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On Decomposition of gr* - closed set in Topological Spaces

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ABSTRACT: *The aim of this paper is to introduced and study the classes of gr* locally closed set and different notions of generalization of continuous functions namely gr*lc-continuity, gr*lc**-continuity and gr*lc**-continuity and their corresponding irresoluteness were studied. Furthermore, the notions of Z-sets, Z_r-sets and Z_{r*}-sets are used to obtain decompositions of gr*-continuity, gr*-open maps and contra gr*-continuity were investigated.*

KEYWORDS: *gr*-separated, gr*-dense, gr*-submaximal, gr*lc-continuity, gr*lc**-continuity gr*lc**-continuity, contra Z- continuity, contra Z_r- continuity and contra Z_{r*}- continuity*

I. INTRODUCTION:

The first step of locally closedness was done by Bourbaki [2]. He defined a set A to be locally closed if it is the intersection of an open and a closed set. In literature many general topologists introduced the studies of locally closed sets. Extensive research on locally closedness and generalizing locally closedness were done in recent years. Stone [10] used the term LC for a locally closed set. Ganster and Reilly used locally closed sets in [4] to define LC-continuity and LC-irresoluteness. Balachandran et al [1] introduced the concept of generalized locally closed sets. The aim of this paper is to introduce and study the classes of gr* locally closed set and different notions of generalization of continuous functions namely gr*lc-continuity, gr*lc**-continuity and gr*lc**-continuity and their corresponding irresoluteness were studied. Furthermore, the notions of Z-sets, Z_r-sets and Z_{r*}-sets are used to obtain decompositions of gr*-continuity, gr*-open maps and contra gr*-continuity were investigated.

II. PRELIMINARY NOTES

Throughout this paper (X,τ), (Y,σ) are topological spaces with no separation axioms assumed unless otherwise stated. Let A⊆X. The closure of A and the interior of A will be denoted by Cl(A) and Int(A) respectively.

Definition 2.1. A Subset S of a space (X,τ) is called
(i) locally closed (briefly lc)[4] if S=U∪F, where U is open and F is closed in (X,τ).
(ii) r-locally closed (briefly rlc) if S=U∪F, where U is r-open and F is r-closed in (X,τ).
(iii) generalized locally closed (briefly glc) [1] if S=U∪F, where U is g-open and F is g-closed in (X,τ).

Definition 2.2. [8] For any subset A of (X,τ), RCl(A)=∩{G: G⊇A, G is a regular closed subset of X}.

Definition 2.3. [5] A subset A of a topological space (X,τ) is called a generalized regular star closed set [briefly gr*-closed] if Rcl(A)⊆U whenever A⊆U and U is g-open subset of X.

Definition 2.4. [6] For a subset A of a space X, gr*-cl(A) = ∩{F: A⊆F, F is gr* closed in X} is called the gr*-closure of A.

Remark 2.5. [5] Every r-closed set in X is gr*-closed in X.

Remark 2.6. For a topological space (X,τ), the following statements hold:
(1) Every closed set is gr*-closed but not conversely [5].
(2) Every gr*-closed set is g-closed but not conversely [5].
(3) Every gr*-closed set is sg-closed but not conversely [5].
(4) A subset A of X is gr*-colsed if and only if gr*-cl(A)=A.
(5) A subset A of X is gr*-open if and only if gr*-int(A)=A.

Corollary 2.7. If A is a gr*-closed set and F is a closed set, then A∩F is a gr*-closed set.

Definition 2.8. A function f:(X,τ)→(Y,σ) is called

i) LC-continuous [4] if $f^{-1}(V)\in LC(X,\tau)$ for every $V\in\sigma$.
ii) GLC-continuous [1] if $f^{-1}(V)\in GLC(X,\tau)$ for every $V\in\sigma$.

Definition 2.9. A subset S of a space (X,τ) is called
(i) submaximal [3] if every dense subset is open.
(ii) g-submaximal [1] if every dense subset is g-open.
(iii) rg-submaximal [2] if every dense subset is rg-open.

Definition 2.10. A subset A of a space (X,τ) is called
(i) an α*-set [7] if int(A) = int(cl(int(A)).
(ii) an A-set [11] if $A = G\cap F$ where G is open and F is regular closed in X.
(iii) a t-set [12] if int (A) = int(cl(A)).
(iv) a C-set [9] if $A = G\cap F$ where G is open and F is a t-set in X.
(v) a C_r-set [9] if $A = G\cap F$ where G is rg-open and F is a t-set in X.
(vi) a C_{r*}-set [9] if $A = G\cap F$ where G is rg-open and F is an α*-set in X.

III. GR* LOCALLY CLOSED SET

Definition 3.1. A subset A of (X,τ) is said to be generalized regular star locally closed set (briefly gr^*lc) if $S=L\cap M$ where L is gr^*-open and M is gr^*-closed in (X,τ).

Definition 3.2. A subset A of (X,τ) is said to be gr^*lc^*set if there exists a gr^*-open set L and a closed set M of (X,τ) such that $S=L\cap M$.

