A New Method to Compute the Adjoint and Inverse of a 3 × 3 non – singular Matrices

K. Thirumurugan

Pgt in Mathematics, ghss valuthavur, Villupuram Dt, Tamilnadu, India.

ABSTRACT: *In this paper, I   present a new method to compute the determinant, adjoint and inverse of 3 × 3 non – singular matrices is considered. In this method to find the determinant value, adjoint of matrix is very quick when comparing to other known method. This is an easy method which helps to find the solution of inverse matrix.*

**KEYWORDS:** *Determinant, non – singular, adjoint, Inverse*

I.   A NEW METHOD TO COMPUTE THE DETERMINANT FOR 3×3 MATRIX

The new method to compute the determinant of a 3×3 matrix may be one of the easiest methods to compute the determinant of the third order. I have given three types of method to evaluate the determinants value.

$$Let\ A = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

This method will be formed of six diagonals with three different elements of determinants. The elements products in three diagonals in left side will be '+' sign, in other hand the elements product in three other different diagonals in right side will be het '-' sign.
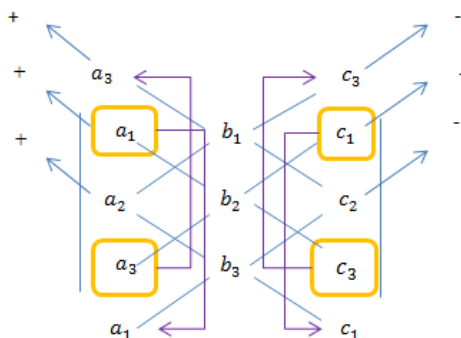
Type (1)
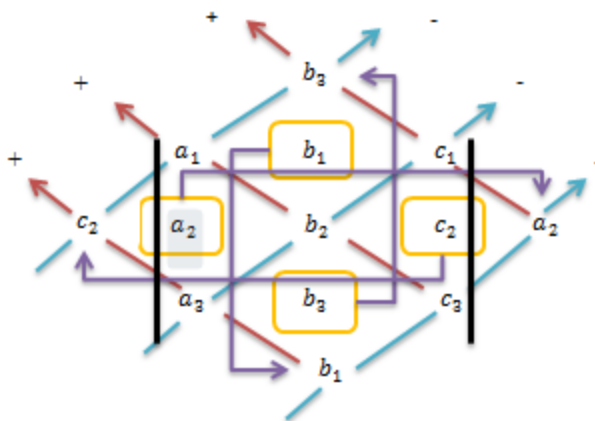
While applying the new method, to compute the determinants of the third, we have
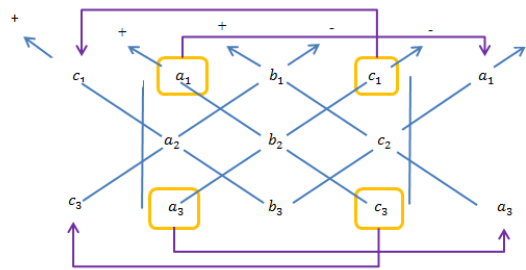
$$|A| = a_2 b_1 c_2 + a_1 b_2 c_3 + a_2 b_3 c_1 - a_1 b_3 c_2 - a_3 b_2 c_1 - a_2 b_1 c_3 \qquad (1)$$

Type (2)

$$|A| = a_3 b_1 c_2 + a_1 b_2 c_3 + a_2 b_3 c_1 - a_2 b_1 c_3 - a_3 b_2 c_1 - a_1 b_3 c_2$$
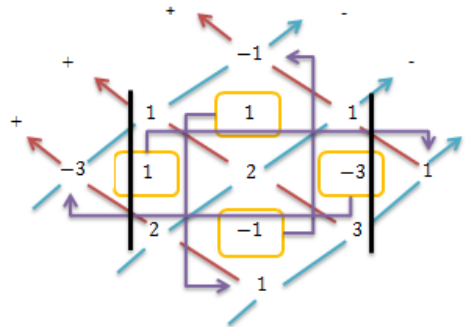
Type (3)

$$|A| = a_3 b_1 c_2 + a_1 b_2 c_3 + a_2 b_3 c_1 - a_2 b_1 c_3 - a_3 b_2 c_1 - a_1 b_3 c_2$$

