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On A Sequential Probit Model of Infant Mortality in Nigeria

K. T. Amzat[1], S. A. Adeosun[1]
[1] *Department of Mathematical Sciences, Crescent University, Abeokuta, Nigeria.*

ABSTRACT: *This paper presents and analyzed the nature of relationship between infant mortality and some socioeconomic and demographic variables, and the proximate covariate that influence the survival of an infant using the 2003 Nigeria Demographic and Health Survey Data (NDHS). We used sequential probit model to examine the relationship between the dependent variables (infant's death and age at death) and predictor variables for both correlated and uncorrelated error terms. The results of the analysis showed that in both of the situation with correlated and uncorrelated error terms, infant's being alive or death is positively affected by education, birth order number, duration of breast feeding and negatively affected by both total children born and place of delivery. There are significant differences among the predictor variables on the probability of infant's death at neonatal and post neonatal period. The correlation between the error terms is significant. It is needed to be examined two stages together.*

KEYWORDS: *Convergence, Discrete choice, Homoscedasticity, Infant Mortality, Sequential probit model.*

I. INTRODUCTION

By nature, individual enters the human world by birth and leaves by death. Births and deaths are two facts opposite to one another. In statistical terms, a distinction exists between births and infant mortality. They have two things in common; they are both events that have a date and they occur only once for every man. Infant mortality refers to the death of an infant during the first year of life (number of deaths among infants under one year old per 1,000 live births in a given year). Historically, there has been a negative relationship between infant mortality rates (IMR) and economic factors as rightly pointed by some scholars (Zerai [1], Suwal [2]). This relationship is likely caused indirectly by several variables both exogenous and endogenous. Since mothers and infants are among the most vulnerable members of society, infant mortality is a measure of a population's health. In addition, disparities in infant mortality by race/ethnicity and socioeconomic status are an important measure of the inequalities in a society.

Some have argued that racial and economic disparities is a reflection of the long-standing disparity between black and white populations, with the infant mortality rate among black Americans consistently twice that of white Americans. Others cite the wide inequalities between the wealthiest and poorest segments of the society. Whereas, in Africa and Nigeria in particular, mortality as an aspect of demographic studies has not been given as much attention as fertility, the reason is not far-fetched, most researchers who ventured into the area are partially if not wholly, discouraged by the poor responses to child mortality questions. People see questions about their mortality experiences as too private. Much more important is the fact that these experiences are interpreted as uncontrollable by human force. There is also the belief that a woman faced with the problem of constants child deaths is being visited by the same child several times, only to be recalled back to the spirit world on each occasion (Fadipe [3]). The Infant Mortality Rate (IMR) is a public health indicator of a complex societal problem. Numerous frameworks have been used to help understand the multiple determinants of infant mortality in a society and to identify interventions to reduce infant mortality. While the root social causes of infant mortality- persistent poverty, pervasive and subtle racism, and the chronic stresses associated with them- may not be easy to address, it is still possible to understand the risks of infant death by examining the biological pathways through which these societal forces act.Therefore, this paper presents the conceptual basis for the sequential probit model. Theoretical background and parameter estimation method are given. More so, the impact of various demographic and socioeconomic attributes on the probability of infant mortality are estimated via two stages (neonatal phase and post neonatal phase) sequential probit model for both correlated and uncorrelated error terms. Thereafter, estimates of parameters for the infant mortality data are given.

Many studies on infant mortality have suggested different variables indirectly affecting infant mortality (Suwal [2], Turrel and Mengersen [4], Agha [5]). However, hypothesis about indirect effect are not adequately represented by conventional methods. Sequential probit model is a more appropriate statistical technique for this type of situation because of its usefulness in the inclusion of dependent variables into a model and provides estimates of independent variables' marginal effect (Grooraert and Patrins [6], Orzlem and Hatice [7]).