Definition 3.3. A subset B of (X,τ) is said to be gr^*lc^{**}set if there exists an open set L and a gr^*-closed set M such that $B=L\cap M$.
The class of all gr^*lc (resp. gr^*lc^* & gr^*lc^{**}) sets in X is denoted by $GR^*LC(X)$.(resp. $GR^*LC^*(X)$ & $GR^*LC^{**}(X)$)
From the above definitions we have the following results.

Proposition 3.4.
i) Every locally closed set is gr^*lc.
ii) Every rlc-set is gr^*lc.
iii) Every gr^*lc^*-set is gr^*lc.
iv) Every gr^*lc^{**}-set is gr^*lc.
v) Every gr^*lc-set is glc.
vi) Every rlc-set is gr^*lc^*.
vii) Every rlc-set is gr^*lc^{**}.
viii) Every rlc^*-set is gr^*lc^{**}.
ix) Every rlc^{**}-set is gr^*lc^*.
x) Every rlc^{**}-set is gr^*lc.
xi) Every gr^*lc^*-set is glc.
xii) Every gr^*-closed set is gr^*lc.
However the converses of the above are not true as seen by the following examples

Example 3.5. Let X={a,b,c,d} with τ={ϕ,{c},{a,b},{a,b,c},X}. Then A={a} is gr^*lc-set but not locally closed.

Example 3.6. Let X={a,b,c,d} with τ={ϕ,{a},{b},{a,b},{a,b,c},X}. Then A={c} is gr^*lc-set but not rlc-set.

Example 3.7. In example 3.5, Let A={d} is gr^*lc-set but not gr^*lc^*-set.

Example 3.8. In example 3.6, Let A={c} is gr^*lc-set but not gr^*lc^{**}-set.

Example 3.9. Let Let X={a,b,c,d} with τ={ϕ,{a,d},{b,c},X}. Then A={a} is glc-set but not gr^*lc-set.

Example 3.10. In Example 3.5, Let A={b} is gr^*lc^* set but not rlc-set.

Example 3.11. In example 3.6, Let A={d} is gr^*lc^{**} set but not rlc-set.

Example 3.12. Let X={a,b,c,d} with τ={ϕ,{b},{c},{b,c},{b,c,d},X}. Then A={a,b} is gr^*lc^{**}-set but not rlc^*-set.

Example 3.13. In example 3.12, Let A={d} is gr^*lc^* set but not rlc^{**}-set.

Example 3.14. In example 3.12, Let A={b} is gr^*lc-set but not rlc^{**}-set.

Example 3.15. In example 3.6, Let A={d} is glc-set but not gr^*lc^*-set.

Example 3.16. Let X={a,b,c,d} with τ={ϕ,{a},{c},{a,c},{a,c,d},X}. Then A={a} is gr^*lc-set but not gr^*-closed set.

Remark 3.17. The concepts of gr^*lc^*set and gr^*lc^{**} sets are independent of each other as seen from the following example.

Example 3.18. In example 3.6, Let A={c} is gr^*lc^*-set but not gr^*lc^{**}-set and Let A={d} is gr^*lc^{**}-set but not gr^*lc^*-set.

Remark 3.19. The concepts of gr^*lc^*-set and rlc^*-set are independent of each other as seen form the following example.

Example 3.20. Let X={a,b,c,d} with τ={ϕ,{a},{c},{a,c},{a,c,d},X}. Then A={d} is gr^*lc^*-set but not rlc^*-set and let A={b} is rlc^*-set but not gr^*lc^*-set.

Remark 3.21. The concepts of gr^*lc^{**}-set and rlc^{**}-set are independent of each other as seen from the following example.

Example 3.22. In example 3.20, Let A={b} is gr^*lc^{**}-set but not rlc^{**}-set and let A={a,c} is rlc^{**}-set but not gr^*lc^{**}-set.

Remark 3.23. The concepts of locally closed set and gr^*lc^* are independent of each other as seen from the following example.

Example 3.24. In example 3.20, Let A={b,d} is locally closed set but not gr^*lc^*-set and let A={d} is gr^*lc^*-set but not locally closed set.

Remark 3.25. The concepts of rlc^{**} set and gr^* closed-set are independent of each other as seen from the following example.

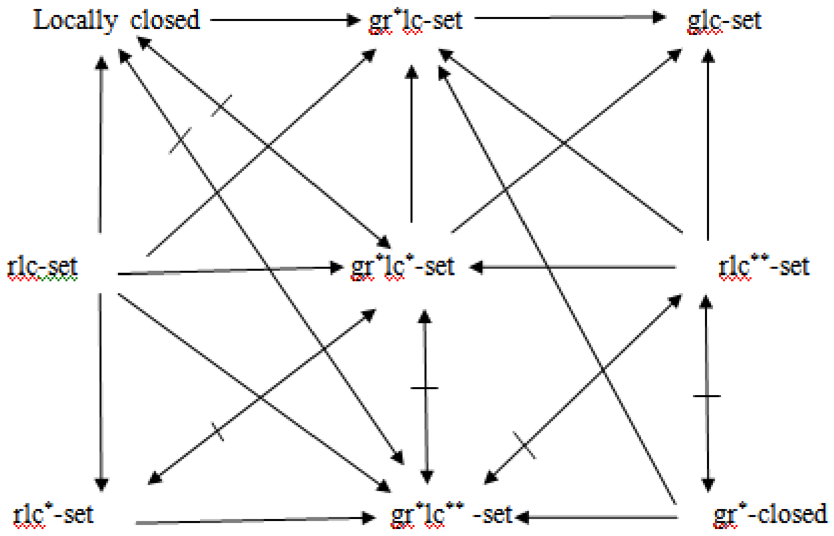
Example 3.26. In example 3.5, Let A={c} is rlc^{**} set but not gr^*closed set and let A={d} is gr^*-closed set but not rlc^{**}-set.

