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# A Class of Seven Point Zero Stable Continuous Block Method for Solution of Second Order Ordinary Differential Equation

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***ABSTRACT:*** *This paper considers the development of a class of seven-point implicit methods for direct solution of general second order ordinary differential equations. We extend the idea of collocation of linear multi-step methods to develop a uniform order 6 seven (7)-step block methods. The single continuous formulation derived is evaluated at grid point of* $x = x_{n+q}, q = 7$ *and its second derivative evaluated at interior points* $q = 2, 3, 4, 5, 6$ *yielding the multi-discrete schemes that form a self starting uniform order 6-block method. Two numerical examples were used to demonstrate the efficiency of the methods.*

***KEYWORDS****: Linear Multistep Method, Seven Point Block Method, Continuous Formulation, Zero Stable, Matrix Inverse, Region of Absolute Stability*.

## I. INTRODUCTION

In this paper, a direct numerical solution to the general second order initial value differential equations of the form:

$$y'' = f(x, y, y'), y(0) = \alpha, y'(0) = \beta \qquad (1)$$

is proposed without recourse to the conventional way of reducing it to a system of first order of equations which has many disadvantages (Awoyemi and Kayode, 2002). Attempts have been made by various authors to solve equation (1) in which the first derivative ($y'$) is absent, (Onumanyi et-al, 2002). This limits the solution to a special class of differential equations. Efforts have also been made to develop method for solving equation (1) directly with little attention at solutions at some grid points (Yahaya and Badmus, 2009; Umar, 2011). In this paper, we construct a uniform order 6, seven-step block method for direct approximation of the solution of equation (1).

## II. DEVELOPMENT OF THE METHOD

We propose an approximate solution to (1) in the form:

$$y(x) = \sum_{g=0}^{r+s-1} a_g\, x^g \qquad (2)$$

$$y''(x) = \sum_{g=0}^{r+s-1} g(g-1)\, a_g\, x^{(g-2)} = f(x, y, y') \qquad (3)$$

Collocating (3) at $x = x_{n+q}, q = 2, 3, \ . \ . \ . \ , 6$ and interpolating (2) at $x = x_{n+q}, q = 7$ leads to a system of equations written in the form:

$$\begin{aligned}
&a_0 + a_1 x_n + a_2 x_n^2 + a_3 x_n^3 + a_4 x_n^4 + a_5 x_n^5 + a_6 x_n^6 + a_7 x_n^7 = y_n \\
&a_0 + a_1 x_{n+1} + a_2 x_{n+1}^2 + a_3 x_{n+1}^3 + a_4 x_{n+1}^4 + a_5 x_{n+1}^5 + a_6 x_{n+1}^6 + a_7 x_{n+1}^7 = y_{n+1} \\
&a_0 + a_1 x_{n+2} + a_2 x_{n+2}^2 + a_3 x_{n+2}^3 + a_4 x_{n+2}^4 + a_5 x_{n+2}^5 + a_6 x_{n+2}^6 + a_7 x_{n+2}^7 = y_{n+2} \\
&a_0 + a_1 x_{n+3} + a_2 x_{n+3}^2 + a_3 x_{n+3}^3 + a_4 x_{n+3}^4 + a_5 x_{n+3}^5 + a_6 x_{n+3}^6 + a_7 x_{n+3}^7 = y_{n+3} \\
&a_0 + a_1 x_{n+4} + a_2 x_{n+4}^2 + a_3 x_{n+4}^3 + a_4 x_{n+4}^4 + a_5 x_{n+4}^5 + a_6 x_{n+4}^6 + a_7 x_{n+4}^7 = y_{n+4} \\
&a_0 + a_1 x_{n+5} + a_2 x_{n+5}^2 + a_3 x_{n+5}^3 + a_4 x_{n+5}^4 + a_5 x_{n+5}^5 + a_6 x_{n+5}^6 + a_7 x_{n+5}^7 = y_{n+5} \\
&a_0 + a_1 x_{n+6} + a_2 x_{n+6}^2 + a_3 x_{n+6}^3 + a_4 x_{n+6}^4 + a_5 x_{n+6}^5 + a_6 x_{n+6}^6 + a_7 x_{n+6}^7 = y_{n+6} \\
&2a_0 + 6a_3 x_{n+7} + 12a_4 x_{n+7}^2 + 20a_5 x_{n+5}^3 + 30a_6 x_{n+7}^4 + 42a_7 x_{n+7}^5 = y_{n+7}
\end{aligned} \qquad (4)$$

