

$S^\#$ - Open and $S^\#$ -Closed Functions in Topological Spaces

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Abstract

The main objective of this paper is to further investigate $S^\#$ -closed sets and introduce the concept of new class of closed functions called $S^\#$ -open and $S^\#$ -closed functions in topological spaces along with their characterizations and properties. The newly defined concepts were compared with the existing maps and also the counter examples were provided. We also define $S^\#$ -Homeomorphism functions and study their fundamental properties. Counter examples are discussed to clarify the limitations of these concepts.

Keywords: $S^\#$ -closed set, $S^\#$ -closed function, $S^\#$ -open function, $S^\#$ -homeomorphism, $S^\#$ -continuous function.
Mathematics Subject Classification (2010): 54A05, 54B05, 54C08, 54D10.

Date of Submission: 08-10-2026

Date of Acceptance: 19-10-2026

I. Introduction

Many researchers and mathematicians contributed their work in the field of general topology, Njastad [17] initiated the idea of α -open sets in topological spaces. Notion of generalized closed (shortly g-closed) sets were initiated as well as studied by Levine [10] in 1970. Later Maki et al [12] established and developed the theme of generalized α -closed ($g\alpha$ -closed) sets as well as α -generalized closed (αg -closed) sets respectively in topological spaces. Many researchers like Arya and Gupta [4], Balachandran, Sundaram and Maki [5], Dontchev [9], Maki, Umehara and Noiri [13], Malghan [15] have developed generalized closed sets, their generalizations along with related themes in general topology. The concept of homeomorphism has been generalized by many topologists. In [14] notion of g-homeomorphisms is introduced and studied in topological spaces.

The purpose of this paper is to investigate and study the concepts of $S^\#$ -closed, $S^\#$ -open functions and $S^\#$ -homeomorphisms in topological space .

II. Prerequisites

Throughout this paper $(X,\tau),(Y,\sigma),(Z,\gamma)$ (or simply X,Y and Z) always means topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of space X . We denote the closure of A and the interior of A by $Cl(A)$ and $Int(A)$ respectively.

In this section we recall definitions of semi-open sets, semi-closed sets, generalized closed sets and related concepts which are used throughout the paper.

Definition 2.1: A map $k: X \rightarrow Y$ is termed as

- (i) α -continuous [6] whenever $k^{-1}(N)$ is $\alpha F(X)$ being each N is $C(Y)$.
- (ii) g-continuous [1] whenever $k^{-1}(D)$ is $GF(X)$ for each D is $C(Y)$.
- (iii) αg -continuous [3] whenever $k^{-1}(D)$ is $\alpha GF(X)$ being each D is $C(Y)$.

Definition 2.2: A map $\lambda: X \rightarrow Y$ is named as

- i) g-closed [15] whenever $\lambda(G)$ is GF(Y) being each G as a $C(X)$ (resp. $O(X)$).
- ii) α g-closed [7] (jointly α g-open) wherever $\lambda(L)$ is α GF(Y) (jointly α GO(Y)) as each L as $C(X)$ (resp. $O(X)$).
- iii) α -open [11] whenever $\lambda(K)$ is $\alpha O(Y)$ for each K as a $O(X)$.

Definition 2.3: A map $k: X \rightarrow Y$ is termed as irresolute[2] if $k^{-1}(G)$ is $SO(X)$ considering each G as $SO(Y)$.

Definition 2.4: The bijective map $h: X \rightarrow Y$ is named as

- i) Homeomorphism if h is both open and continuous
- ii) generalized homeomorphism (g-homeomorphism)[2] if h is both g-continuous and g-open.
- iii) α -generalized homeomorphism(α g-homeomorphism)[8] if h is both α g-continuous and α g-open.

III. S[#]-Open and S[#]-Closed Functions

Definition 3.1: A function $f: X \rightarrow Y$ is named as S[#]-open whenever the image of each $O(X)$ is $S^{\#}O(Y)$.

Definition 3.2: A mapping $k: X \rightarrow Y$ is termed as S[#]-closed whenever the image of each $C(X)$ is $S^{\#}F(Y)$.

