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Stochastic Analysis of Three Levels of Manpower System with Recruitment and Departure Affecting Business

K.Hari Kumar[1], R.Ramanarayanan[2],P.Sekar[3]
[1.] *SRM University, Kattankulathur,Tamilnadu,Chennai, India*
[2.] *VelTech Dr. RR &Dr. SR Technical University, Chennai, India.*
[3.] *C.Kandaswami Naidu College for Men,Chennai,India*

ABSTRACT: *In this paper we consider a business organization in fluctuating condition of availability of Manpower, Business, Recruitment Phase and Departure Phase with a special emphasis given to a new and prevailing idea of frequent changes taking place in Manpower. The different states have been discussed under the assumption that changes from availability to shortage and shortage to availability occur in exponential times with different parameters. An expression for rate of crisis under steady state (C_∞) is derived and steady state cost has also been worked by assuming different costs for the parameters under different conditions.*

KEYWORDS: *Manpower Planning, Crisis State, Steady State Probabilities and Steady state cost.*

I. INTRODUCTION

Nowadays we find that labour has become a buyer market as well as seller market. Any company normally runs on commercial basis wishes to keep only the optimum level of any resources needed to meet company's requirement at any time during the course of the business and manpower is not an exception. This is spelt in the sense that a company does not want to keep manpower more than what is required. Hence, recruitment is done when the business is busy and shed manpower when the business is lean.The worker have the option to switch over to other jobs because of better working condition, better emolument, proximity to their living place or other reasons. Under such situations the company may be there but manpower may not be available. If skilled labourers and technically qualified persons leave the business, the seriousness is much felt and the company has to hire paying heavy price or pay overtime to employees.

Approach to manpower problems have been treated in very many different ways as early as 1947 by Vajda [8] and others. Models in manpower planning has been discussed in depth in Bartholomew [1], Grinold and Marshal [2] and Vajda [8]. The method to compute wastages (Resignation, dismissal and death) and promotion intensities which produce the proportions corresponding to some desire planning proposals have been studied by Lesson [4]. Markov models are designed for wastages and promotion in manpower system by Vassilou [9], V. Subramaniam [8] in his thesis has made an attempt to provide optimal policy for recruitment training, promotion, and wastages in manpower planning models with special provisions such as time bound promotions, cost of training and voluntary retirement scheme. An application of Markov chains in a manpower system with efficiency and seniority and Stochastic structures of graded size in manpower planning systems one may refer to Setlhare [7]. For a system involving manpower, money and machine one may refer to C. Mohan and R. Ramanarayanan [6]. For the study of Semi Markov Models for Manpower planning one may refer to the paper by Sally Meclean [5]. Rao and Talwalker (1990) introduces the concept of setting the clock back to zero property. Stochastic Analysis of Manpower levels affecting business with varying Recruitment rate by K.Hari Kumar, P.Sekar and R.Ramanarayanan [3].In this model we consider four characteristics namely manpower, business, recruitment phase and departure phase and derive a formula for steady state rate of crisis and the steady state probabilities. The different states have been discussed under the assumption that changes from availability to shortage and shortage to availability occur in exponential times with different parameters. An expression for the rate of crisis under steady state (C_∞) is derived and steady state cost has also been worked by assuming different costs for the parameters under different conditions.

II. ASSUMPTIONS

1. There are two levels of business namely.
   - (i) Business is fully available
   - (ii) Business is lean (or) nil.
2. There are three levels of Manpower, Namely
   - (i) Manpower is full and it indicates as 1.

(ii) Manpower is 50% available and it indicates 0.5.

(iii) Manpower is nil and it is indicates 0.

3. The State of the system is given by four Co-ordinate System as follows.
S = [{(0, 0, K, 0) ; K = 1 or 2} ∪ { (0.5, J, K, L), J = 0 or 1, K = 1 or 2 and L = 1 or 2} ∪ { (1, J, 0, L); J = 0 or 1 and L = 1 or 2}]

Here the First Co-ordinate indicates the Manpower level, the Second Co-ordinate indicates business level, the Third Co-ordinate indicates the Phase of the recruitment and the Fourth Co-ordinate indicates the Departure phase. When the Manpower is full there is no recruitment and the recruitment phase is 0. When the Manpower is nil there is no departure and the departure phase is 0. When there is no Manpower the business is Lost and its level is nil. i e. 0.