Remark 3.27. Union of two gr^*lc-sets are gr^*lc-sets.

Remark 3.28. Union of two gr^*lc^*-sets(resp. gr^*lc^{**}-set) need not be an gr^*lc^*-set(resp. gr^*lc^{**}-set) as seen from the following examples.

Example 3.29. In example 3.5, Then the sets {a} and {c,d} are gr^*lc^*-sets, but their union $\{a,c,d\} \notin gr^*lc^*(X)$.

Example 3.30. In example 3.6, then the sets {a} and {b} are gr^*lc^{**}-sets but their union $\{a,b\} \notin gr^*lc^{**}(X)$.
The above discussions are summarized in the following implications..



IV. GR*-DENSE SETS AND GR*-SUBMAXIMAL SPACES

Definition 4.1. A subset A of (X,τ) is called gr*-dense if gr*-cl(A)=X.

Example 4.2. Let X={a,b,c,d}with τ={ϕ,{b},{a,c},{a,b,c},X}. Then the set A={a,b,c} is gr*-dense in (X,τ). Recall that a subset A of a space (X,τ) is called dense if cl(A)=X.

Proposition 4.3. Every gr*-dense set is dense.
Let A be an gr*-dense set in (X,τ). Then gr*-cl(A)=X. Since gr*-cl(A)⊆rcl(A)⊆cl(A). we have cl(A)=X and so A is dense.
The converse of the above proposition need not be true as seen from the following example.

Example 4.4. Let X={a,b,c,d}with τ={ϕ,{b},{c,d},{b,c,d},X}. Then the set A={b,c} is a dense in (X,τ) but it is not gr*-dense in (X,τ).

Definition 4.5. A topological space (X,τ) is called gr*-submaximal if every dense subset in it is gr*-open in (X,τ).

Proposition 4.6. Every submaximal space is gr*-submaximal.

Proof. Let (X,τ) be a submaximal space and A be a dense subset of (X,τ). Then A is open. But every open set is gr*-open and so A is gr*-open. Therefore (X,τ) is gr*-submaximal.
The converse of the above proposition need not be true as seen from the following example.

Example 4.7. Let X = {a,b,c}with τ = {ϕ,X}. Then gr*O(X) = P(X). we have every dense subset is gr*-open and hence (X,τ) is gr*-submaximal. However, the set A={c} is dense in (X,τ), but it is not open in (X,τ). Therefore (X,τ) is not submaximal.

Proposition 4.8. Every gr*-submaximal space is g-submaximal.

Proof. Let (X,τ) be a gr*-submaximal space and A be a dense subset of (X,τ). Then A is gr*-open. But every gr*-open set is g-open and A is g-open. Therefore (X,τ) is g-submaximal.
The converse of the above proposition need not be true as seen from the following example.

Example 4.9. Let X = {a,b,c,d}with τ = {ϕ,{d},{a,b,c},X}. Then GO(X) = P(X) and gr*O(X)={ϕ,{d},{a,b,c},X}. we have every dense subset is g-open and hence (X,τ) is g-submaximal. However, the set A={a} is dense in (X,τ), but it is not gr*-open in (X,τ). Therefore (X,τ) is not gr*-submaximal.

Proposition 4.10. Every gr*-submaximal space is rg-submaximal.
The converse of the above proposition need not be true as seen from the following example.

Example 4.11. Let $X=\{a,b,c,d\}$with $\tau=\{\phi,\{a,d\},\{b,c\},X\}$. Then $RO(X)=P(X)$ and $gr^*O(X)=\{\phi,\{b,c\},\{a,d\},X\}$.Every dense subset is rg-open and hence (X,τ) is rg-submaximal. However the set $A=\{a\}$ is dense in (X,τ), but it is not gr^*-open in (X,τ). Therefore (X,τ) in not gr^*-submaximal.

Remark 4.12. From propositions 4.6, 4.8 and 4.10, we have the following diagram.

Submaximal $\longrightarrow$ gr^*-submaximal $\longrightarrow$ g-submaximal $\longrightarrow$ rg-maximal

Theorem 4.13. Assume that $GR^*C(X)$ is closed under finite intersections. For a subset A of (X,τ) the following statements are equivalent:

(1) $A\in GR^*LC(X)$,

(2) $A=S\cap gr^*\text{-}cl(A)$ for some gr*-open set S,

(3) $gr^*\text{-}cl(A)-A$ is gr*-closed,

(4) $A\cup(gr^*\text{-}cl(A))^c$ is gr*-open,

(5) $A\subseteq gr^*\text{-}int(A\cup(gr^*\text{-}cl(A))^c)$.

