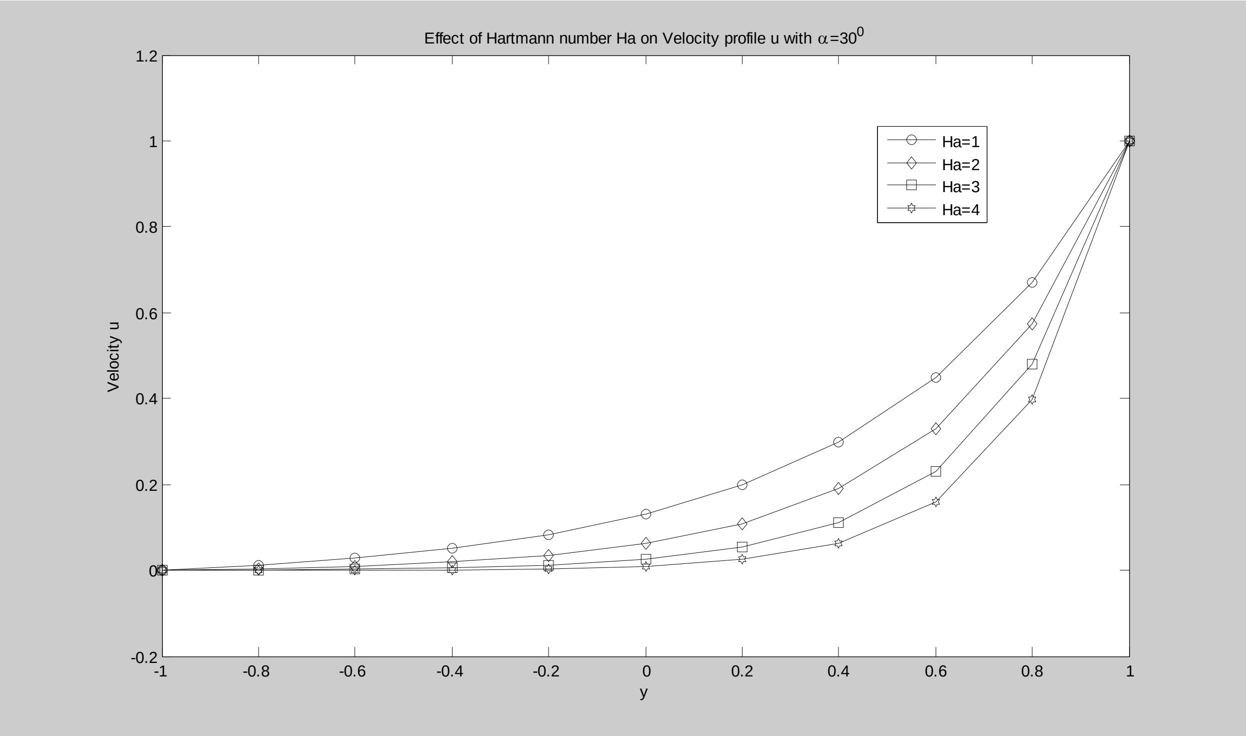


Figure 2: Effect of Hartmann number $Ha$ on velocity profile $u$ with $\alpha = 30^0$ , $A = 1, t = 0.5, and\ \lambda = 1.$