Basic Probit Model:

Probit regression, also called a probit model, is used to model dichotomous or binary outcome variables. In the model, the inverse standard normal distribution of the probability is model as a linear combination of the predictors. The model can be represented as follow:

$$\Psi^{-1}(P) = Z = \beta_0 + \beta_1 x_1 + \cdots + \beta_k x_k + u_k \text{, where } P = \Psi(Z) = \int_{-\infty}^{Z} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}}\, du \qquad (1)$$

and Ψ^{-1} is the inverse of the cumulative distribution function (cdf) of the standard normal distribution. Binary response models directly describe the response probability $\mathbb{P}(y_i = 1)$ of the dependent variable y_i. The probability that the dependent variables take value 1 is modeled as $\mathbb{P}(y_i = 1|x_i) = \mathcal{F}(Z_i) = \mathcal{F}(x_i\beta)$ where β is a $k \times 1$ dimensional column vector of parameters. The probit model assumes that the transformation function $\mathcal{F}$ is the cumulative density function of the standard normal distribution. The response probabilities are then $\mathbb{P}(y_i = 1|x_i) = \Phi(x_i\beta)$. The model also assumes that there is a latent or unobserved variable that is linearly depends on x_i given as $\hat{y} = x_i\beta + u_i$, $\mathbb{E}(u_i) = 0$. This latent variable is the utility difference between choosing

$$y_i = \begin{cases} 1, & \hat{y} > 0 \\ 0, & otherwise \end{cases}. \qquad (2)$$

Sequential Probit Model:

A sequential probit model is used in analyzing discrete choice problems. In the sequential model, each decision is made sequentially according to a binary probit model. Whether or not an alternative j is selected is determined before an alternative k $(> j)$ is considered. The latent variable model is given by

$$\hat{y}_{tj} = \Phi_j + x_t\beta_j + \xi_{tj} \qquad (3)$$

where $\xi_{tj} \sim \text{NID}(0,1)$ for $j = 1,2,\ldots,J-1$, and Φ_j is a scalar constant term, β_j is a $k \times 1$ coefficient vector. Then the sequential probit model is defined as follows:

$$\begin{aligned} y_t &= 1 \text{ if } \hat{y}_{t1} \geq 0 \\ y_t &= 2 \text{ if } \hat{y}_{t1} < 0 \text{ and } \hat{y}_{t2} \geq 0 \\ &\vdots \\ y_t &= j \text{ if } \hat{y}_{t1} < 0, \hat{y}_{t2} < 0, \ldots, \hat{y}_{tj-2} < 0 \text{ and } \hat{y}_{tj-1} < 0. \end{aligned}$$

For this paper, the sequential probit model consists in assuming that infant's age at death occurs about three options in a sequential manner, namely:

α_0 = Infant lives

α_1 = Infant is died between 0 – 1 month (neonatal)

α_2 = Infant is died between 1 – 12 months (post neonatal).

A sequential model is considered with two qualitative variables Y_1 and Y_2, which are observed sequentially. This is illustrated in the figure 1.1 below (Ozlem and Hatice [7]).

Figure 1.1: Sequential probit model

If $y_1 = 0$, outcome α_0 is observed, otherwise depending on the value of y_2, there are two additional outcomes:

$$\alpha_1 = \{y_1 = 1, y_2 = 0\}$$
$$\alpha_2 = \{y_1 = 1, y_2 = 1\}$$

In other words, the first n_1 observation corresponds to outcome $\alpha_0(y_1 = 0)$ and the next n_2 observation corresponds to outcome α_1 or α_2 depending on the value of y_2. We associate with stage j $(1 \text{ or } 2)$ a latent variable $\hat{y}_{j,i}$ such that

$$\hat{y}_{j,i} = \begin{cases} 1, & \hat{y}_{j,i} > 0 \\ 0, & otherwise \end{cases} \tag{4}$$

The continuous latent variables are modeled as follows:

$$\begin{aligned} \hat{y}_{1,i} &= x_{1,i}\beta_1 + \xi_{1,i} \quad \text{for} \quad i = 1,2,3, \dots, n \\ \hat{y}_{2,i} &= x_{2,i}\beta_2 + \xi_{2,i} \quad \text{for} \quad i = n_1 + 1, \dots, n \end{aligned} \tag{5}$$

where $k_2 \times 1$, β_1 and β_2 are vectors of the parameters to be estimated, $\xi_{1,i}$ and $\xi_{2,i}$ are vectors of the error terms (Ozarci [8], Waelbroeck [9]).