Theorem 3.3: Each open function is S[#]-open function, however reverse is not possible.

Proof: Allow $f: X \rightarrow Y$ be an open function. Make W as an $O(X)$. Thereupon $f(W)$ is $O(Y)$. And therefore $f(W)$ is $S^{\#}O(Y)$. Henceforth f is S[#]-open function.

Example 3.4: Taking $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{a, b\}\}$. At that time Id. function $k: X \rightarrow Y$ is not an open, due to $\{b\}$ an $O(X)$, $k(\{b\}) = \{b\}$ is not an $O(Y)$. However f is S[#]-open.

Theorem 3.5: Whenever $\mu: X \rightarrow Y$ is S[#]-open function together with Y as a T_S -space, thereupon f is an open function.

Proof: Enable P be an $O(X)$. Then $\mu(P)$ is $S^{\#}O(Y)$, as μ is S[#]-open function. Subsequently $\mu(P)$ is $O(Y)$ as Y is T_S -space. Hence μ is open function.

Theorem 3.6: Each α g-open function is S[#]-open function. However, reverse is impossible.

Proof: Entitle $f: X \rightarrow Y$ be a α g-open function. Allow R be an $O(X)$. Then $f(R)$ is $\alpha GO(Y)$ as f is α g-open function. Therefore $f(R)$ is in $S^{\#}O(Y)$. Hence f is S[#]-open function.

Example 3.7: Taking $X = Y = \{p, q, r, s\}$ with $\tau = \{X, \phi, \{p, r\}\}$, $\sigma = \{Y, \phi, \{p, q\}\}$. $S^{\#}F(Y) = \{Y, \phi, \{q\}, \{r\}, \{s\}, \{p, r\}, \{p, s\}, \{q, r\}, \{q, s\}, \{r, s\}, \{p, q, r\}, \{p, q, s\}, \{p, r, s\}, \{q, r, s\}\}$. Accordingly, $\{q\}$ is $C(X)$, $f(\{q\})$ is $S^{\#}F(Y)$ though it isn't $\alpha GF(Y)$.

Theorem 3.8: Each g-open function is S[#]-open function.

Proof: Permit $h: X \rightarrow Y$ be a g-open function. Allow W be an $O(X)$. At that time $h(W)$ is $GO(Y)$. Accordingly, $h(W)$ is $S^{\#}O(Y)$. Hence f is S[#]-open.

The example makes clear that reverse of the above theorem is impossible.

Example 3.9: Considering $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{a, b\}, \{a, c\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{a, b\}\}$. Thereupon an Id. function $h: X \rightarrow Y$ is $S^\#$ -open however not g-open, as for the $\{a, c\}$ an $O(X)$, $h(\{a, c\}) = \{a, c\}$ isn't $GO(Y)$ yet it is $S^\#O(Y)$.

Theorem 3.10: Each closed function is $S^\#$ -closed function though not conversely.

Proof: Consider $f: X \rightarrow Y$ be a closed function. Allow K be a $C(X)$. At that time $f(K)$ is $C(Y)$. So $f(G)$ is $S^\#F(Y)$. Accordingly, f is $S^\#$ -closed function.

Example 3.11: In the Example 3.4, the function f is $S^\#$ -closed though not a closed function, as for the $\{a, c\}$ a $C(X)$, $f(\{a, c\}) = \{a, c\}$ is not a $C(Y)$ yet it is $S^\#F(Y)$.

Theorem 3.12: Each αg -closed function is $S^\#$ -closed function, still converse is untrue.

Proof: Allow $f: X \rightarrow Y$ be a αg -closed function. Entitle G be a $C(X)$. Thereupon $f(G)$ is $\alpha GF(Y)$. So $f(G)$ is $S^\#F(Y)$. Henceforth, f is $S^\#$ -closed function.

Example 3.13: In Example 3.7, the function f is $S^\#$ -closed still it isn't αg -closed, as for $\{b\}$ a $C(X)$, $f(\{b\}) = \{b\}$ is not $\alpha GF(Y)$.