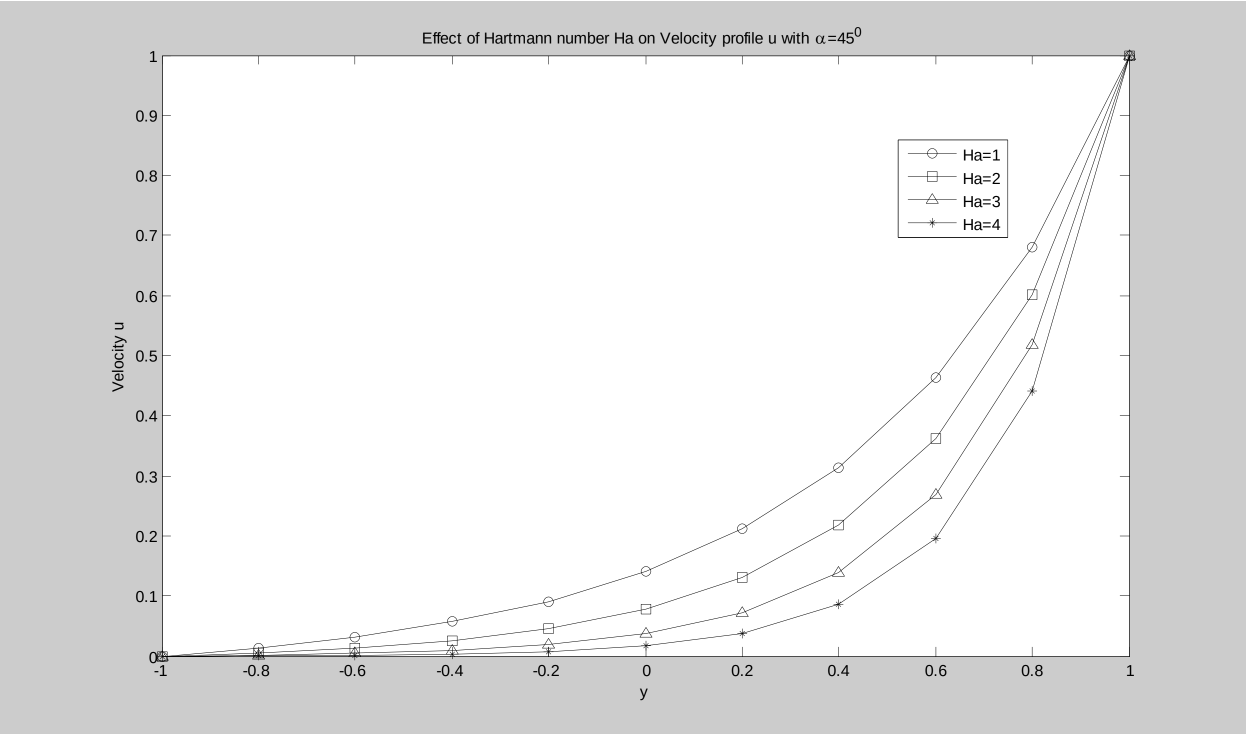


Figure 3: Effect of Hartmann number $Ha$ on velocity profile $u$ with $\alpha = 45^0$ , $A = 1, t = 0.5, and\ \lambda = 1$