Proof. (1)⇒(2). Let $A\in GR^*LC(X)$. Then $A=S\cap G$ where S is gr*-open and G is gr*-closed. Since $A\subseteq G$, $gr^*\text{-}cl(A)\subseteq G$ and so $S\cap gr^*\text{-}cl(A)\subseteq A$. Also $A\subseteq S$ and $A\subseteq gr^*\text{-}cl(A)$ implies $A\subseteq S\cap gr^*\text{-}cl(A)$ and therefore $A=S\cap gr^*\text{-}cl(A)$.

(2)⇒(3). $A=S\cap gr^*\text{-}cl(A)$ implies $gr^*\text{-}cl(A)-A= gr^*\text{-}cl(A)\cap S^c$ which is gr*-closed since S^c is gr*-closed and $gr^*\text{-}cl(A)$ is gr*-closed.

(3)⇒(4). $A\cup(gr^*\text{-}cl(A))^c=(gr^*\text{-}cl(A)-A)^c$ and by assumption, $(gr^*\text{-}cl(A)-A)^c$ is gr*-open and so is $A\cup(gr^*\text{-}cl(A))^c$.

(4)⇒(5). By assumption, $A\cup(gr^*\text{-}cl(A))^c= gr^*\text{-}int(A\cup(gr^*\text{-}cl(A))^c)$ and hence $A\subseteq gr^*\text{-}int(A\cup(gr^*\text{-}cl(A))^c)$.

(5)⇒(1). By assumption and since $A\subseteq gr^*\text{-}cl(A)$, $A=gr^*\text{-}int(A\cup(gr^*\text{-}cl(A))^c)\cap gr^*\text{-}cl(A)$. Therefore, $A\in GR^*LC(X)$.

Theorem 4.14. For a subset A of (X,τ), the following statements are equivalent:

(1) $A\in GR^*LC^*(X)$,

(2) $A=S\cap cl(A)$ for some gr*-open set S,

(3) $cl(A)-A$ is gr*-closed,

(4) $A\cup(cl(A))^c$ is gr*-open.

Proof. (1)⇒(2). Let $A\in GR^*LC^*(X)$. There exist an gr*-open set S and a colsed set G such that $A=S\cap G$. Since $A\subseteq S$ and $A\subseteq cl(A)$, $A\subseteq S\cap cl(A)$. Also since $cl(A)\subseteq G$, $S\cap cl(A)\subseteq S\cap G=A$. Therefore $A=S\cap cl(A)$.

(2)⇒(1). Since S is gr*-open and cl(A) is a closed set, $A=S\cap cl(A)\in GR^*LC^*(X)$.

(2)⇒(3). Since $cl(A)-A=cl(A)\cap S^c$, $cl(A)-A$ is gr*-closed by Remark 2.6.

(3)⇒(2). Let $S=(cl(A)-A)^c$. Then by assumption S is gr*-open in (X,τ) and $A=S\cap cl(A)$.

(3)⇒(4). Let $G=cl(A)-A$. Then $G^c=A\cup(cl(A))^c$ and $A\cup(cl(A))^c$ is gr*-open.

(4)⇒(3). Let $S=A\cup(cl(A))^c$. Then S^c is gr*-closed and $S^c=cl(A)-A$ and so $cl(A)-A$ is gr*-closed.

Theorem 4.15. A space (X,τ) is gr*-submaximal if and only if $P(X)=GR^*LC^*(X)$.

Proof. Necessity. Let $A\in P(X)$ and let $V=A\cup(cl(A))^c$. This implies that $cl(V)=cl(A)\cup(cl(A))^c=X$. Hence $cl(V)=X$. Therefore V is a dense subset of X. Since (X,τ) is gr*-submaximal, V is gr*-open. Thus $A\cup(cl(A))^c$ is gr*-open and by theorem 4.14, we have $A\in GR^*LC^*(X)$.

Sufficiency. Let A be a dense subset of (X,τ). This implies $A\cup(cl(A))^c=A\cup X^c=A\cup\phi=A$. Now $A\in GR^*LC^*(X)$ implies that $A=A\cup(cl(A))^c$ is gr*-open by Theorem 4.14. Hence (X,τ) is gr*-submaximal.

Theorem 4.16. Let A be a subset of (X,τ). Then $A\in GR^*LC^{**}(X)$ if and only if $A=S\cap gr^*\text{-}cl(A)$ for some open set S.

Proof. Let $A\in GR^*LC^{**}(X)$. Then $A=S\cap G$ where S is open and G is gr*-closed. Since $A\subseteq G$, $gr^*\text{-}cl(A)\subseteq G$. We obtain $A=A\cap gr^*\text{-}cl(A)=S\cap G\cap gr^*\text{-}cl(A)=S\cap gr^*\text{-}cl(A)$.

Converse part is trivial.

Theorem 4.17. Let A be a subset of (X,τ). If $A\in GR^*LC^{**}(X)$, then $gr^*\text{-}cl(A)-A$ is gr*-closed and $A\cup(gr^*\text{-}cl(A))^c$ is gr*-open.

Proof. Let $A\in GR^*LC^{**}(X)$. Then by theorem 4.16, $A=S\cap gr^*\text{-}cl(A)$ for some open set S and $gr^*\text{-}cl(A)-A=gr^*\text{-}cl(A)\cap S^c$ is gr*-closed in (X,τ). If $G=gr^*\text{-}cl(A)-A$, then $G^c=A\cup(gr^*\text{-}cl(A))^c$ and G^c is gr*-open and so is $A\cup(gr^*\text{-}cl(A))^c$.