Theorem 3.14: Each g-closed function is $S^\#$ -closed function, however reverse is untrue.

Proof: Approve $f: X \rightarrow Y$ be a g-closed function. Make E as a $C(X)$. Then $f(E)$ is $GF(Y)$. Accordingly, $f(E)$ is $S^\#F(Y)$. Consequently, f is $S^\#$ -closed function.

Example 3.15: In Example 3.9, the function f is $S^\#$ -closed though not g-closed, as $\{b\}$ is $C(X)$, $f(\{b\}) = \{b\}$ isn't $GF(Y)$ yet it is $S^\#F(Y)$.

Theorem 3.16: A mapping $h: X \rightarrow Y$ is $S^\#$ -closed iff considering any member S of Y along with an open set U including $h^{-1}(S)$ there is V as a $S^\#O(Y)$ so as $S \subseteq V$ and $h^{-1}(V) \subseteq U$.

Proof: Presume f is $S^\#$ -closed function. Entitle S be a member of Y along with U be an $O(X)$ hereupon $h^{-1}(S) \subseteq U$. Thereupon $V = Y - h(X - U)$ is $S^\#$ -open set including S so as $h^{-1}(V) \subseteq U$.

Contrarily, predict W is a $C(X)$. Thereupon $h^{-1}(Y - h(W)) = X - W$ is open. Referring to hypothesis, there is V as a $S^\#O(Y)$ so as $Y - h(W) \subseteq V$ with $h^{-1}(V) \subseteq X - W$. Accordingly $W \subseteq X - h^{-1}(V)$. Hence $Y - V \subseteq h(W) \subseteq f(X - h^{-1}(V)) \subseteq Y - V$, so $h(W) = Y - V$. As $Y - V$ is $S^\#$ -closed, $h(W)$ is $S^\#$ -closed. Consequently, h is $S^\#$ -closed function.

Remark 3.17: Composition of two $S^\#$ -closed maps not at all a $S^\#$ -closed.

Example 3.18: Considering $X = Y = Z = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{a, c\}, X\}$, $\sigma = \{\phi, \{a\}, Y\}$ and $\gamma = \{\phi, \{a\}, \{b\}, \{a, b\}, Z\}$. Taking $f: X \rightarrow Y$ as well as $g: Y \rightarrow Z$ as Id. functions. So f along with g are both $S^\#$ -closed functions yet $gof: X \rightarrow Z$ isn't a $S^\#$ -closed function, due to $\{b\}$ a $C(X)$, $(gof)(\{b\}) = g(f(\{b\})) = g\{b\} = \{b\}$ isn't $S^\#F(Z)$.

Theorem 3.19: When $\alpha: X \rightarrow Y$ is closed as well as $\beta: Y \rightarrow Z$ are $S^\#$ -closed maps, thereupon $(\beta \circ \alpha): X \rightarrow Z$ is $S^\#$ -closed function.

Proof: Consider V be any $C(X)$. As α is closed, $\alpha(V)$ is $C(Y)$. Again β is $S^\#$ -closed function, $\beta(\alpha(V))$ is $S^\#F(Z)$. However, $\beta(\alpha(V)) = (\beta \circ \alpha)(V)$ is $S^\#F(Z)$. Accordingly, $(\beta \circ \alpha) : X \rightarrow Z$ is $S^\#$ -closed function.

Theorem 3.20: Whenever $\beta : X \rightarrow Y$ is $S^\#$ -closed and Y is T_S -space, subsequently β is closed function.

Proof: Allow $\beta : X \rightarrow Y$ be a $S^\#$ -closed function. Make G be a $C(X)$. At that time $\beta(G)$ is $S^\#F(Y)$. As Y is T_S -space, $\beta(G)$ is $C(Y)$. Hence, β is closed function.

Theorem 3.21: Whenever $f : X \rightarrow Y$ as well as $g : Y \rightarrow Z$ are two $S^\#$ -closed functions with Y as T_S -space, thereupon $g \circ f : X \rightarrow Z$ is $S^\#$ -closed function.