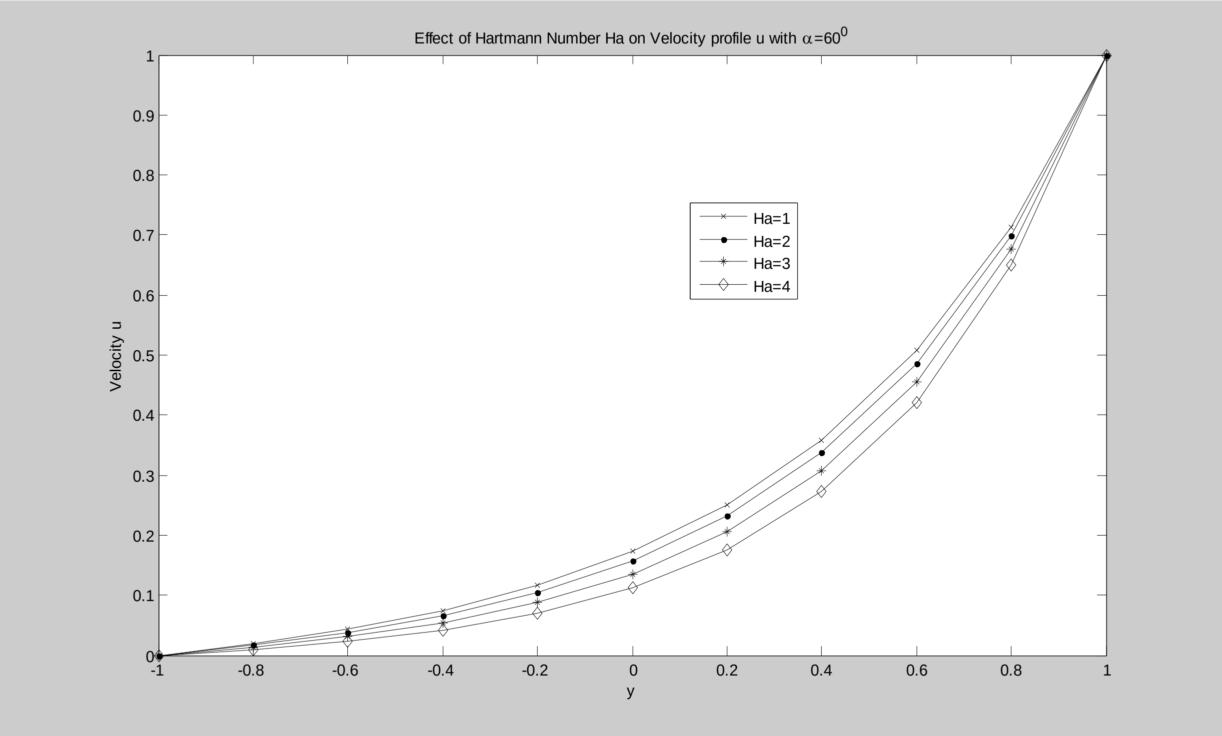


Figure 4: Effect of Hartmann number $Ha$ on velocity profile $u$ with $\alpha = 60^0$ , $A = 1, t = 0.5, and\ \lambda = 1$

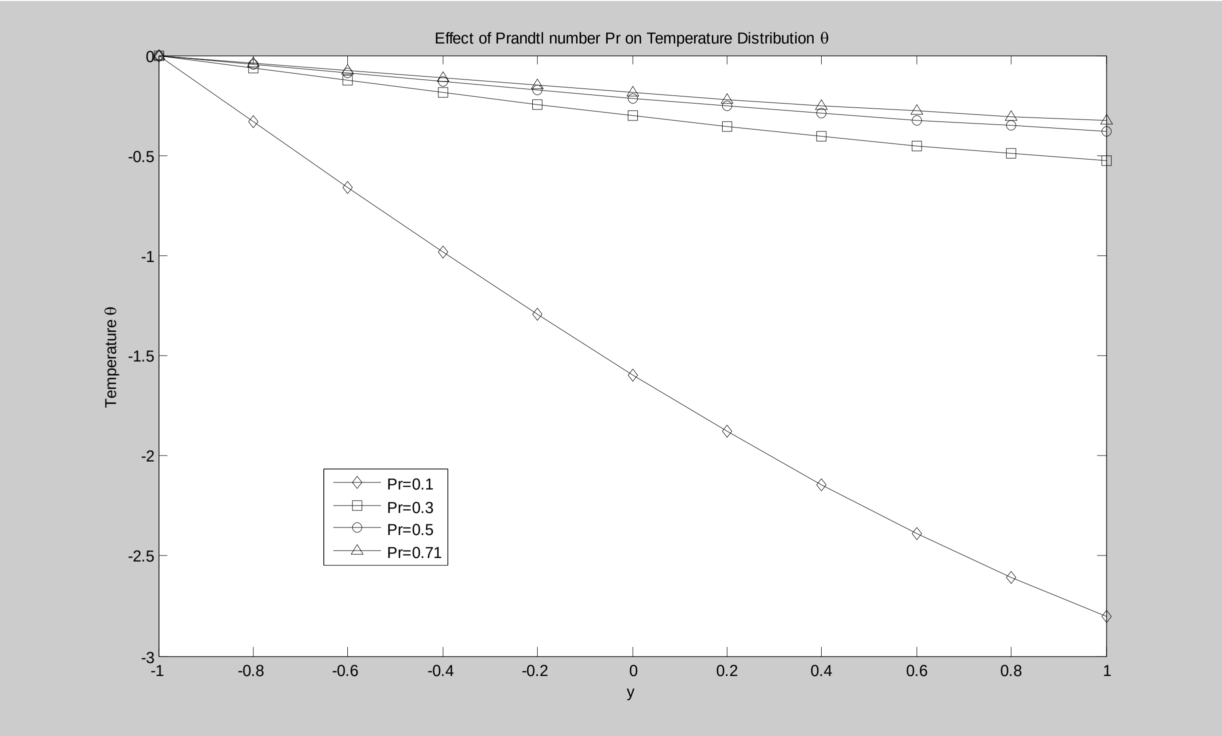


Figure 5: Effect of Prandtl number $Pr$ on temperature distribution $\theta$ with $\lambda = 0.5, t = 0.5$.