Illustration:

Evaluate the determinant $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{vmatrix}$

Solution:

By applying Type (1) method, easily we get the value of determinant

$\Delta = (-3 \times 2 \times 1) + (1 \times 2 \times 3) + (-1 \times 1 \times 1) - (-3 \times 1 \times -1) - (2 \times 2 \times 1) - (1 \times 3 \times 1)$

$\Delta = -6 + 6 - 1 - 3 - 4 - 3$

$\Delta = -11$

(1.1)

The results acquired by using the new method are entirely equal with the results acquired by the other known method. In base of this, we can conclude that this new method to compute the determinants of third order is true and can be used for third order determinant.

II.   A NEW METHOD TO COMPUTE THE ADJOINT OF A 3×3 MATRIX

$$A = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

To find adjoint of A, a new method can be formed by using following scheme.
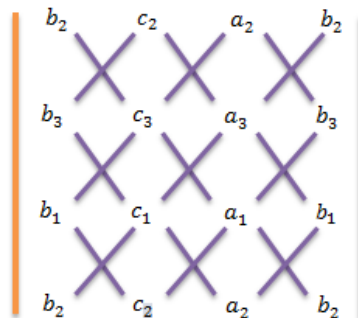
$$= \begin{bmatrix} b_2c_3 - b_3c_2 & a_3c_2 - a_2c_3 & a_2b_3 - a_3b_2 \\ b_3c_1 - b_1c_3 & a_1c_3 - a_3c_1 & a_3b_1 - a_1b_3 \\ b_1c_2 - b_2c_1 & a_2c_1 - a_1c_2 & a_1b_2 - a_2b_1 \end{bmatrix}^T$$

$$adj\ A = \begin{bmatrix} b_2c_3 - b_3c_2 & b_3c_1 - b_1c_3 & b_1c_2 - b_2c_1 \\ a_3c_2 - a_2c_3 & a_1c_3 - a_3c_1 & a_2c_1 - a_1c_2 \\ a_2b_3 - a_3b_2 & a_3b_1 - a_1b_3 & a_1b_2 - a_2b_1 \end{bmatrix} \tag{2}$$

Illustration:

Find the adjoint of matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$ by using new method.

Solution:

To find adjoint of A, we can write matrix A which can be formed by using following scheme.
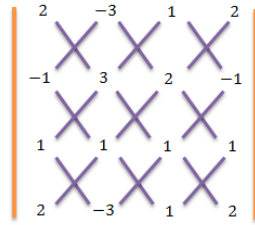
$$adj\ A = \begin{bmatrix} (6-3) & (-6-3) & (-1-4) \\ (-1-3) & (3-2) & (2+1) \\ (-3-2) & (1+3) & (2-1) \end{bmatrix}^T$$

$$adj\ A = \begin{bmatrix} 3 & -9 & -5 \\ -4 & 1 & 3 \\ -5 & 4 & 1 \end{bmatrix}^T$$

$$adj\ A = \begin{bmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{bmatrix} \tag{2.1}$$

III. INVERSE OF MATRIX

If A is a non-singular matrix, there exists inverse which is given by

$$A^{-1} = \frac{1}{|A|}(adj\ A) \tag{3}$$

Applying equations (1) and (2) into (3), we have

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} b_2c_3 - b_3c_2 & b_3c_1 - b_1c_3 & b_1c_2 - b_2c_1 \\ a_3c_2 - a_2c_3 & a_1c_3 - a_3c_1 & a_2c_1 - a_1c_2 \\ a_2b_3 - a_3b_2 & a_3b_1 - a_1b_3 & a_1b_2 - a_2b_1 \end{bmatrix} \tag{3.1}$$

Hence the equation (3.1) is obtaining the inverse of 3×3 non – singular matrix.