Proposition 4.18. Assume that $GR^*O(X)$ forms a topology. For subsets A and B in (X,τ), the following are true:

(1) If $A, B\in GR^*LC(X)$, then $A\cap B\in GR^*LC(X)$.

(2) If A, B∈GR*LC*(X), then A∩B∈GR*LC*(X).
(3) If A, B∈GR*LC**(X), then A∩B∈GR*LC**(X).
(4) If A∈GR*LC(X) and B is gr*-open (resp. gr*-closed), then A∩B∈GR*LC(X).
(5) If A∈GR*LC*(X) and B is gr*-open (resp. closed), then A∩B∈GR*LC*(X).
(6) If A∈GR*LC**(X) and B is gr*-closed (resp. open), then A∩B∈GR*LC**(X).
(7) If A∈GR*LC*(X) and B is gr*-closed, then A∩B∈GR*LC(X).
(8) If A∈GR*LC**(X) and B is gr*-open, then A∩B∈GR*LC(X).
(9) If A∈GR*LC**(X) and B∈GR*LC*(X), then A∩B∈GR*LC(X).
Proof. By Remark 2.5 and Remark 2.6, (1) to (8) hold.
(9). Let A=S∩G where S is open and G is gr*-closed and B=P∩Q where P is gr*-open and Q is closed. Then A∩B=(S∩P)∩(G∩Q) where S∩P is gr*-open and G∩Q is gr*-closed, by Remark 2.6. Therefore A∩B∈GR*LC(X).

Definition 4.19 . Let A and B be subsets of (X,τ). Then A and B are said to be gr*-separated if A∩gr*-cl(B)=ϕ and gr*-cl(A)∩B=ϕ.

Example 4.20. Let X={a,b,c}with τ={ϕ,{a},{c},{a,c},{b,c},X}. Let A={a} and B={b}. Then gr^*-cl(A)={a} and gr^*-cl(B)={b} and so the sets A and B are gr^*-separated.

Proposition 4.21. Assume that GR*O(X) forms a topology. For a topological space (X,τ), the following are true:
(1) Let A, B∈ GR*LC(X). If A and B are gr*-separated then A∪B∈ GR*LC(X).
(2) Let A, B∈ GR*LC*(X). If A and B are separated (i.e., A∩cl(B)=ϕ and cl(A)∩B=ϕ), then A∪B∈ GR*LC*(X).
(3) Let A, B∈ GR*LC**(X). If A and B are gr*-separated then A∪B∈ GR*LC**(X).
Proof. (1) Since A, B∈ GR*LC(X). by theorem 4.13, there exists gr*-open sets U and V of (X,τ) such that A=U∩ gr*-cl(A) and B=V∩ gr*-cl(B). Now G=U∩(X− gr*-cl(B)) and H=V∩(X− gr*-cl(A)) are gr*-open subsets of (X,τ). Since A∩ gr*-cl(B)=ϕ, A⊆(gr*-cl(B))^c. Now A=U∩ gr*-cl(A) becomes A∩(gr*-cl(B))^c=G∩ gr*-cl(A). Then A=G∩ gr*-cl(A). Similarly B=H∩ gr*-cl(B). Moreover G∩ gr*-cl(B)=ϕ and H∩ gr*-cl(A)=ϕ. Since G and H are gr*-open sets of (X,τ), G∪H is gr*-open. Therefore A∪B=(G∪H)∩ gr*-cl(A∪B) and hence A∪B∈ GR*LC(X).
(2) and (3) are similar to (1), using Theorems 4.13 and 4.14.

Lemma 4.22. If A is gr^*-closed in (X,τ) and B is gr^*-closed in (Y,σ), then A×B is gr^*-closed in (X×Y, τ×σ).

Theorem 4.23. Let (X,τ) and (Y,σ) be any two topological spaces. Then
i) If A∈GR*LC(X,τ) and B∈GR*LC(Y,σ), then A×B∈GR*LC(X×Y, τ×σ).
ii) If A∈GR*LC*(X,τ) and B∈GR*LC*(Y,σ), then A×B∈GR*LC*(X×Y, τ×σ).
iii) If A∈GR*LC**(X,τ) and B∈GR*LC**(Y,σ), then A×B∈GR*LC**(X×Y, τ×σ).
Proof. Let A∈GR*LC(X,τ) and B∈GR*LC(Y,σ). Then there exists gr^*-open sets V and V' of (X,τ) and (Y,σ) respectively and gr^*-closed sets W and W' of (X,τ) and (Y,σ) respectively such that A=V∩W and B=V'∩W'. Then A×B= (V∩W)×(V'∩W')=(V×V')∩(W×W') holds and hence A×B∈GR*LC(X×Y, τ×σ).
The proofs of (ii) and (iii) are similar to (i).

V. GR^*LC -CONTINUOUS AND GR^*LC-IRRESOLUTE FUNCTIONS

In this section, we define gr^*LC-continuous and gr^*LC-irresolute functions and obtain a pasting lemma for gr^*LC**-continuous functions and irresolute functions.