Proof: Permit Q be a $C(X)$. Thereupon $f(Q)$ is $S^\#F(Y)$ as f is $S^\#$ -closed function. As Y is T_S -space, $f(Q)$ is $C(Y)$. Again g is $S^\#$ -closed, $g(f(Q))$ is $S^\#F(Z)$. Though $g(f(Q)) = (g \circ f)(Q)$ is $S^\#F(Z)$. Henceforth, $g \circ f : X \rightarrow Z$ is $S^\#$ -closed function.

Theorem 3.22: Consider $\alpha : X \rightarrow Y$ as well as $\beta : Y \rightarrow Z$ be two functions and $\beta \circ \alpha : X \rightarrow Z$ is $S^\#$ -closed. Listed properties are true.

- i) Whenever α is continuous as well as surjective, thereupon β is $S^\#$ -closed.
- ii) Whenever β is $S^\#$ -irresolute and injective, thereupon α is $S^\#$ -closed.

Proof: i) Consider F be a $C(Y)$. Thereupon, $\alpha^{-1}(F)$ is $C(X)$ as α is continuous. Since $(\beta \circ \alpha)$ is $S^\#$ -closed and f is surjective, $(\beta \circ \alpha)(\alpha^{-1}(F))$ is $S^\#F(Z)$. That is $(\beta \circ \alpha)(\alpha^{-1}(F)) = \beta(\alpha(\alpha^{-1}(F))) = \beta(F)$ is $S^\#F(Z)$. Hence β is $S^\#$ -closed function.

ii) Make H as a $C(X)$. As $(\beta \circ \alpha)$ is closed, $(\beta \circ \alpha)(H)$ is a $S^\#F(Z)$. As β is $S^\#$ -irresolute and injective, $\beta^{-1}((\beta \circ \alpha)(H)) = \beta^{-1}(\beta(\alpha(H))) = \alpha(H)$ is $S^\#F(Y)$. So α is $S^\#$ -closed function.

Theorem 3.23: A function $\beta : X \rightarrow Y$ is bijection. Accordingly, results are analogous.

- i) $\beta^{-1} : Y \rightarrow X$ is $S^\#$ -continuous
- ii) β is $S^\#$ -open
- iii) β is $S^\#$ -closed

Proof: (i) $\Rightarrow$ (ii): Entitle G be an $O(X)$. By (i), $(\beta^{-1})^{-1}(G) = \beta(G)$ is $S^\#O(Y)$ and so β is $S^\#$ -open.

(ii) $\Rightarrow$ (iii): Enable K be a $C(X)$. Thence $X - K$ is $O(X)$. Due to hypothesis (ii), $\beta(X - K)$ is $S^\#O(Y)$. That is $\beta(X - K) = Y - \beta(K)$ is $S^\#O(Y)$. Accordingly, $\beta(K)$ is $S^\#F(Y)$. Hence β is $S^\#$ -closed function.

(iii) $\Rightarrow$ (i): Allow J be a $C(X)$. At that time by (iii), $\beta(J)$ is $S^\#F(Y)$. However $\beta(J) = (\beta^{-1})^{-1}(J)$ is $S^\#F(Y)$. Accordingly, $\beta^{-1} : Y \rightarrow X$ is $S^\#$ -continuous function.

Theorem 3.24: A function $\beta : X \rightarrow Y$ is $S^\#$ -open iff for any member S of Y and as long as any closed set F including $\beta^{-1}(S)$, there arises K as a $S^\#F(Y)$ comprising S thereby $\beta^{-1}(K) \subseteq F$.

Proof: Imagine $\beta : X \rightarrow Y$ is $S^\#$ -open function. Permit S be a subset of Y also F be a $C(X)$ comprising $\beta^{-1}(S)$. Thereupon $K = Y - \beta(X - F)$ is a $S^\#$ -closed set involving S so as $\beta^{-1}(K) \subseteq F$.