Table 1: Variation of skin friction $\tau_1$ *and* $\tau_2$ with different values of Hartmann number $Ha$ at the inclination $\alpha = 15^0$

| $Ha$ | $\tau_1$ | $\tau_2$ |
|---|---|---|
| 1.0 | 0.5058 | 1.5028 |
| 2.0 | 0.0566 | 1.9883 |
| 3.0 | 0.0241 | 2.5761 |
| 4.0 | 0.0090 | 3.1957 |

Table 2: Variation of skin friction $\tau_1$ *and* $\tau_2$ with different values of Hartmann number $Ha$ at the inclination $\alpha = 30^0$

| $Ha$ | $\tau_1$ | $\tau_2$ |
|---|---|---|
| 1.0 | 0.0793 | 1.7340 |
| 2.0 | 0.0232 | 2.5997 |
| 3.0 | 0.0051 | 3.5473 |
| 4.0 | $3.6189 \times 10^{-4}$ | 4.5149 |

Table 3: Variation of skin friction $\tau_1$ *and* $\tau_2$ with different values of Hartmann number $Ha$ at the inclination $\alpha = 45^0$

| $Ha$ | $\tau_1$ | $\tau_2$ |
|---|---|---|
| 1.0 | 0.0901 | 1.6332 |
| 2.0 | 0.0340 | 2.3452 |
| 3.0 | 0.0098 | 3.1490 |
| 4.0 | 0.0024 | 3.9763 |

Table 4: Variation of skin friction $\tau_1$ *and* $\tau_2$ with different values of Hartmann number $Ha$ at the inclination $\alpha = 60^0$

| $Ha$ | $\tau_1$ | $\tau_2$ |
|---|---|---|
| 1.0 | 0.1273 | 1.3442 |
| 2.0 | 0.1088 | 1.4793 |
| 3.0 | 0.0851 | 1.6789 |
| 4.0 | 0.0621 | 1.9195 |

Table 5: Variation of Nusselt number $Nu_1$ with different values of Prandtl number $Pr$ and time $t$.

| $t$ | $Pr = 0.1$ | $Pr = 0.3$ | $Pr = 0.5$ | $Pr = 0.71$ |
|---|---|---|---|---|
| 0.0 | $-0.1366$ | $-0.1366$ | $-0.1366$ | $-0.1366$ |
| 0.5 | $-1.6638$ | $-0.3143$ | $-0.2252$ | $-0.1942$ |
| 1.0 | $-20.2696$ | $-0.7231$ | $-0.3713$ | $-0.2762$ |
| 1.5 | $-246.9345$ | $-1.6638$ | $-0.6121$ | $-0.3928$ |
| 2.0 | $-3.0083 \times 10^3$ | $-3.8284 \times 10^3$ | $-1.0092$ | $-0.5585$ |

Table 6: Variation of Nusselt number $Nu_2$ with different values of Prandtl number $Pr$ and time $t$.

| $t$ | $Pr = 0.1$ | $Pr = 0.3$ | $Pr = 0.5$ | $Pr = 0.71$ |
|---|---|---|---|---|
| 0.0 | $-0.0477$ | $-0.0477$ | $-0.0477$ | $-0.0477$ |
| 0.5 | $-0.5814$ | $-0.1098$ | $-0.0787$ | $-0.0679$ |
| 1.0 | $-7.0825$ | $-0.2527$ | $-0.1297$ | $-0.0965$ |
| 1.5 | $-86.228$ | $-0.5814$ | $-0.2139$ | $-0.1372$ |
| 2.0 | $-1.0511 \times 10^3$ | $-1.3377$ | $-0.3526$ | $-0.1952$ |

## V. SUMMARY AND CONCLUSION

In this section we studied the effect of inclined Hartmann in an unsteady MHD couette flow between two infinite parallel porous plates with heat transfer.

The momentum and energy equations are written in a dimensionless form using the dimensionless parameters.

Variable separable technique was employed to solved the velocity profile and temperature distribution.