When re-arranging (4) in a matrix form $Ax = B$, we obtained

$$\begin{pmatrix}
1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6 & x_n^7 \\
1 & x_{n+1} & x_{n+1}^2 & x_{n+1}^3 & x_{n+1}^4 & x_{n+1}^5 & x_{n+1}^6 & x_{n+1}^7 \\
1 & x_{n+2} & x_{n+2}^2 & x_{n+2}^3 & x_{n+2}^4 & x_{n+2}^5 & x_{n+2}^6 & x_{n+2}^7 \\
1 & x_{n+3} & x_{n+3}^2 & x_{n+3}^3 & x_{n+3}^4 & x_{n+3}^5 & x_{n+3}^6 & x_{n+3}^7 \\
1 & x_{n+4} & x_{n+4}^2 & x_{n+4}^3 & x_{n+4}^4 & x_{n+4}^5 & x_{n+4}^6 & x_{n+4}^7 \\
1 & x_{n+5} & x_{n+5}^2 & x_{n+5}^3 & x_{n+5}^4 & x_{n+5}^5 & x_{n+5}^6 & x_{n+5}^7 \\
1 & x_{n+6} & x_{n+6}^2 & x_{n+6}^3 & x_{n+6}^4 & x_{n+6}^5 & x_{n+6}^6 & x_{n+6}^7 \\
0 & 0 & 2 & 6x_{n+7} & 12x_{n+7}^2 & 20x_{n+7}^3 & 30x_{n+7}^4 & 42x_{n+7}^5
\end{pmatrix}
\begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \end{pmatrix}
=
\begin{pmatrix} y_n \\ y_{n+1} \\ y_{n+2} \\ y_{n+3} \\ y_{n+4} \\ y_{n+5} \\ y_{n+6} \\ f_{n+7} \end{pmatrix} \qquad (5)$$

where the $\alpha_g$ 's are the coefficients to be determined, and are obtained as continuous coefficients of $\alpha_j(x)$ and $\beta_j(x)$.

Specifically, the proposed solution takes the form:

$$y(x) = \alpha_0(x)y_n + \alpha_1(x)y_{n+1} + \alpha_2(x)y_{n+2} + \alpha_3(x)y_{n+3} + \alpha_4(x)y_{n+4} + \alpha_5(x)y_{n+5} + \alpha_6(x)y_{n+6} + h^2[\beta_7(x)f_{n+7}] \quad (6)$$

A mathematical software (maple 15) is used to obtained the inverse of the matrix D in equation (5) were values for $\alpha_g$ 's were established. After some manipulation to the inverse of the matrix, we obtain the continuous formulation of the method as:

$y(x) :=$

$$+\left(-\frac{324479}{22512}\frac{\xi^3}{h^3}+\frac{\frac{95925}{13132}x_n-\frac{95925}{13132}x}{h}-\frac{389}{78792}\frac{\xi^7}{h^7}\right.$$
$$\left.+\frac{18839}{3216}\frac{\xi^4}{h^4}-\frac{9141}{7504}\frac{\xi^5}{h^5}+\frac{2803}{22512}\frac{\xi^6}{h^6}+\frac{4545}{268}\frac{\xi^2}{h^2}\right)y_{n+4}$$
$$+\left(\frac{174911}{28140}\frac{\xi^3}{h^3}+\frac{-\frac{98331}{32830}x_n+\frac{98331}{32830}x}{h}+\frac{3929}{1575840}\frac{\xi^7}{h^7}\right.$$
$$\left.-\frac{16879}{6432}\frac{\xi^4}{h^4}+\frac{42747}{75040}\frac{\xi^5}{h^5}-\frac{19023}{2680}\frac{\xi^2}{h^2}-\frac{13663}{225120}\frac{\xi^6}{h^6}\right)$$
$$y_{n+5}+\left(\frac{32147}{24120}\frac{\xi^2}{h^2}-\frac{401557}{337680}\frac{\xi^3}{h^3}+\frac{4967}{9648}\frac{\xi^4}{h^4}\right.$$
$$-\frac{319}{590940}\frac{\xi^7}{h^7}+\frac{4297}{337680}\frac{\xi^6}{h^6}+\frac{\frac{10939}{19698}x_n-\frac{10939}{19698}x}{h}$$
$$\left.-\frac{7787}{67536}\frac{\xi^5}{h^5}\right)y_{n+6}+\left(\left(-\frac{90}{3283}x_n+\frac{90}{3283}x\right)h\right.$$
$$-\frac{15}{536}\frac{\xi^4}{h^2}+\frac{25}{3752}\frac{\xi^5}{h^3}+\frac{29}{469}\frac{\xi^3}{h}-\frac{3}{3752}\frac{\xi^6}{h^4}-\frac{9}{134}\xi^2$$
$$\left.+\frac{1}{26264}\frac{\xi^7}{h^5}\right)f_{n+7}$$

Evaluating the continuous formulation at $x = x_{n+q}$, $q = 7$ and its second derivative evaluated at $q = 2, \ldots, 6$ and its first derivative evaluated at $x = x_n$ we obtained the following discrete equations:

$$\frac{67}{90}y_{n+7}-\frac{223}{70}y_{n+6}+\frac{879}{140}y_{n+5}-\frac{949}{126}y_{n+4}+\frac{41}{7}y_{n+3}-\frac{201}{70}y_{n+2}+\frac{1019}{1260}y_{n+1}-\frac{1}{10}=\frac{\frac{2}{h}}{7}[f_{n+7}]$$

$$\frac{349}{18}y_{n+6}-\frac{387}{4}y_{n+5}+\frac{675}{5}y_{n+4}+\frac{6835}{18}y_{n+3}-1005y_{n+2}+\frac{2253}{4}y_{n+1}-\frac{1057}{36}y_n=\frac{2}{h}[f_{n+7}+469f_{n+7}]$$
$$\frac{1}{90}y_{n+6}-\frac{3}{20}y_{n+5}+\frac{3}{2}y_{n+4}-\frac{49}{18}y_{n+3}+\frac{3}{2}y_{n+2}+\frac{3}{20}y_{n+1}-\frac{1}{90}y_n=\frac{2}{h}[f_{n+3}]$$
$$\frac{961}{20}y_{n+6}-\frac{13191}{20}y_{n+5}+1224y_{n+4}-\frac{1325}{2}y_{n+3}-\frac{201}{4}y_{n+2}+\frac{9}{20}y_{n+1}-\frac{7}{10}y_n=\frac{2}{h}[f_{n+7}-469f_{n+4}]$$
$$\frac{78289}{90}y_{n+6}-\frac{29727}{20}y_{n+5}+\frac{195}{2}y_{n+4}+\frac{16091}{17}y_{n+3}-\frac{1005}{2}y_{n+2}+\frac{2913}{20}y_{n+1}-\frac{1631}{90}y_n=\frac{2}{h}[11f_{n+7}-938f_{n+5}]$$
$$\frac{7859}{45}y_{n+6}-\frac{11529}{20}y_{n+5}+\frac{3171}{4}y_{n+4}-\frac{11131}{18}y_{n+3}+\frac{603}{2}y_{n+2}-\frac{1689}{20}y_{n+1}+\frac{1871}{180}y_n=\frac{2}{h}[67f_{n+6}-9f_{n+7}]$$
$$\frac{3282}{90}hz_n+\frac{10939}{540}y_{n+6}-\frac{32777}{300}y_{n+5}+\frac{6395}{24}y_{n+4}-\frac{20767}{54}y_{n+3}+\frac{4355}{12}y_{n+2}-\frac{14981}{60}y_{n+1}+\frac{56329}{600}y_n=\frac{2}{h}[f_{n+7}] \quad (7)$$