Definition 5.1. A function f:(X,τ)→(Y,σ) is called
i) gr^*LC-continuous if f^{-1}(V)∈ gr^*LC(X,τ) for every V∈σ.
ii) gr^*LC^*-continuous if f^{-1}(V)∈ gr^*LC^*(X,τ) for every V∈σ.
iii) gr^*LC^{**}-continuous if f^{-1}(V)∈ gr^*LC^{**}(X,τ) for every V∈σ.
iv) gr^*LC-irresolute if f^{-1}(V)∈gr^*LC(X,τ) for every V∈gr^*LC(Y,σ).
v) gr^*LC^*-irresolute if f^{-1}(V)∈gr^*LC^*(X,τ) for every V∈gr^*LC*(Y,σ).
vi) gr^*LC^{**}-irresolute if f^{-1}(V)∈gr^*LC^{**}(X,τ) for every V∈gr^*LC**(Y,σ).

Proposition 5.2. If f:(X,τ)→(Y,σ) is gr^*LC-irresolute, then it is gr^*LC-continuous.
Proof. Let V be open in Y. Then V∈gr^*LC(Y,σ). By assumption, f^{-1}(V)∈gr^*LC(X,τ). Hence f is gr^*LC-continuous.

Proposition 5.3. Let f:(X,τ)→(Y,σ) be a function, then
1. If f is LC-continuous, then f is gr^*LC-continuous.
2. If f is gr^*LC^*-continuous, then f is gr^*LC-continuous.
3. If f is gr^*LC^{**}-continuous, then f is gr^*LC-continuous.
4. If f is gr^*LC-continuous, then f is glc-continuous.

Remark 5.4. The converses of the above are not true may be seen by the following examples.

Example 5.5. 1. Let X=Y={a,b,c,d}, τ={ϕ,{c},{a,b},{a,b,c},X} and σ={ϕ,{a},{a,d},X}. Let f: X→Y be the identity map. Then f is gr^*LC-continuous but not LC-continuous. Since for the open set {a,d}, f^{-1}{a,d}= {a,d} is not locally closed in X.
2. Let X=Y={a,b,c,d}, τ={ϕ,{c},{a,b},{a,b,c},X} and σ={ϕ,{d},{a,d},{a,c,d},X}. Let f: X→Y be the identity map. Then f is gr^*LC-continuous but not gr^*LC^*-continuous. Since for the open set {a,c,d}, f^{-1}{a,c,d} = {a,c,d}is not gr^*LC^*-closed in X.
3. Let X=Y={a,b,c,d}, τ={ϕ,{c},{a,b},{a,b,c},X} and σ={ϕ,{a},{a,c},X}. Let f: X→Y be the identity map. Then f is gr^*LC-continuous but not gr^*LC^{**}-continuous. Since for the open set {a,c}, f^{-1}{a,c} = {a,c} is not gr^*LC^{**}-closed in X.
4. Let X=Y={a,b,c,d}, τ={ϕ,{a,d},{b,c},X} and σ={ϕ,{a},{a,b},{a,b,d},X}. Let f: X→Y be the identity map. Then f is glc-continuous but not gr^*LC-continuous. Since for the open set {a,b,d}, f^{-1}{a,b,d} = {a,b,d} is not gr^*LC-set in X.

We recall the definition of the combination of two funtions: Let X=A∪B and f:A→Y and h:B→Y be two functions. We say that f and h are compatible if f/A∩B=h/A∩B. If f:A→Y and h:B→Y are compatible, then the functions (f∆h)(X)=h(X) for every x∈B is called the combination of f and h.
Pasting lemma for gr^*LC^{**}-continuous (resp. gr^*LC^{**}-irresolute) functions.

Theorem 5.6. Let X=A∪B, where A and B are gr^*-closed and regular open subsets of (X,τ) and f:(A, τ/B)→(Y,σ) and h:(B,τ/B)→(Y,σ) be compatible functions.
a) If f and h are gr^*LC^{**}-continuous, then (f∆h):X→Y is gr^*LC^{**}-continuous.
b) If f and h are gr^*LC^{**}-irresolute, then (f∆h):X→Y is gr^*LC^{**}-irresolute.

Next we have the theorem concerning the composition of functions.

Theorem 5.7. Let f:(X,τ)→(Y,σ) and g:(Y,σ)→(Z,η) be two functions, then
a) g∘f is gr^*LC-irresolute if f and g are gr^*LC-irresolute.
b) g∘f is gr^*LC^*-irresolute if f and g are gr^*LC^*-irresolute.
c) g∘f is gr^*LC^{**}-irresolute if f and g are gr^*LC^{**}-irresolute.
d) g∘f is gr^*LC-continuous if f is gr^*LC-irresolute and g is gr^*LC-continuous.
e) g∘f is gr^*LC^*-continuous if f is gr^*LC^*-continuous and g is continuous.
f) g∘f is gr^*LC-continuous if f is gr^*LC-continuous and g is continuous.
g) g∘f is gr^*LC^*-continuous if f is gr^*LC^*-irresolute and g is gr^*LC^*-continuous.
h) g∘f is gr^*LC^{**}-continuous if f is gr^*LC^{**}-irresolute and g is gr^*LC^{**}-continuous.