4. The departure time distribution of Manpower is exponential with parameter $\lambda_{.5,\ J,\ K,1}$ when the Manpower level is 50% for J = 0 or 1, K = 1 or 2 in departure phase 1. The departure time distribution of Manpower is exponential with parameter $\lambda_{1,\ J,\ 0,\ 1}$ when the Manpower level is full, for J = 0 or 1 in departure phase 1. The parameter $\lambda_{.5,J,\ K,\ 1}$ changes to $\lambda_{.5,\ J,\ K,\ 2}$. When the departure phase 1 changes to phase 2 in an exponential time with parameter β..
The Parameter $\lambda_{1,\ J,\ 0,\ 1}$ change to $\lambda_{1,\ J,\ 0,\ 2}$ when the departure phase 1 changes to phase 2 in an exponential time with parameter β. Phase 2 changes to Phase 1, when departure occurs with Manpower level is full and it changes to Phase 0, when departure occurs with 50% Manpower level
5. The Manpower recruitment time distribution is exponential with $\mu_{0,\ 0,\ 1,\ 0}$ where there is nil Manpower (and lean parameter business), recruitment phase is 1. The parameter $\mu_{0,\ 0,\ 1,\ 0}$ changes to $\mu_{0,\ 0,\ 2,\ 0}$,when the recruitment phase 1 changes to phase 2 in an exponential time with parameter α. The Manpower recruitment time distribution is exponential with parameter $\mu_{.5,\ J,\ 1,\ L}$, when the Manpower level is 50%, J = 0 or 1, L = 1 or 2 in recruitment phase 1. The Parameter $\mu_{.5,\ J,\ 1,\ L}$ changes to $\mu_{.5,\ J,\ 2,\ L}$, when the recruitment phase 1 changes to phase 2 in an exponential time with parameter α. Phase 2 changes to Phase 1 when recruitment occurs from nil Manpower level and it changes to Phase 0 when the recruitment occurs with 50% Manpower level.
6. Whenever recruitment occurs,50% of Manpower is filled up. Whenever departure occurs 50% of Manpower departs.
7. Whenever recruitment (departure) occurs in recruitment phase.

III. SYSTEM ANALYSIS

a. The Stochastic Process $X(t)$ describing the state of the systems is a continuous time Markov Chain with 14 points state space as given below in the order of Manpower, Business, Recruitment Phase and Departure Phase.

S = {(0, 0, 1, 0) (0, 0, 2, 0) (0.5, 0, 1, 1) (0.5, 0, 1, 2) (0.5, 0, 2, 1) (0.5, 0, 2, 2) (0.5, 1, 1, 1) (0.5, 1, 1, 2) (0.5, 1, 2, 1) (0.5, 1, 2, 2) (1, 0, 0, 1) (1, 0, 0, 2) (1, 1, 0, 1) (1, 1, 0, 2) }

The Infinitesimal generator of the continuous time Markov Chain of the state space is given below which is a matrix of order 14.

Here

$Q =$

$M/B/R/D$	(0,0,1,0)	(0,0,2,0)	(.5,0,1,1)	(.5,0,1,2)	(.5,0,2,1)	(.5,0,2,2)	(.5,1,1,1)	(.5,1,1,2)	(.5,1,2,1)	(.5,1,2,2)	(1,0,0,1)	(1,0,0,2)	(1,1,0,1)	(1,1,0,2)
(0,0,1,0)	$-\varepsilon_1$	α	$\mu_{0,0,1,0}$	0	0	0	0	0	0	0	0	0	0	0
(0,0,2,0)	0	$-\varepsilon_2$	$\mu_{0,0,2,0}$	0	0	0	0	0	0	0	0	0	0	0
(.5,0,1,1)	$\lambda_{.5,0,1,1}$	0	$-\varepsilon_3$	β	α	0	b	0	0	0	$\mu_{.5,0,1,1}$	0	0	0
(.5,0,1,2)	$\lambda_{.5,0,1,2}$	0	0	$-\varepsilon_4$	0	α	0	b	0	0	$\mu_{.5,0,1,2}$	0	0	0
(.5,0,2,1)	$\lambda_{.5,0,2,1}$	0	0	0	$-\varepsilon_5$	β	0	0	b	0	$\mu_{.5,0,2,1}$	0	0	0
(.5,0,2,2)	$\lambda_{.5,0,2,2}$	0	0	0	0	$-\varepsilon_6$	0	0	0	b	$\mu_{.5,0,2,2}$	0	0	0
(.5,1,1,1)	$\lambda_{.5,1,1,1}$	0	a	0	0	0	$-\varepsilon_7$	b	α	0	0	0	$\mu_{.5,1,1,1}$	0
(.5,1,1,2)	$\lambda_{.5,1,1,2}$	0	0	a	0	0	0	$-\varepsilon_8$	0	α	0	0	$\mu_{.5,1,1,2}$	0
(.5,1,2,1)	$\lambda_{.5,1,2,1}$	0	0	0	a	0	0	0	$-\varepsilon_9$	β	0	0	$\mu_{.5,1,2,1}$	0
(.5,1,2,2)	$\lambda_{.5,1,2,2}$	0	0	0	0	a	0	0	0	$-\varepsilon_{10}$	0	0	$\mu_{.5,1,2,2}$	0
(1,0,0,1)	0	0	$\lambda_{1,0,0,1}$	0	0	0	0	0	0	0	$-\varepsilon_{11}$	β	b	0
(1,0,0,2)	0	0	$\lambda_{1,0,0,2}$	0	0	0	0	0	0	0	0	$-\varepsilon_{12}$	0	b
(1,1,0,1)	0	0	0	0	0	0	$\lambda_{1,1,0,1}$	0	0	0	a	0	$-\varepsilon_{13}$	β
(1,10,2)	0	0	0	0	0	0	$\lambda_{1,1,0,2}$	0	0	0	0	a	0	$-\varepsilon_{14}$