Equation (7) is the proposed seven-step block method from the continuous formulation. The application of the block integrators (7) with $n = 0$, give values of $y_1$, $y_2$, $y_3$, $y_4$, $y_5$, $y_6$ and $y_7$ directly without the use of starters.

## III. ANALYSIS OF THE METHODS

**Order, Consistency and zero-stability**

1.1 Order of a LMM

A linear multistep method (LMM) is said to be of order $p$ if $C_o = 0$, $C_1 = 0, \ldots, C_{p+1} = 0$ but $C_{p+2} \neq 0$ where $C_{p+2}$ is called the error constant.

1.2 Consistency of LMM

A linear multistep method (LMM) is consistent if it has order $P \geq 1$.

1.3 Zero Stability of LMM

A LMM is said to be zero-stable if no root of the 1st characteristic polynomial has modulus greater than one, and if every root with modulus 1 is simple.

1.4 Fundamental theorem of Dahlquist on LMM

The necessary and sufficient conditions for a LMM to be convergent are that, it be consistent and zero-stable.

Thus, equation (7) shows that it has uniform order $[6, 6, 6, 6, 6, 6, 6]^T$ with error constants

$$C_{p+2} = \left[\frac{-363}{3920}, \frac{-2531}{504}, \frac{1}{560}, \frac{-52}{35}, \frac{-80023}{5040}, \frac{521}{56}, \frac{-3281}{630}\right]^T \tag{8}$$

hence, equation (8) therefore satisfies definitions (1.1) and (1.2), (Fatunla, 1992).

## IV. CONVERGENCE ANALYSIS

A desirable property for a numerical integrator is that its solution behaves similar to the theoretical solution to a given problem at all times. Thus, several definitions, which call for the method to posses some "adequate" region of absolute stability, can be found in several literatures. See Fatunla [6, 7], Lambert [11, 12] etc. Following Fatunla [6, 7], the seven block integrators in equation (7) are put in matrix form as:

$$\overset{A^{(0)}}{\begin{pmatrix} \frac{1019}{1260} & \frac{-201}{70} & \frac{41}{7} & \frac{-949}{126} & \frac{879}{140} & \frac{-223}{70} & \frac{67}{90} \\ \frac{-1689}{20} & \frac{603}{2} & \frac{-1131}{18} & \frac{3171}{4} & \frac{-11529}{20} & \frac{7859}{45} & 0 \\ \frac{2913}{20} & \frac{-1006}{2} & \frac{16091}{18} & \frac{195}{2} & \frac{-29727}{20} & \frac{78289}{90} & 0 \\ \frac{9}{20} & \frac{201}{4} & \frac{-1325}{2} & 1224 & \frac{-13191}{20} & \frac{961}{20} & 0 \\ \frac{-3}{20} & \frac{3}{2} & \frac{-49}{18} & \frac{3}{2} & \frac{-3}{20} & \frac{1}{90} & 0 \\ \frac{2253}{4} & -1005 & \frac{6835}{18} & \frac{675}{4} & \frac{-387}{4} & \frac{349}{18} & 0 \\ \frac{-14981}{60} & \frac{4355}{12} & \frac{-20767}{54} & \frac{6395}{24} & \frac{-32777}{300} & \frac{10939}{540} & 0 \end{pmatrix}} \begin{pmatrix} y_{n+1} \\ y_{n+2} \\ y_{n+3} \\ y_{n+4} \\ y_{n+5} \\ y_{n+6} \\ y_{n+7} \end{pmatrix} =$$