For estimation of the parameters, maximum likelihood estimation (MLE) was adopted. The probabilities of the different options are written as follows:

$$\begin{aligned} \mathbb{P}(y_{1,i} = 0) &= \mathbb{P}(\xi_{1,i} \le -x_{1,i}\beta_1) = \Phi_1(-x_{1,i}\beta_1) \\ \mathbb{P}(y_{1,i} = 1, y_{2,i} = 0) &= \mathbb{P}(\xi_{1,i} > -x_{1,i}\beta_1,\ \xi_{2,i} \le -x_{2,i}\beta_2, \rho) = \Phi_2(x_{1,i}\beta_1, -x_{2,i}\beta_2, \rho) \\ \mathbb{P}(y_{1,i} = 1, y_{2,i} = 1) &= \mathbb{P}(\xi_{1,i} > -x_{1,i}\beta_1,\ \xi_{2,i} > -x_{2,i}\beta_2, \rho) = \Phi_2(x_{1,i}\beta_1, -x_{2,i}\beta_2, \rho) \end{aligned} \tag{6}$$

where Φ_1 and Φ_2 are cumulative distribution function of the univariate and bivariate standard normal distribution respectively. If we assume that ξ_1 and ξ_2 are independent, their joint probability is the product of their marginal probabilities. We then have

$$\begin{aligned} \mathbb{P}(y_{1,i} = 1, y_{2,i} = 0) &= \mathbb{P}(\xi_{1,i} > -x_{1,i}\beta_1) \cdot \mathbb{P}(\xi_{2,i} \le -x_{2,i}\beta_2) = \Phi(x_{1,i}\beta_1)\Phi(x_{2,i}\beta_2) \\ \mathbb{P}(y_{1,i} = 1, y_{2,i} = 1) &= \mathbb{P}(\xi_{1,i} > -x_{1,i}\beta_1) \cdot \mathbb{P}(\xi_{2,i} > -x_{2,i}\beta_2) = \Phi(x_{1,i}\beta_1)\Phi(x_{2,i}\beta_2) \end{aligned} \tag{7}$$

Using the probabilities above, the likelihood function of the sequential probit model is given by

$$L(\beta_1, \beta_2, \rho) = \prod_{i=1}^{n} \mathbb{P}_{00,i}^{\,1-y_{1,i}} \cdot \mathbb{P}_{10,i}^{\,y_{1,i}(1-y_{2,i})} \cdot \mathbb{P}_{11,i}^{\,y_{1,i}y_{2,i}} \tag{8}$$

Taking the natural logarithm of the likelihood function $L(\beta_1, \beta_2, \rho)$, we obtain (see Eklof and Kalsson [10])

$$\sum_{i=1}^{n} \{(1 - y_{1,i}) \cdot \ln \mathbb{P}_{00,i} + y_{1,i}(1 - y_{2,i}) \cdot \ln \mathbb{P}_{10,i} + y_{1,i}y_{2,i} \ln \mathbb{P}_{11,i}\} \tag{9}$$

If the error terms are independent $(\rho = 0)$, natural logarithm of the likelihood functions yields

$$nL(\beta_1, \beta_2) = \sum_{i=1}^{n} (1 - y_{1,i}) \cdot \ln \Phi(-x_{1,i}\beta) + y_{1,i}(1 - y_{2,i}) \ln\{\Phi(x_{1,i}\beta) - \Phi(x_{1,i}\beta) \cdot \Phi(x_{2,i}\beta)\}$$

$$+ y_{1,i}y_{2,i} \ln\{\Phi(x_{1,i}\beta_1) \cdot \Phi(x_{2,i}\beta_2)\} \tag{10}$$