$\varepsilon_1 = \left(\mu_{0,0,1,1} + \alpha\right)$ $\varepsilon_2 = \left(\mu_{0,0,2,0}\right)$

$\varepsilon_3 = \left(\mu_{.5,0,1,1} + \lambda_{.5,0,1,1} + \alpha + \beta + b\right)$ $\varepsilon_4 = \left(\mu_{.5,0,1,2} + \lambda_{.5,0,1,2} + \alpha + b\right)$

$\varepsilon_5 = \left(\mu_{.5,0,2,1} + \lambda_{5,0,2,1} + \beta + b\right)$ $\varepsilon_6 = \left(\mu_{.5,0,2,2} + \lambda_{5,0,2,2} + b\right)$

$\varepsilon_7 = \left(\mu_{.5,1,1,1} + \lambda_{5,1,1,1} + \alpha + \beta + a\right)$ $\varepsilon_8 = \left(\mu_{.5,1,1,2} + \lambda_{5,1,1,2} + \alpha + a\right)$

$\varepsilon_9 = \left(\mu_{.5,1,2,1} + \lambda_{5,1,2,1} + \beta + a\right)$ $\varepsilon_{10} = \left(\mu_{.5,1,2,2} + \lambda_{5,1,2,2} + a\right)$

$\varepsilon_{11} = \left(\lambda_{1,0,0,1} + \beta + b\right)$ $\varepsilon_{12} = \left(\lambda_{1,0,0,2} + b\right)$

$\varepsilon_{13} = \left(\lambda_{1,1,0,1} + a + \beta\right)$ $\varepsilon_{14} = \left(\lambda_{1,1,0,2} + a\right)$

The matrix Q of Order 14 is partitioned as

$$Q = \begin{pmatrix} Q' & \underline{c} \\ \underline{r} & -\varepsilon_{14} \end{pmatrix}$$

Where Q' is the Submatrix of Q without considering the last row and last column of Q

Let $\underline{\pi} = \left(\underline{\pi}', \pi_{14}\right)$ be the steady state probability vector such $\underline{\pi}$ Q=0 and $\underline{\pi}\ \underline{e} = 1$ .....(1)

Where $\underline{\pi} = ($ $\pi_{0,0,1,0}, \pi_{0,0,2,0}, \pi_{.5,0,1,1}, \pi_{.5,0,1,2}, \pi_{.5,0,2,1}, \pi_{.5,0,2,2}, \pi_{.5,1,1,1}, \pi_{.5,1,1,2}, \pi_{.5,1,2,1}$ $\pi_{.5,1,2,2}, \pi_{1,0,0,1}, \pi_{1,0,0,2}, \pi_{1,1,0,1}, \pi_{1,1,0,2}$) and $\underline{e} = (1,1,1,1,...,1)^t$ are vectors of order 1×14 and 14×1 respectively.