$$\overset{A^{(1)}}{\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{10} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1871}{180} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1631}{90} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{7}{10} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{90} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1057}{36} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{-56326}{600} \end{pmatrix}} \begin{pmatrix} y_{n-6} \\ y_{n-5} \\ y_{n-4} \\ y_{n-3} \\ y_{n-2} \\ y_{n-1} \\ y_n \end{pmatrix} + h^2$$

$$\overset{B^{(0)}}{\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{7} \\ 0 & 0 & 0 & 0 & 0 & 67 & -9 \\ 0 & 0 & 0 & 0 & 938 & 0 & 11 \\ 0 & 0 & 0 & -469 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 469 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}} \begin{pmatrix} f_{n+1} \\ f_{n+2} \\ f_{n+3} \\ f_{n+4} \\ f_{n+5} \\ f_{n+6} \\ f_{n+7} \end{pmatrix} \tag{9}$$

For easy analysis, the expression in (9) was normalized to obtained

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} y_{n+1} \\ y_{n+2} \\ y_{n+3} \\ y_{n+4} \\ y_{n+5} \\ y_{n+6} \\ y_{n+7} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} y_{n-6} \\ y_{n-5} \\ y_{n-4} \\ y_{n-3} \\ y_{n-2} \\ y_{n-1} \\ y_n \end{pmatrix}$$

$$\begin{pmatrix} 0 & \frac{18637}{2520} & \frac{-235183}{10080} & \frac{10754}{315} & \frac{-135713}{5040} & \frac{5603}{504} & \frac{-19087}{10080} \\ 0 & \frac{13613}{630} & \frac{-20449}{315} & \frac{29467}{315} & \frac{-23062}{315} & \frac{18953}{630} & \frac{-1609}{315} \\ 0 & \frac{20799}{560} & \frac{-24105}{224} & \frac{43263}{280} & \frac{-67563}{560} & \frac{27723}{560} & \frac{-1881}{224} \\ 0 & \frac{16616}{315} & \frac{-47072}{315} & \frac{13576}{63} & \frac{-52928}{315} & \frac{21704}{315} & \frac{-736}{63} \\ 0 & \frac{17225}{252} & \frac{-385375}{2016} & \frac{69875}{252} & \frac{-217025}{1008} & \frac{11125}{126} & \frac{-30175}{2016} \\ 0 & \frac{5877}{77} & \frac{-8151}{35} & \frac{11871}{35} & \frac{-9162}{35} & \frac{1509}{14} & \frac{-639}{35} \\ 0 & \frac{71687}{720} & \frac{-395479}{1440} & \frac{144403}{360} & \frac{-221921}{720} & \frac{92267}{720} & \frac{-30919}{1440} \end{pmatrix} \begin{pmatrix} f_{n+1} \\ f_{n+2} \\ f_{n+3} \\ f_{n+4} \\ f_{n+5} \\ f_{n+6} \\ f_{n+7} \end{pmatrix} \quad (10)$$

Equation (10) is the 1-block 7 point method. The first characteristics polynomial of the 1-block 7-step block method is thereby given as:

$\rho(R) = det\left[RA^{(0)} - A^{(1)}\right]$

$$= \det\left\{ R \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \right\}$$

$$= \det\left\{ \begin{pmatrix} R & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & R & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & R & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & R & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & R & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & R & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & R \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \right\}$$

$$= \det \begin{pmatrix} R & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & R & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & R & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & R & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & R & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & R & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & R-1 \end{pmatrix} \quad (11)$$

$\rho(R) = R^6(R-1)$.This implies that $R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = 0,\ \ R_7 = 1$

The 1-block 7 point is zero stable and is also consistent as its order $[6, 6, 6, 6, 6, 6, 6]^T > 1$.Thus, it is convergent, following Henrici [9].

