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Some New Class of Continuous Functions in Topological Spaces

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Abstract:*The purpose of this paper is to introduce and study new class of continuous functionsnamely $\widehat{S_P}^*$- continuous function,$\widehat{S_P}^*$ - irresolute function. Additionally some properties of these functions are investigated.*
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Key words :$\widehat{S_P}^*$ *- closed sets,* $\widehat{S_P}^*$ *- open sets,* $\widehat{S_P}^*$ *- continuous function,* $\widehat{S_P}^*$ *- irresolute function.*

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I. Introduction

One of the important and basic concepts in topology and several branches of mathematics which have been researched by many authors is the continuity functions. N.Levine [9] introduced the concept of semi open P.Battacharya and B.K.Lahiri [1] introduced the concept of semi generalized closed sets in 1987. Continuing this research work in 1991,P.Sundaram et al [18] introduced semi generalized continuous functions and semi generalized irresolute functions in topological spaces J.ArulJesti et al[17] has introduced and studied $S_g{}^*$-closed sets, $S_g{}^*$-open sets, $S_g{}^*$-continuous function , $S_g{}^*$- irresolute function. S.PiousMissier and SiluvaiA[16] have introduced the concept of $\widehat{S_P}^*$-open sets, $\widehat{S_P}^*$- closed sets and studied their properties in topological spaces.Extending this work, we introduced a new continuous functions called, $\widehat{S_P}^*$continuous function, $\widehat{S_P}^*$- irresolute functions in topological spaces.

II. Preliminaries

Throughout this paper, *X*, *Y* and *Z* always denote topological spaces (*X*, τ), (*Y*, σ) and (*Z*, η) on which no separation axioms are assumed, unless explicitly stated.

Definition 2.1[18] Let A be a subset of a topological space (X,τ). Then
(a) A is called a semi generalized star open set (briefly $S_g{}^*$-open) if there is an open set U in X such that $U \subseteq A \subseteq$ Scl *(U).
(a) A is called a semi generalizedstar closed set (briefly $S_g{}^*$-closed) if its complement is a semi generalizedstar open set in (X,τ).

Definition 2.2 [16]A subset A of a topological space (X,τ) is called a $\widehat{S_P}^*$-open set, if here is an open set U such that $U \subseteq A \subseteq PCl^*(U)$. The collection of all $\widehat{S_P}^*$-open sets in (X,τ) is denoted by $\widehat{S_P}^*O(X,\tau)$ or $\widehat{S_P}^*O(X)$.
Theorem 2.3 [16] Arbitrary union of $\widehat{S_P}^*$-open sets is $\widehat{S_P}^*$-open

Definition 2.4[16] A subset A of a Space X is called $\widehat{S_P}^*$-closed set if its complement $(X\backslash A)$ is $\widehat{S_P}^*$-open in X.The class of all $\widehat{S_P}^*$closed sets in (X,τ) is denoted by $\widehat{S_P}^* \subseteq (X,\tau)$ or simply $\widehat{S_P}^*$ is a collection of X in (X,τ)

Theorem. 2.5[16] :
(i) Every open set is a $\widehat{S_P}^*$-open set and Every closed set is $\widehat{S_P}^*$-closed.

(ii) Every α-open set in X is a $\widehat{S_P}^*$- open set and Every α-closedset is$\widehat{S_P}^*$- closed.
(iii) Every semi α-open set in X is a $\widehat{S_P}^*$- open set.
(iv) Every $semi^*$-open set is $\widehat{S_P}^*$-open and Every$semi^*$-closed set is $\widehat{S_P}^*$-closed.
(v) Every S_g^*-open set is a $\widehat{S_P}^*$-open set and EverySg^*-closed set is $\widehat{S_P}^*$-closed.

Theorem 2.6 [16] IfA is a subset of a topological space X, the following statements are equivalent
(i) A is $\widehat{S_P}^*$closed
(ii) There is a pre-closed F in X such that $PInt^*(F) \subseteq A \subseteq F$
(iii) $PInt^*(Cl(A)) \subseteq A$
(iv) $PInt^*(Cl(A)) = PInt^*(A)$
(v) $PInt^*(A \cup Cl(A)) = PInt^*(A)$

Theorem 2.7 [16] If A is any subset of a topological space X,A is $\widehat{S_P}^*$-closed if and only if $\widehat{S_P}^* Cl(A) = A$.