Also we have

$$\underline{\pi}'\ Q' + \pi_{14}\ \underline{r} = 0 \quad ......(2)$$

and

$$\underline{\pi}'\underline{c} - \pi_{14}\varepsilon_{14} = 0 \quad ....(3)$$

Using equation (1),(2) and (3), we get

$$\underline{\pi}' = \frac{\underline{r}\left(-Q'\right)^{-1}}{1 + \underline{r}\left(-Q'\right)^{-1}\underline{e}} \qquad ....(4)$$

The Critical States are given by

$$C = \left\{(.5,1,1,1),(.5,1,1,2),(.5,1,2,1),(.5,1,2,2)\right\}$$

The rate of crisis in the steady state is

$$C_\infty = \lambda_{.5,1,1,1}\pi_{.5,1,1,1} + \lambda_{.5,1,1,2}\pi_{.5,1,1,2} + \lambda_{.5,1,2,1}\pi_{.5,1,2,1} + \lambda_{.5,1,2,2}\pi_{.5,1,2,2} \quad ....(5)$$

Numerical Illustration

We assume that

$\lambda_{.5,0,1,1} = 3,\ \lambda_{.5,0,1,2} = 4,\ \lambda_{.5,0,2,1} = 5,\ \lambda_{.5,0,2,2} = 6,\ \lambda_{.5,1,1,1} = 7,\ \lambda_{.5,1,1,2} = 8$

$\lambda_{.5,1,2,1} = 9,\ \lambda_{.5,1,2,2} = 10,\ \lambda_{1,0,0,1} = 11,\ \lambda_{1,0,0,2} = 12,\ \lambda_{1,1,0,1} = 13,\ \lambda_{1,1,0,2} = 14$

$\mu_{0,0,1,0} = 1,\ \mu_{0,0,2,0} = 2,\ \mu_{.5,0,1,1} = 3,\ \mu_{.5,0,1,2} = 4,\ \mu_{.5,0,2,1} = 5,\ \mu_{.5,0,2,2} = 6$

$\mu_{.5,1,1,1} = 7,\ \mu_{.5,1,1,2} = 8,\ \mu_{.5,1,2,1} = 9,\ \mu_{.5,1,2,2} = 10$

α=1, β=2, a=1 and b=2

Using the above values in equation (4), we get the steady state probability vector

$\underline{\pi} = ($ 0.4684, 0.2342, 0.1356, 0.0252, 0.0098, 0.0032, 0.0384, 0.0070, 0.0027
0.0009, 0.0403, 0.0060, 0.0275, 0.00448)

From (5), we get the rate of crisis

$$C_{\infty} = 0.3587$$

Steady State Cost
Numerical Illustration 1

The steady state cost of the system in different situations and the cost with respect to the coordinates (i,j,k,l) in the i-th environment is

$$C_{ijkl} = \pi_{ijkl}\left[C_B^{ij} + C_R^{ik} + C_D^{il}\right] \quad ....(6)$$

Here C_B^{ij} Stands for the cost of Business with respect to Manpower at the states of i=0,1/2,1 and j=1,2

C_R^{ik} Stands for the cost of Recruitment Phase with respect to Manpower at the states of i=0,1/2,1 and k=1,2

C_D^{il} Stands for the cost of Departure Phase with respect to Manpower at the states of i=0,1/2,1 and l=1,2

We assume the cost and arrive the expected total cost

$$C_B^{0,0} = 0,\ C_B^{0.5,0} = 5,\ C_B^{0.5,1} = 3,\ C_B^{1,0} = 8,\ C_B^{1,1} = 2$$

$$C_R^{0,1} = 6,\ C_R^{0,2} = 9,\ C_R^{0.5,1} = 5,\ C_R^{0.5,2} = 10,\ C_R^{1,0} = 2$$

$$C_D^{0,0} = 0,\ C_D^{0.5,2} = 12,\ C_D^{0.5,1} = 11,\ C_D^{1,1} = 10,\ C_D^{1,2} = 14$$

Steady State Probability	Steady State Cost	Steady State Probability	Steady State Cost
$\pi_{0,0,1,0} = 0.4684$	2.8104	$\pi_{.5,1,1,2} = 0.0070$	0.14
$\pi_{0,0,2,0} = 0.2342$	2.1078	$\pi_{.5,1,2,1} = 0.0027$	0.648
$\pi_{.5,0,1,1} = 0.1356$	2.8476	$\pi_{.5,1,2,2} = 0.0009$	0.0225
$\pi_{.5,0,1,2} = 0.0252$	0.5544	$\pi_{1,0,0,1} = 0.0403$	0.806
$\pi_{.5,0,2,1} = 0.0098$	0.2548	$\pi_{1,0,0,2} = 0.0060$	0.144
$\pi_{.5,0,2,2} = 0.0032$	0.0864	$\pi_{1,1,0,1} = 0.0275$	0.385
$\pi_{.5,1,1,1} = 0.0384$	0.7296	$\pi_{1,1,0,2} = 0.00448$	0.0806