Equation (7) can also be reformulated to give:

## V. REGION OF ABSOLUTE STABILITY

To compute and plot region of absolute stability of the block methods, we reformulate (7) to obtain equation (12) and express it as a general linear methods in the form:

$$\begin{bmatrix} Y \\ y_{n+1} \end{bmatrix} = \begin{bmatrix} A & U \\ B & V \end{bmatrix} \begin{bmatrix} h\,f(y) \\ y_{i-1} \end{bmatrix}$$

Where:

$$A=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-60}{14981} \\ 0 & 0 & \frac{-467}{1005} & 0 & 0 & 0 & 0 & \frac{-1}{1005} \\ 0 & 0 & 0 & \frac{-18}{49} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-469}{1224} & 0 & 0 & \frac{1}{1224} \\ 0 & 0 & 0 & 0 & 0 & \frac{-18760}{29727} & 0 & \frac{-220}{29727} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{3015}{7859} & \frac{-405}{7859} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{90}{469} \end{pmatrix}$$

$$U=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{10939}{134829} & \frac{-589986}{1348290} & \frac{31975}{29962} & \frac{-207670}{134829} & \frac{21775}{14981} & 0 & \frac{56329}{149810} \\ \frac{349}{18090} & \frac{-387}{4020} & \frac{45}{268} & \frac{1367}{3618} & 0 & \frac{751}{1340} & \frac{-1057}{36180} \\ \frac{1}{245} & \frac{-27}{490} & \frac{27}{49} & 0 & \frac{27}{49} & \frac{-27}{490} & \frac{1}{245} \\ \frac{-961}{24480} & \frac{4397}{8160} & 0 & \frac{1325}{2448} & \frac{-67}{1632} & \frac{-1}{2720} & \frac{7}{12240} \\ \frac{156578}{267543} & 0 & \frac{650}{9909} & \frac{160910}{267543} & \frac{-3350}{9909} & \frac{971}{9909} & \frac{-3262}{267543} \\ 0 & \frac{103761}{31436} & \frac{-142695}{31436} & \frac{55655}{15718} & \frac{-27135}{15718} & \frac{15201}{31436} & \frac{-1871}{31436} \\ \frac{2007}{469} & \frac{-7911}{938} & \frac{4735}{469} & \frac{-3690}{469} & \frac{27}{7} & \frac{-1091}{938} & \frac{9}{67} \end{pmatrix}$$

$$B=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{90}{469} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{3015}{7859} & \frac{-405}{7859} \\ 0 & 0 & 0 & 0 & 0 & \frac{-1876}{29727} & 0 & \frac{-220}{29727} \\ 0 & 0 & 0 & 0 & \frac{-469}{1224} & 0 & 0 & \frac{1}{1224} \\ 0 & 0 & 0 & \frac{-18}{49} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-469}{1005} & 0 & 0 & 0 & 0 & \frac{-1}{1005} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-60}{14981} \end{pmatrix}$$

and

$$V=\begin{pmatrix} \frac{2007}{469} & \frac{-7911}{938} & \frac{4745}{469} & \frac{-3690}{469} & \frac{27}{7} & \frac{-1091}{938} & \frac{9}{67} \\ 0 & \frac{103761}{31436} & \frac{-142695}{31436} & \frac{55655}{15718} & \frac{-27135}{15718} & \frac{15201}{31436} & \frac{-1871}{31436} \\ \frac{156578}{267543} & 0 & \frac{650}{9909} & \frac{160910}{267543} & \frac{-3350}{9909} & \frac{971}{9909} & \frac{-3262}{267543} \\ \frac{-961}{24480} & \frac{4397}{8160} & 0 & \frac{1325}{2448} & \frac{-67}{1632} & \frac{-1}{2720} & \frac{7}{12240} \\ \frac{1}{245} & \frac{-27}{490} & \frac{27}{49} & 0 & \frac{27}{49} & \frac{-27}{490} & \frac{1}{245} \\ \frac{349}{18090} & \frac{-387}{4020} & \frac{45}{268} & \frac{1367}{3618} & 0 & \frac{751}{1340} & \frac{-1057}{36180} \\ \frac{10939}{134829} & \frac{-589986}{1348290} & \frac{31975}{29962} & \frac{-207670}{134829} & \frac{21775}{14981} & 0 & \frac{56329}{149810} \end{pmatrix}$$