The natural logarithm of maximum likelihood function with correlated error term is as follows:

$$nL(\beta_1, \beta_2, \rho) = \sum_{i=1}^{n} (1 - y_{1,i}) \cdot \ln \Phi(-x_{1,i}\beta) + y_{1,i}(1 - y_{2,i}) \ln\{\Phi(x_{1,i}\beta) \cdot \Phi_2(x_{1,i}\beta, x_{2,i}\beta_2, \rho)\}$$

$$+ y_{1,i}y_{2,i} \ln\{\Phi(x_{1,i}\beta_1) \cdot \Phi(x_{2,i}\beta_2, \rho)\} \tag{11}$$

The estimators of sequential probit model were derived by maximizing likelihood function given above. However, the maximum likelihood estimator possesses a number of attractive asymptotic properties, for many problems; they are:

- Consistency: the estimator converges in probability to the value being estimated.
- Asymptotic normality: as the sample size increases, the distribution of the MLE tends to the Gaussian distribution with mean θ and covariance matrix equal to the inverse of Fisher information matrix (Myung [10])

- Efficiency: it achieves the Cramer-Rao lower bound when the sample size tends to infinity. This means that no asymptotically unbiased estimator has lower asymptotic mean square error than the MLE.

Remark: In a sequential probit model, the log likelihood is globally concave and the latent error term is normally distributed and homoscedastic. The maximum likelihood is inconsistent in the presence of Heteroscedasticity. Typically, we used a bivariate normal distribution for two standard normally distributed errors and the joint density would be

$$\Phi(u_1, u_2) = \frac{1}{2\pi}\sigma_{u_1}\sigma_{u_2}\sqrt{1-\rho^2}e^{[-u_1^2 + u_2^2 - 2\rho u_1 u_2 / 1-\rho^2]}$$

where ρ is a “correlation parameter” denoting the extent to which the two errors covary.

Application and Analysis:

At the first stage, the factors affecting both the mortality and survival of infants born within 1998 – 2003 in Nigeria were examined. At the second stage of the sequential process, we investigated the factors affected the infant’s age at death. The data used in the study obtained from the answers of the married women, between 15 – 48 years, who joined the 2003 Nigeria Demographic and Health Survey Data (NDHS) conducted by the Federal Office of Statistics with the aim of gathering reliable information on fertility, family planning, infant and child mortality, vaccination status, breastfeeding and nutrition, etc. In the sequential process, data of 5138 infants have been used for each of the two stages. The variables used in this paper are given in Table 1 and subsequently followed with explainable and necessary tables.

Table 1: Definitions of Variables

Variables	Type of Variable	Stage
Infant’s being alive or death	Dependent	1
Infant’s birth order number	Independent	1
Woman’s total children born	Independent	1
Duration of breastfeeding	Independent	1
Woman’s highest education	Independent	1
Place of delivery	Independent	1
Infant’s age at death	Dependent	2
Duration of breastfeeding	Independent	2
Place of delivery	Independent	2
Delivery by Caesarian Section (CS)	Independent	2
Age of woman at first birth	Independent	2

Table 2: The estimation results of sequential probit model when $\rho = 0$

Variable	Coefficient	Standard error	Z	$P>\|Z\|$	95% Confidence Interval	
First Stage						
Total children born	-0.4122	0.0069	-11.70	0.0000	-0.4813	-0.3432
Birth order number	0.3784	0.0353	10.73	0.0000	0.3093	0.4476
Place of delivery	-0.0110	0.0017	-6.54	0.0000	-0.0143	-0.0077
Highest education	0.2213	0.0286	7.74	0.0000	0.1653	0.2774
Duration of breastfeeding	0.0054	0.0007	8.05	0.0000	0.0042	0.0068
Second Stage						
Age of first birth	0.0070	0.0069	-1.10	0.312	-0.0066	0.0205
Place of delivery	-0.0066	0.0018	-3.75	0.000	-0.0101	-0.0032
Delivery by CS	-0.0036	0.0285	-0.13	0.895	-0.0594	0.0522

From TABLE 2 above, at the first stage of the model, the dependent variable (infant’s being alive or death) is affected by total children born, birth order number, place of delivery, highest education and duration of breastfeeding. At the second stage, infant’s age at death is affected by place of delivery. To determine the magnitude of these effects, marginal effects can be calculated which indicate that for every one unit increase in a variable causes an increase on the probability of the dependent variable.