VI. DECOMPOSITION OF GR^*-CLOSED SETS

In this section, we introduce the notions of Z-sets, Z_r-sets and Z_{r*}-sets to obtain decompostions of gr^*-closed sets.

Definition 6.1. A subset S of (X,τ) is called a
1) Z-set if S=L∩M where L is gr^*-open and M is a t-set.
2) Z_r-set if S=L∩M where L is gr^*-open and M is a α^*-set.
3) Z_{r*}-set if S=L∩M where L is gr^*-open and M is a A-set.

Proposition 6.2.
1. Every Z-set is a C-set.
2. Every Z-set is a C_r-set.
3. Every Z-set is a C_{r*}-set.
4. Every Z_r-set is a C-set.
5. Every Z_r-set is a C_r-set.
6. Every Z_r-set is a C_{r*}-set.
7. Every Z_{r*}-set is a C-set.
8. Every Z_{r*}-set is a C_r-set.

9. Every Z_{r*}-set is a C_{r*}-set.
10. Every Z_{r*}-set is a Z-set.
11. Every Z_{r*}-set is a Z_r-set.
12. Every A-set is a Z-set.
13. Every A-set is a Z_r-set.
14. Every Z-set is a Z_r-set.
15. Every t-set is a Z_r-set.

Remark 6.3. The converses of the above are not true may be seen by the following examples.

Example 6.4. Let X={a,b,c}with τ={ϕ,{a},X}. Let A={a,b}. Then A is a C-set, C_r-set and C_{r*}-set but A is not a Z-set.

Example 6.5. Let X={a,b,c}with τ={ϕ,{b},{a,b},X}. Let A={b}. Then A is a C-set, C_r-set and C_{r*}-set but A is not a Z_r-set.

Example 6.6. Let X={a,b,c}with τ={ϕ,{b,c},X}. Let A={a}. Then A is a C-set, C_r-set and C_{r*}-set but A is not a Z_{r*}-set.

Example 6.7. Let X={a,b,c}with τ={ϕ,{a},{c},{a,c},X}. Let A={b}. Then A is a Z-set, Z_r-set but A is not a Z_{r*}-set.

Example 6.8. Let X={a,b,c}with τ={ϕ,{a},{b},{a,b},{b,c},X}. Let A={c}. Then A is a Z-set, Z_r-set but A is not a A-set.

Example 6.9. In example 6.6, Let A={a,b}. Then A is Z_r-set but A is not a Z-set.

Example 6.10. In example 6.7, Let A={a}. Then A is Z_r-set but A is not a t-set.

Remark 6.11. The concepts of Z-set and α^*-set are independent of each other as seen from the following example.

Example 6.12. Let X={a,b,c}with τ={ϕ,{a,c},X}. Let A={a,c} is a Z-set but not a α^*-set and let A={a,b} is a α^*-set but not a Z-set.

Remark 6.13. The concepts of Z_{r*}-set and t-set are independent of each other as seen from the following example.

Example 6.14. In example 6.5. Let A={b} is a Z_{r*}-set but not a t-set and let A={a} is a t-set but not a Z_{r*}-set.

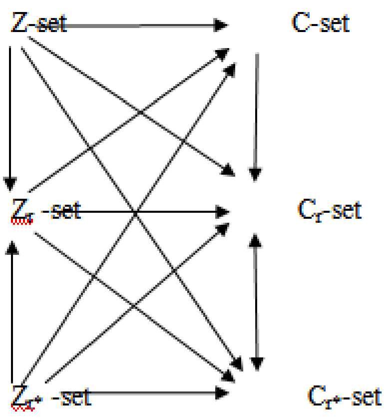
Remark 6.15. The concepts of Z_{r*}-set and α^*-set are independent of each other as seen from the following example.

Example 6.16. In Example 6.12, Let A={a.c} is a Z_{r*}-set but not a α^*-set and let A={b} is a α^*-set but not a Z_{r*}-set.

Proposition 6.17. If S is a gr^*-open set, then
i) S is a Z-set.
ii) S is a Z_r-set.
iii) S is a Z_{r*}-set.

Remark 6.18. The converse of the above proposition are not true may be seen by the following examples.

Example 6.19. In example 6.7. Let A={a,b} is a Z-set, Z_r-set and Z_{r*}-set but not a gr^*-open set.
The above discussions are summarized in the following diagram



Proposition 6.20. Let A and B are Z-sets in X. Then A∩B is a Z-set in X.

Proof. Since A, B are Z-sets. Let $A=L_1 \cap M_1$, $B=L_2 \cap M_2$ where L_1,L_2 are gr^*-open sets and M_1 , M_2 are t-sets. Since intersection of two gr^*-open sets is gr*-open sets and intersection of t-sets is t-set it follows that A∩B is a Z-set in X.

Remark 6.21. a) The union of two Z_r-sets need not be a Z_r-set.
b) Complement of a Z_r-sets need not be a Z_r-set.

Example 6.22. In example 6.4.
a) A={a} and B={b} are Z_r-sets but A∪B={a,b} is not a Z_r-set.
b)X−{b}={a,c} is not a Z_r-set.