Using a matlab program, the values of the following matrix of A, B, U and V are used to produce the absolute stability region of the seven step block method as shown in fig.1

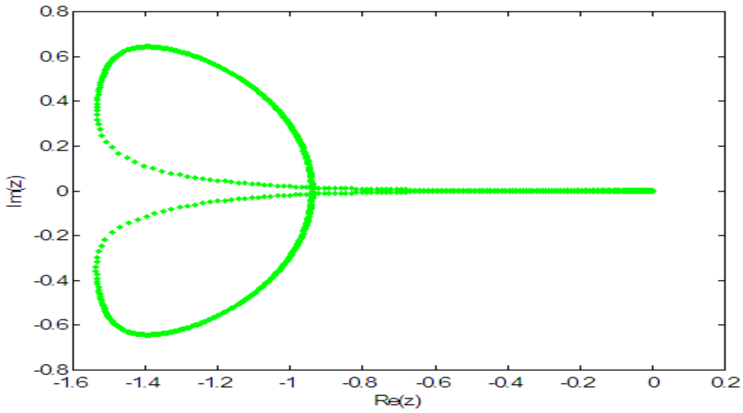


Fig.1 Absolute Stability Region of the Seven Step Block Method

## VI. NUMERICAL EXPERIMENT

Two numerical examples are solved to demonstrate the efficiency and accuracy of our block methods for values of $x$, $y(x)$ being the numerical solution at $x$. Our results from block method(8) is compared with results obtained by other scholars:

1. $y'' - 100y = 0,\ y(0) = 1,\ y'(0) = -10,\ 0 \le x \le 1.0,\quad h = 0.01$

   Theoretical solution: $y'(x) = e^{-10x}$

2. $y'' + y = 0,\ y(0) = 1,\ y'(0) = 1,\ 0 \le x \le 1.2,\quad h = 0.1$

   Theoretical solution: $y(x) = Cosx + Sinx$

Table I: Numerical solution of the methods for problem 1

| $N$ | $x$ | Theoretical Solution $y(x)$ | Our Proposed Seven Point Block Method (7) | J.O.Ehigie et-al [10] |
|---|---|---|---|---|
| 0 | 0 | 1.0000000000 | 1.0000000000 | 1.0000000000 |
| 1 | 0.1 | 0.9048374180 | 0.9048374036 | 0.9048333333 |
| 2 | 0.2 | 0.8187307531 | 0.8187307146 | 0.8187225417 |
| 3 | 0.3 | 0.7408182207 | 0.7408181574 | 0.7408057996 |
| 4 | 0.4 | 0.6703200460 | 0.6703199572 | 0.6703032900 |
| 5 | 0.5 | 0.6065306597 | 0.6065305446 | 0.6065094003 |
| 6 | 0.6 | 0.5488116361 | 0.5488114934 | 0.5487856598 |
| 7 | 0.7 | 0.4965853038 | 0.4965851322 | 0.4965543500 |
| 8 | 0.8 | 0.4493289641 | 0.4493287845 | 0.4492927225 |
| 9 | 0.9 | 0.4065696597 | 0.4065694656 | 0.4065277670 |
| 10 | 1.0 | 0.3678794412 | 0.3678792303 | 0.3678314776 |