Conversely, speculate that U is an $O(X)$. Then $\beta^{-1}(Y - \beta(U)) \subseteq X - \beta^{-1}[\beta(U)] \subseteq X - U$ and $X - U$ is $C(X)$. By hypothesis, there is K as a $S^{\#}F(Y)$ so as $Y - \beta(U) \subseteq K$ and $\beta^{-1}(K) \subseteq X - U$. Therefore $U \subseteq X - \beta^{-1}(K)$. Henceforth, $Y - K \subseteq \beta(U) \subseteq f[X - \beta^{-1}(K)] \subseteq Y - K$. Which indicates $\beta(U) \subseteq Y - K$. As $Y - K$ is $S^{\#}$ -open, $\beta(U)$ is $S^{\#}$ -open and thus β is $S^{\#}$ -open function.

Theorem 3.25: Whenever $\beta : X \rightarrow Y$ is a bijective $S^{\#}$ -open function. When X is a $S^{\#}$ - T_1 -space with T_S -space, subsequently Y is a $S^{\#}$ - T_1 -space.

Proof: Permit y_1 and y_2 be two unlike points of Y . As f is bijective, there occurs separate points x_1 and x_2 of X thereby $\beta(x_1) = y_1$ and $\beta(x_2) = y_2$. As X is a $S^{\#}$ - T_1 -space, there prevails $S^{\#}$ -open sets G as well as H thereby $x_1 \in G$ along with $x_2 \notin G$ and $x_1 \notin H$ and $x_2 \in H$. Again as X is T_S -space, G as well as H are open sets in X . Here f is $S^{\#}$ -open function, $\beta(G)$ and $\beta(H)$ are $S^{\#}$ -open sets so as $y_1 = \beta(x_1) \in \beta(G)$, $y_2 = \beta(x_2) \notin \beta(G)$ and $y_1 = \beta(x_1) \notin \beta(H)$, $y_2 = \beta(x_2) \in \beta(H)$. Henceforth, Y is $S^{\#}$ - T_1 -space.

Theorem 3.26: Whenever $f: X \rightarrow Y$ is a bijective $S^{\#}$ -open function. If X is $S^{\#}$ - T_2 -space and T_S -space, thereupon Y is also $S^{\#}$ - T_2 -space.

Proof: is analogous to 3.25.

IV. $S^{\#}$ -Homeomorphisms in Topological Spaces

Definition 4.1: A bijective function $h: X \rightarrow Y$ is termed as $S^{\#}$ -homeomorphism whenever h and h^{-1} are $S^{\#}$ -continuous functions.

Example 4.2: Enable $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{a, b\}\}$ and $\sigma = \{Y, \phi, \{a, b\}\}$. Id. function $h: X \rightarrow Y$ is $S^{\#}$ -homeomorphism as h along with h^{-1} are $S^{\#}$ -continuous functions.

Theorem 4.3: Every homeomorphism is a $S^{\#}$ -homeomorphism, however reverse is not possible.

Proof: Let $\beta: X \rightarrow Y$ be a homeomorphism. Then β and β^{-1} are continuous functions. As each continuous function is $S^{\#}$ -continuous. So β as well as β^{-1} are $S^{\#}$ -continuous functions. Henceforth β is $S^{\#}$ -homeomorphism.

Example 4.4: In Example 4.2, the function f is $S^{\#}$ -homeomorphism but not a homeomorphism.

Theorem 4.5: Whenever $f: X \rightarrow Y$ is αg -homeomorphism thereupon $S^{\#}$ -homeomorphism.

Proof: Consider $f: X \rightarrow Y$ is αg -homeomorphism. At that time f as well as f^{-1} are αg -continuous functions. As each αg -continuous is $S^{\#}$ -continuous. Consequently, f and f^{-1} are $S^{\#}$ -continuous functions. Henceforth f is $S^{\#}$ -homeomorphism.

Remark 4.6: The contrary of the above theorem isn't at all possible.