Illustration:

Find the inverse of $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$

By applying equations (1.1) and (2.1) in equation (3), we obtain the inverse of matrix

$$A^{-1} = \frac{1}{-11}\begin{bmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{bmatrix}$$

Computation of Determinant, Adjoint and Inverse of 3×3 matrices

Problem	Matrix	Determinant	Adjoint	Inverse
1	$\begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$	2	$\begin{bmatrix} 2 & -1 & -2 \\ 2 & -2 & -2 \\ 6 & -5 & -8 \end{bmatrix}$	$\frac{1}{2}\begin{bmatrix} 2 & -1 & -2 \\ 2 & -2 & -2 \\ 6 & -5 & -8 \end{bmatrix}$
2	$\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$	9	$\begin{bmatrix} 0 & 3 & 3 \\ 3 & 2 & -7 \\ 3 & -1 & -1 \end{bmatrix}$	$\frac{1}{9}\begin{bmatrix} 0 & 3 & 3 \\ 3 & 2 & -7 \\ 3 & -1 & -1 \end{bmatrix}$
3	$\begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$	35	$\begin{bmatrix} -4 & 11 & -5 \\ -1 & -6 & 25 \\ 6 & 1 & -10 \end{bmatrix}$	$\frac{1}{35}\begin{bmatrix} -4 & 11 & -5 \\ -1 & -6 & 25 \\ 6 & 1 & -10 \end{bmatrix}$
4	$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$	5	$\begin{bmatrix} 4 & -2 & -1 \\ -1 & 3 & -1 \\ -1 & -2 & 4 \end{bmatrix}$	$\frac{1}{5}\begin{bmatrix} 4 & -2 & -1 \\ -1 & 3 & -1 \\ -1 & -2 & 4 \end{bmatrix}$
5	$\begin{bmatrix} -1 & 2 & -2 \\ 4 & -3 & 4 \\ 4 & -4 & 5 \end{bmatrix}$	-1	$\begin{bmatrix} 1 & -2 & 2 \\ -4 & 3 & -4 \\ -4 & 4 & -5 \end{bmatrix}$	$\frac{1}{-1}\begin{bmatrix} 1 & -2 & 2 \\ -4 & 3 & -4 \\ -4 & 4 & -5 \end{bmatrix}$

IV.   CONCLUSION

This is one of the new methods with great application value in finding quickly and easily the third order determinant, adjoint and Inverse of the matrices.

REFERENCES

[1]   Dodgson, C,L. " Condensation of Determinants Being a New and Brief Method for Computing and their Arithmetic Values." *Proc.Roy.Soc.Ser.*A15, (1886), 150 – 155.
[2]   Hannus, Pauk Henry: An elementary treatise on the theory of determinants. A text-book for colleges, Ithaca, New York: Cornell University Library, Boston, Ginn and company (1886), 13, 14, 18.
[3]   Wels, Laenas Gifford: A short course in the theory of determinants, Ithaca, New York: Cornell University Library, New York, London: Macmillan and Co, (1893), 8.
[4]   Scott, Robert Forsyth: The theory of determinants and their applications, Ithaca, New York: Cornell University Library, Cambridge: University Press,(1905) 3 – 5.
[5]   Wield, Laenas Gifford: Determinants, Ithaca, New York: Cornell University Librart, New York: J. Wiley and Sons;[etc..], (1906), 14 -15.
[6]   Jos. V. Collins: Advanced algebra, American Book Company (1913), 281, 286.
[7]   Dardna Hajrizaj. "New Method to compute the Determinant", International journal of Algebra, (2009), 3, 211 - 219.