Table II: Numerical solution of the methods for problem 2

| $N$ | $x$ | Theoretical Solution $y(x)$ | Our Proposed Seven Point Block Method (7) | J.O.Ehigie et-al [10] |
|---|---|---|---|---|
| 0 | 0 | 1.0000000000 | 1.0000000000 | 1.0000000000 |
| 1 | 0.1 | 1.0948375819 | 1.0948375542 | 1.0948333333 |
| 2 | 0.2 | 1.1787359086 | 1.1787358353 | 1.1787274551 |
| 3 | 0.3 | 1.2508566958 | 1.2508565766 | 1.2508441229 |
| 4 | 0.4 | 1.3104793363 | 1.3104791726 | 1.3104627711 |
| 5 | 0.5 | 1.3570081005 | 1.3570078938 | 1.3569877099 |
| 6 | 0.6 | 1.3899780883 | 1.3899778407 | 1.3899540774 |
| 7 | 0.7 | 1.4090598745 | 1.4090595886 | 1.4090324847 |
| 8 | 0.8 | 1.4140628003 | 1.4140624870 | 1.4140323067 |
| 9 | 0.9 | 1.4049368779 | 1.4049365218 | 1.4049035867 |
| 10 | 1.0 | 1.3817732907 | 1.3817728944 | 1.3817375360 |
| 11 | 1.1 | 1.3448034815 | 1.3448030490 | 1.3448425826 |
| 12 | 1.2 | 1.2943968404 | 1.2943963760 | 1.2943557871 |

| $N$ | $x$ | Our Proposed Seven Point Block Method (7) | J.O.Ehigie et-al [10] |
|---|---|---|---|
| 0 | 0 | 0.000E-00 | 0.000E-00 |
| 1 | 0.1 | 1.440E-08 | 4.080E-06 |
| 2 | 0.2 | 3.850E-08 | 8.210E-06 |
| 3 | 0.3 | 6.330E-08 | 1.240E-05 |
| 4 | 0.4 | 8.880E-08 | 1.680E-05 |
| 5 | 0.5 | 1.151E-07 | 2.130E-05 |
| 6 | 0.6 | 1.427E-07 | 2.600E-05 |
| 7 | 0.7 | 1.716E-07 | 3.100E-05 |
| 8 | 0.8 | 1.796E-07 | 3.620E-05 |
| 9 | 0.9 | 1.941E-07 | 4.190E-05 |
| 10 | 1.0 | 2.109E-07 | 4.800E-05 |

Table III: Table of Absolute Errors for problem 1

| N | $x$ | Our Proposed Seven Point Block Method (7) | J.O.Ehigie et-al [10] |
|---|---|---|---|
| 0 | 0 | 0.000E-00 | 0.000E-00 |
| 1 | 0.1 | 2.770E-08 | 4.250E-06 |
| 2 | 0.2 | 7.330E-08 | 8.450E-06 |
| 3 | 0.3 | 1.192E-07 | 1.257E-05 |
| 4 | 0.4 | 1.637E-07 | 1.657E-05 |
| 5 | 0.5 | 2.067E-07 | 2.039E-05 |
| 6 | 0.6 | 2.476E-07 | 2.401E-05 |
| 7 | 0.7 | 2.859E-07 | 2.739E-05 |
| 8 | 0.8 | 3.133E-07 | 3.049E-05 |
| 9 | 0.9 | 3.561E-07 | 3.329E-05 |
| 10 | 1.0 | 3.963E-07 | 3.576E-05 |
| 11 | 1.1 | 4.325E-07 | 3.765E-05 |
| 12 | 1.2 | 4.644E-07 | 3.987E-05 |

Table IV: Table of Absolute Errors for problem 2

## VII. CONCLUSION

We conclude that our new block method is of uniform order 6 and is suitable for direct solution of general second order ordinary differential equations. All the discrete equations derived in this work were obtained from a single continuous formulations and its combination with the main method form the block method which is self starting.

Analytical solutions were obtained in block form which tends to speed up computation process. Our method was applied to two numerical problems and results obtained converges to the theoretical solution.

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