Table 3: Marginal effects of independent variables of sequential probit model

Variable	Coefficient	Standard error	Z	$P>\|Z\|$	95% Confidence Interval	
First Stage						
Total children born	-0.0874	0.0073	-12.00	0.0000	-0.1006	-0.0718
Birth order number	0.0803	0.0073	10.96	0.0000	0.0648	0.0936
Place of delivery	-0.0023	0.0004	-6.58	0.0000	-0.0030	-0.0016
Highest education	0.0470	0.0061	7.78	0.0000	0.0341	0.0580
Duration of breastfeeding	0.0012	0.0001	8.09	0.0000	0.0009	0.0014

Second Stage					
Age of first birth	0.0011	0.0010	1.01	0.3120	-0.0008 0.0026
Place of delivery	-0.0010	0.0003	-3.76	0.0000	-0.0013 -0.0004
Delivery by CS	-0.0005	0.0043	-0.13	0.8990	-0.0076 0.0067

According to TABLE 3, at the first stage of the model, place of delivery and total children born, negatively affected the probability of infant's death. The total children born decrease the probability of infant's death by 8.7%. Birth order number is the most effective positive variable which indicates that increase of one unit in this variable (birth order number) causes an increase in the probability of infant's death by 8.03%, closely followed by highest education with 4.7% and duration of breastfeeding. At the second stage of the model, place of delivery negatively affect the probability of infant's age at death.

Table 4: Estimates of parameters of sequential probit model for $\rho \neq 0$

Variable	Coefficient	Standard error	Z	$P>\|Z\|$	95% Confidence Interval
First Stage					
Total children born	-0.2951	0.0301	-9.78	0.0000	-0.3542 -0.2359
Birth order number	0.2753	0.0300	9.18	0.0000	0.2165 0.3340
Place of delivery	-0.0108	0.0016	-6.64	0.0000	-0.0140 -0.0076
Highest education	0.1959	0.0240	8.16	0.0000	0.1488 0.2429
Duration of breastfeeding	0.0053	0.0006	9.47	0.0000	0.0042 0.0064
Second Stage					
Age of first birth	-0.0025	0.0069	-0.37	0.7120	-0.0159 0.0109
Place of delivery	-0.0072	0.0018	-3.98	0.0000	-0.0107 -0.0036
Delivery by CS	-0.0357	0.0321	-1.11	0.2660	-0.0985 0.0271
rho (ρ)	0.9100	0.0142			

From the TABLE 4, at the first stage of the model, education, total number of children born, place of delivery, birth order number and duration of breastfeeding are significant. At the second stage, place of delivery significantly affect infant's age at death.

Table 5: Marginal effect of independent variable for $\rho \neq 0$ (Neonatal)

Variable	Coefficient	Standard error	Z	$P>\|Z\|$	95% Confidence Interval
First Stage					
Total children born	-0.0338	0.0165	-2.05	0.040	-0.0660 -0.0015
Birth order number	0.0311	0.0155	2.00	0.045	0.0007 0.0615
Place of delivery	-0.0021	0.0023	-1.81	0.071	-0.0043 0.0002
Highest education	0.0189	0.0085	2.22	0.026	0.0022 0.0356
Duration of breastfeeding	0.0007	0.0003	2.30	0.021	0.0001 0.0012
Second Stage					
Age of first birth	-0.0001	0.0001	-1.09	0.275	-0.0001 0.0000
Place of delivery	-0.0001	0.0011	-1.78	0.073	-0.0032 0.0001
Delivery by CS	-0.0002	0.0004	-0.49	0.623	-0.0010 0.0006

Table 6: Marginal effect of independent variable for $\rho \neq 0$ (Post neonatal)