Proposition 6.23. Let A and B be Z_r-sets in X. Then A∩B is also a Z_r-sets.

Remark 6.24. The union of two Z_{r^*}-sets is also a Z_{r^*}-sets and the complement of a Z_{r^*}-set need not be a Z_{r^*}-sets follows from the following example.

Example 6.25. In example 6.6. Let A={b} and B={c} are Z_{r^*}-sets and A∪B={b,c} is also a Z_{r^*}-sets. Let X−{b}={a,b} is not a Z_{r^*}-set.

VII. DECOMPOSITION OF GR*-CONTINUITY

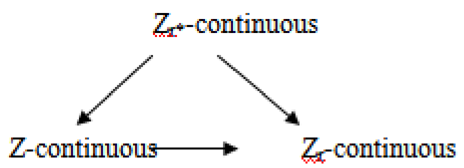
Definition 7.1. A function f: X→Y is said to be
i) Z-continuous if $f^{-1}(V)$ is a Z-set for every open set V in Y.
ii) Z_r-continuous if $f^{-1}(V)$ is a Z_r-set for every open set V in Y.
iii) Z_{r^*}-continuous if $f^{-1}(V)$ is a Z_{r^*}-set for every open set V in Y.

Proposition 7.2.
i) Every Z_{r^*}-continuous function is Z-continuous.
ii) Every Z_{r^*}-continuous function is Z_r-continuous.
iii) Every Z-continuous function is Z_r-continuous.

Proof. Follows from 6.2 and 7.1

Remark 7.3. Converses of the above proposition are not true may be seen by the following example.

Example 7.4.
The above discussions are summarized in the following implications..



Defintion 7.5. A map f: X→Y is said to be
i) Z-open if f(U) is a Z-set in Y for each open set V in X.
ii) Z_r-open if f(U) is a Z_r-set in Y for each open set V in X.
iii) Z_{r*}-open if f(U) is a Z_{r*}-set in Y for each open set V in X.

Definition 7.6. A map f: X→Y is said to be
i) contra Z-continuous if $f^{-1}(V)$ is a Z-set for every closed set V in Y.
ii) contra Z_r-continuous if $f^{-1}(V)$ is a Z_r-set for every closed set V in Y.
i) contra Z_{r*}-continuous if $f^{-1}(V)$ is a Z_{r*}-set for every closed set V in Y.

Theorem 7.7. A subset A of X is
i) gr^*-open if only if it is both g^*-open and a Z-set in X.
ii) gr^*-open if only if it is both g^*-open and a Z_r -set in X.
iii) gr^*-open if only if it is both rg-open and a Z_{r*}-set in X.

Proof. Necessity: obvious
Sufficiency: Assume that A is both g^*-open and a Z-set in X. By assumption, A is a Z-set in X implies A=L∩M, where L is gr^*-open and M is a t-set. Let F be a g-closed such that F⊂A, since A is gr^*-open, F⊂A implies F⊂ rint(A)⊂ int(A). Then A is gr^*-open and F⊂A⊂L implies F⊂int(A). Hence F⊂int(L)∩int(M)=int(L∩M)=int(A). Hence A is gr^*-open.
ii)Necessity: obvious
sufficiency: Assume that A is both g^*-open and a Z_r-set in X. By assumption, A is a Z_r-set in X implies A=L∩M, where L is gr^*-open and M is a α^*-set. Let F be a g-closed such that F⊂A, since A is gr^*-open, F⊂A implies F⊂ rint(A)⊂ int(A). Then A is gr^*-open and F⊂A⊂L implies F⊂int(A). Hence F⊂int(L)∩int(M)=int(L∩M)=int(A). Hence A is gr^*-open.
iii)Necessity: obvious
sufficiency: Assume that A is both g^*-open and a Z_{r*}-set in X. By assumption, A is a Z_{r*}-set in X implies A=L∩M, where L is gr^*-open and M is a A-set. Let F be a r-closed such that F⊂A, since A is gr^*-open, F⊂A implies F⊂ rint(A)⊂ int(A). Then A is gr^*-open and F⊂A⊂L implies F⊂int(A). Hence F⊂int(L)∩int(M)=int(L∩M)=int(A). Hence A is gr^*-open.

Theorem 7.8. A mapping f: X→Y is
i) gr^*-continuous if and only if it is both g^*-continuous and Z-continuous.
ii) gr^*-continuous if and only if it is both g^*-continuous and Z_r-continuous.
iii) gr^*-continuous if and only if it is both rg-continuous and Z_{r*}-continuous.

Proof. Follows from theorem 7.7.

Theorem 6.35. A map f: X→Y is
i) gr^*-open if and only if it is both g^*-open and Z-open.
ii) gr^*-open if and only if it is both g^*-open and Z_r-open.
iii) gr^*-open if and only if it is both rg-open and Z_{r*}-open.

Proof. Follows from theorem 7.7.

Theorem 6.36. A mapping f: X→Y is
i) contra gr^*-open if and only if f is both contra g^*-continuous and contra Z-continuous.
ii) contra gr^*-open if and only if f is both contra g^*-continuous and contra Z_r-continuous.
iii) contra gr^*-open if and only if f is both contra rg-continuous and contra Z_{r*}-continuous.

Proof. Follows from theorem 7.7.

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