However, at high Hartmann number, the velocity as well as the skin friction decreases. When the magnetic field is high, it reduces the energy loss through the plates. But large Nusselt number corresponds to more active convection. Also, when the Prandlt number increases, the temperature distribution decreases.

Thus, it shows that magnetic field has significant effect to the flow of an unsteady MHD couette flow between two infinite parallel porous plates in an inclined magnetic field with heat transfer.

This work can be applied in electric power generator, extrusion of plastics in the manufacture of Rayon and Nylon etc.

## REFERENCES

[1.] A.K. Al-Hadhrami, L. Elliot, M.D. Ingham, X. Wen, (2003): Flow through horizontal channels of porous materials, Inter. J. of Energy Research **27**, 875-889.

[2.] A. Raptis, C. Perkidis, (1999): Radiation and Free Convention flow past a moving plate, Int, J. of Applied Mechanics and Engineering, **4**(4): 817-821.

[3.] C.B. Singh, (1993): MHD Steady flow of liquid between two parallel plates, In: Proc. of First Conference of Kenya Mathematical Society, 24-26.

[4.] C.B. Singh, P.c. Ram, (1978): Unsteady MHD Fluid Flow through a channel: Journal of Scientific Research, **28**(2).

[5.] D.G. Drake, (1965): Flow in a channel due to a periodic pressure gradient, Quart. Journal of Mech. And Appl. Maths., **18**(1).

[6.] Dileep Singh Chauchan and Priyanka Rastogi (2012): Heat transfer effects on rotating MHD couette flow in a channel partially field by a porous medium with hall current. Journal of Appl. Sci. Eng. **15** (3): 281 – 290.

[7.] E.M. Sparrow, R.D. Cess, (1961): The effect of a magnetic field on free convention heat transfer, Int. J. heat Mass Tran., **3**(4): 267-274.

[8.] H. Alfven, (1942): Existence of electromagnetic-hydrodynamics waves, Nature, **150(**3805)**,** 405-406.

[9.] J.A. Shercliff, (1956): Entry of conducting and non-conducting fluids in pipes, Journal of Mathematical Proc. of the Cambridge Philosophical Soc., **52**, 573-583.

[10.] John Mooney, Nick Stokes, (1997): Time-varying MHD flows with free surfaces, Inter. Conf. on CFD in Mineral Metal Processing and Power Generation CSIRO.

[11.] N. Sandeep and V. Sugunamma (2013): Effect of an inclined magnetic field on unsteady free convection flow of a dusty viscous fluid between two infinite flat plates filled by a porous medium. Int. Journal of Appl. Maths. Modelling, **1** (1): 16 – 33.

[12.] P.A. Davidson, (2001): An introduction to MHD, First Edition, Cambrige University Press, UK.

[13.] P.C. Ram, C.B. Singh, U. Singh, (1984): Hall effects on Heat and Mass Transfer flow through porous medium, Astrophysics Space Science, 100: 45-51.

[14.] S. Ganesh, S. Krishnambal, (2007): Unsteady MHD Stokes flow of viscous fluid between two parallel porous plates, Journal of Applied Sciences, **7**, 374-379.

[15.] Stamenkovic M. Zivojin, Dragiza D. Nikodijevic, Bratislav D. Blagojevic, Slobodan R. Savic, (2010): MHD flow and Heat Transfer of two immiscible fluids between moving plates, Transaction of the Canadian Society for Mechanical Engineering, 4(34): 351-372.

[16.] U.S. Rajput, P.K. Sahu, (2011): Transient free convection MHD flow between two long vertical parallel plates with constant Temperature and variable mass diffusion, Int. Journal of Math. Analysis, 34(5): 1665-1671.

[17.] V.M. Soundalgekar, Mohammed Abdulla-Ali, (1986): Free Convention effects on MHD flow past an impulsively started infinite vertical isothermal plate, Reg. J. Energy Heat and Mass Transfer **8**(2): 119-125.

[18.] W.A. Manyonge, D.W. Kiema, C.C.W. Iyaya, (2012): Steady MHD Poiseuille Flow between two infinite parallel porous plates in an inclined magnetic field, Inter. Journal of Pure and Applied Maths., **76**(5): 661-668.