Variable	Coefficient	Standard error	Z	$P>\|Z\|$	95% Confidence Interval
First Stage					
Total children born	-0.0273	0.0045	-6.01	0.000	-0.0363 -0.0184
Birth order number	0.0252	0.0042	5.92	0.000	0.0169 0.0335
Place of delivery	-0.0018	0.0004	-4.98	0.000	-0.0025 -0.0011
Highest education	0.0153	0.0027	5.65	0.000	0.0010 0.0206
Duration of breastfeeding	0.0005	0.0001	9.44	0.000	0.0004 0.0006
Second Stage					
Age of first birth	-0.0001	0.0001	-1.00	0.315	-0.0004 0.0001
Place of delivery	-0.0003	0.0002	-4.68	0.075	-0.0026 -0.0011
Delivery by CS	-0.0005	0.0011	-0.49	0.625	-0.0026 0.0016

From TABLE 5 and TABLE 6 above, at the first stage, all the variables are significantly affect the infant's age at death (neonatal) except place of delivery. Birth order number is the most effective positive variable that increases the probability of infant's age at death (neonatal) by 3.11%, followed by education with 1.89%. Total number of children decreases the probability of infant's age at death (neonatal) by 3.36%. Similarly, at the second stage of the model, delivery by caesarian section and age of the woman at first birth are not significant while that of duration of breastfeeding positively affect the probability of infant's age at death. Place of delivery decreases the probability of infant's age at death (post neonatal).

II. CONCLUSION

In this paper, the nature of relationship between infant mortality and some demographic socioeconomic and health variables were studied. The data on infant's being alive or death and age at death used for the study were obtained from 2003 data files. In both of the situations with correlated and uncorrelated error terms, infant's being alive or death is positively affected by education, birth order, duration of breastfeeding and negatively by both total children born and place of delivery. Also, infant's age at death is affected negatively by place of delivery. This indicated that the related independent variables decrease the probability of infant's age at death at post neonatal period. More so, the correlation coefficient, ρ, is statistically significant ($\rho = 0.9100$). That is, infant's age at death and infant's being alive or death are related. There are significant differences among the variables on the probability of infant's death at neonatal and post neonatal period.

This paper provides assessment of the relative importance of factors associated with neonatal and post neonatal in Nigeria. The results showed that; there is a relationship between infant's death and age at death. Birth order number, mother's education and duration of breastfeeding are the most possible variables that influence the survival of a child. To alleviate fears of women towards caesarian section and to support medical experts' point of view, the study indicated that age at first birth and delivery by caesarian section not effective in determining infant's age at death. More so, breastfeeding should be encouraged among nursing mothers and especially working class mothers, as it helps in bringing down infant mortality. It is hope that the findings of this paper will guide the key policy planners to achieve the goals of Nigeria's development plan. All government parastatals and private organizations responsible for different survey should be consistent and make necessary follow up at regular intervals, so that there will be no chance of missing events.

REFERENCES

[1] Zerai, A., Preventive health strategies and infant survival in Zimbabwe, *African Population Studies 11(1),* 1996, 29-62.
[2] Suwal, J.V., 2001. The Main Determinants of Infant Mortality in Nepal. Social Sci. Med., 53: 1667-1681.
[3] Fadipe, N. A. 1970. The Sociology of the Yoruba. Ibadan: Ibadan University Press.
[4] Turrel, G. and Mengersen K., 2000. Socioeconomic Status and Infant Mortality in Australia. A national Study of Small Urban Areas 1985-89. Social Sci. Med., 50: 1209-1225.
[5] Agha, S., 2000.The Determinant of Infant Mortality in Pakistan. Social Sci. Med., 51: 199-208
[6] Grooraert, C. and Patrinos H. A., 1999. A Four Country Comparative Study of Child labour, Policy Analysis of Child Labour: AComparative Study, St. Martin Press, New York.
[7] Ozlem Alpu and Hatice Fidan, Sequential Probit Model for Infant Mortality Modeling in Turkey, *Journal of Applied Sciences4(4),* 2004, 590-595.
[8] Ozarici, O., *Bivariate probit model with full observability and Heteroscedasticity and an application*. Ph.D. Thesis, Osmangazi University, Turkey. 2002.
[9] Waelbroeck, P., 2000. Econometric Analysis of the Sequential Probit Model, GREQUAM, 25.