Example 4.7: Taking $X = \{m, b, p, q\}$ with $\tau = \{X, \phi, \{m, n\}\}$. $S^{\#}F(X) = \{X, \phi, \{m\}, \{p\}, \{q\}, \{m, p\}, \{m, q\}, \{n, p\}, \{n, q\}, \{p, q\}, \{m, n, p\}, \{m, n, q\}, \{m, p, q\}, \{n, p, q\}\}$. Accordingly, $\{n\}$ is $S^{\#}F(X)$ though it isn't $\alpha GF(X)$. along with $Y = \{a, b, c, d\}$, $\sigma = \{Y, \phi, \{b\}\}$. Id. function f is $S^{\#}$ -homeomorphism however not αg -homeomorphism as it isn't αg -continuous, as for $\{b\} \in C(Y)$, $f^{-1}(\{b\}) = \{b\}$ is not $\alpha GF(X)$ though it is $S^{\#}F(X)$.

Theorem 4.8: Every g-homeomorphism is S[#]-homeomorphism though reverse is impossible.

Proof: Let $k: X \rightarrow Y$ be a g-homeomorphism. Accordingly, k and k^{-1} are g-continuous functions. As each g-continuous function is S[#]-continuous function, it follows that, k and k^{-1} are S[#]-continuous functions. So, k is S[#]-homeomorphism.

Example 4.9: In example 3.7, f is S[#]-homeomorphism however g-homeomorphism as it isn't g-continuous, as $\{b, c\} \in \mathcal{O}(Y)$ in Y , $f^{-1}(\{b, c\}) = \{b, c\}$ isn't $\mathcal{GO}(X)$ yet it is $S^{\#}\mathcal{O}(X)$.

Theorem 4.10: Consider $k: X \rightarrow Y$ be a bijective and S[#]-continuous function. Thereupon the statements listed below are analogous.

- a) k is S[#]-open
- b) k is S[#]-homeomorphism
- c) k is S[#]-closed

Proof: follows from Theorem 3.23

Remark 4.11: Composition of two S[#]-homeomorphisms need not be a S[#]-homeomorphism.

Remark 4.12: Allow $f: X \rightarrow Y$ as well as $g: Y \rightarrow Z$ be S[#]-homeomorphism along with Y is T_s -space, thereupon $(g \circ f)$ is S[#]-homeomorphism.

V. Concluding Remarks

The concluding remarks of the paper are as follows

- **Inclusion of Open and Closed Mappings:** Every standard open function is an S[#]-open function, and every standard closed function is an S[#]-closed function, though the converse statements do not hold true in general.
- **Relationship with Generalized Open Functions:** Every g-open function qualifies as an S[#]-open function, but the reverse assertion is false.
- **Relationship with Generalized Closed Functions:** Every g-closed function and every alpha-g-closed function is an S[#]-closed function, though neither converse relationship holds true.
- **Composition Behavior of S[#]-Closed Maps:** The composition of two S[#]-closed functions is not necessarily an S[#]-closed function.
- **Characterization Condition for S[#]-Closed Functions:** A mapping h from X to Y is an S[#]-closed function if and only if for every subset S of Y and open set U containing the inverse image of S , there exists an S[#]-open set V in Y containing S such that the inverse image of V is contained in U .
- **Hierarchical Placement of S[#]-Homeomorphisms:** Every standard homeomorphism, alpha-g-homeomorphism, and g-homeomorphism is an S[#]-homeomorphism, but none of these relationships can be reversed.
- **Equivalence Criteria for Bijective Maps:** For a bijective and S[#]-continuous mapping k from X to Y , being an S[#]-open function, an S[#]-closed function, and an S[#]-homeomorphism are completely equivalent properties.

➤ **Composition of $S^\#$ -Homeomorphisms:** The composition of two $S^\#$ -homeomorphisms does not generally result in an $S^\#$ -homeomorphism, unless the intermediate space satisfies specialized topological conditions, such as being a T_s -space.

Acknowledgment

The authors would like to thank the Editor and an anonymous reviewer for the useful comments, which improved the quality of the article. The authors (Sujata Mookanagoudar & Sujata Tallur) acknowledges with gratitude the University Grants Commission (UGC) and the Department of Collegiate Education, Government of Karnataka (DCE, GoK), for their support and encouragement. The author (MBP) also thanks the Management of KLE Technological University for their continuous support and encouragement in carrying out this research.

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