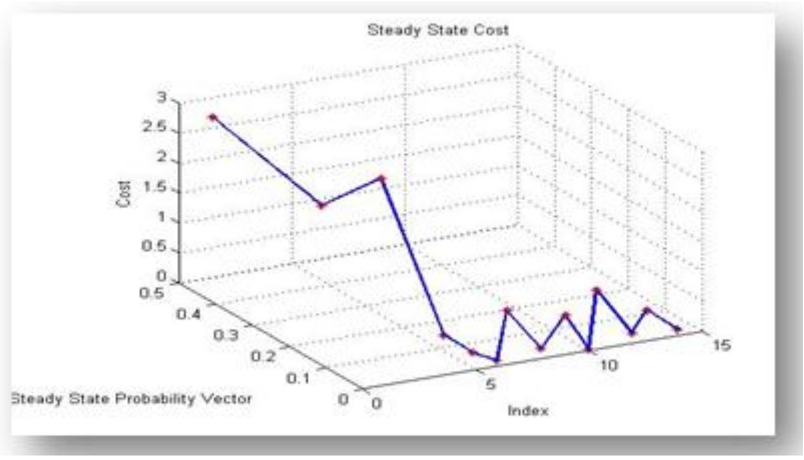

Figure 1

Numerical Illustration 2

The steady state cost of the system in different situations are

$$C_{ijkl} = \pi_{ijkl}\left[C_{MP}^{i} + C_{B}^{j} + C_{R}^{k} + C_{D}^{l}\right] \quad ....(7)$$

Here C_{MP}^{i} Stands for the cost of Manpower at the states of i =0,1/2,1

C_{B}^{j} Stands for the cost of Business at the states of j =1,2

C_{R}^{k} Stands for the cost of Recruitment Phase at the states of k =1,2

C_{D}^{l} Stands for the cost of Departure Phase at the states of l =1,2

We assume the cost and arrive the expected total cost

$$C_{MP}^{0} = 0,\ C_{MP}^{0.5} = 5,\ C_{MP}^{1} = 10,\ C_{B}^{0} = 6,\ C_{B}^{1} = 20$$

$$C_{R}^{1} = 10,\ C_{R}^{2} = 15,\ C_{D}^{1} = 15,\ C_{D}^{2} = 20$$

Steady State Probability	Steady State Cost	Steady State Probability	Steady State Cost
$\pi_{0,0,1,0} = 0.4684$	4.684	$\pi_{.5,1,1,2} = 0.0070$	0.385
$\pi_{0,0,2,0} = 0.2342$	3.513	$\pi_{.5,1,2,1} = 0.0027$	0.1485
$\pi_{.5,0,1,1} = 0.1356$	4.068	$\pi_{.5,1,2,2} = 0.0009$	0.054
$\pi_{.5,0,1,2} = 0.0252$	0.882	$\pi_{1,0,0,1} = 0.0403$	1.0075
$\pi_{.5,0,2,1} = 0.0098$	0.343	$\pi_{1,0,0,2} = 0.0060$	0.18
$\pi_{.5,0,2,2} = 0.0032$	0.128	$\pi_{1,1,0,1} = 0.0275$	1.2375
$\pi_{.5,1,1,1} = 0.0384$	1.92	$\pi_{1,1,0,2} = 0.00448$	0.224

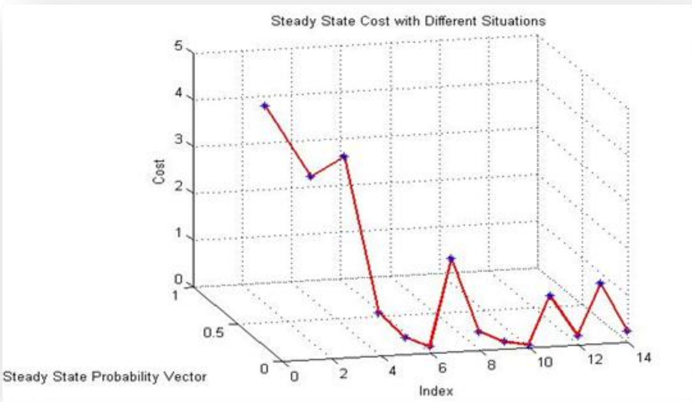

Figure 2

From the above figure 1 and figure 2 , the downward trend of the steady state cost indicates,there is no business during the period.When the business starts, there is an oscillation takes place in the steady state cost.

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