Theorem 2.8 [16]IfA is a subset of a topological space (X, τ), then $PCl^*(Int(A)) = PCl^*(A)$

Theorem 2.9 [15] Let$f: X \to Y$ be a function. Then
(i) $Int^*\left(PCl(f^{-1}(F))\right) = Int^*(f^{-1}(F))$ for every closed set F in Y
(ii) $PCl^*(Int(f^{-1}(V)) = PCl^*(f^{-1}(V))$ for every open set V in Y.

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Theorem 2.10 [15] Let$f: (X, \tau) \to (Y, \sigma)$ be a function. Then the following are equivalent
(i) f is contra $\widehat{S_P}^*$irresolute
(ii) $f^{-1}(F)$ is $\widehat{S_P}^*$open in X for each $\widehat{S_P}^*$closed set F in Y.

III. $\widehat{S_P}^*$ContinuousFunctions

In this section, we define $\widehat{S_P}^*$-continuous functions and study their properties. We find characterizations for these functions and discuss the relationship with other similar functions.

Definition 3.1 A function $f: X \to Y$ is said to be $\widehat{S_P}^*$-continuous if $f^{-1}(V)$ is $\widehat{S_P}^*$-open in X for every open set V in Y.

Definition 3.2 A function $f: X \to Y$ is said to be $\widehat{S_P}^*$-continuous at a point x in X if for each open set V in Y containing $f(x)$, there is $\widehat{S_P}^*$-open set U in X such that $x \in U$ and $f(U) \subset V$.

Theorem 3.3 Let $f: X \to Y$ be a function. Then the following are equivalent.
(iii) f is $\widehat{S_P}^*$-continuous
(iv) f is $\widehat{S_P}^*$-continuous at each point x in X
(v) $f^{-1}(F)$ is $\widehat{S_P}^*$-closed in X for every closed F in Y
(vi) $f(\widehat{S_P}^* Cl(A) \subseteq Cl(f(A))$ for every subset A of X.
(vii) $\widehat{S_P}^* Cl(f^{-1}(B)) \subseteq f^{-1}(Cl(B))$ for every subset B of Y.
(viii) $f^{-1}(Int(B)) \subseteq \widehat{S_P}^* Int(f^{-1}(B))$ for every subset B of Y
(ix) $Int^*\left(PCl(f^{-1}(F))\right) = Int^*(f^{-1}(F))$ for every closed set F in Y
(x) $PCl^*(Int(f^{-1}(V)) = PCl^*(f^{-1}(V))$ for every open set V in Y.

Proof: (i) → (ii) Let $f: X \to Y$ be $\widehat{S_P}^*$-continuous. Let $x \in X$ and V be an open set in Y containing $f(x)$. Then $x \in f^{-1}(V)$. Since f is $\widehat{S_P}^*$-continuous, $U = f^{-1}(V)$ is $\widehat{S_P}^*$-open set containing x such that $f(U) \subset V$. This proves (ii).

(ii) → (i) Let $f: X \to Y$ be $\widehat{S_P}^*$-continuous at each point of X. Let V be an open set in Y. Let $x \in f(V)$. Then V is an open set in Y containing $f(x)$. By (ii), there is $\widehat{S_P}^*$-open set U_x in X containing x such that $f(x) \in f(U_x) \subseteq V$. Therefore, $U_x \subseteq f^{-1}(V)$. Hence $f^{-1}(V) = U\{U_x: x \in f^{-1}(V)\}$. By theorem 2.3, $f^{-1}(V)$ is $\widehat{S_P}^*$-open in X. Hence f is $\widehat{S_P}^*$-continuous.

(ii) → (iii) Let $f: X \to Y$ be $\widehat{S_P}^*$-continuous. Let F be a closed set in Y. Then, $V = (X \backslash F)$ is open in Y. Then by $\widehat{S_P}^*$-continuity of f, $f^{-1}(V) = f^{-1}(X \backslash F) = X \backslash f^{-1}(F)$ is $\widehat{S_P}^*$-open in X. Hence $f^{-1}(F)$ is $\widehat{S_P}^*$-closed in X.

(iii) → (iv) Let $A \subset X$. Let F be a closed set containing $f(A)$. Then $f^{-1}(F)$ is a $\widehat{S_P}^*$-closed set containing A and this implies $\widehat{S_P}^* Cl(A) \subseteq f^{-1}(F)$ and hence $f\left(\widehat{S_P}^* Cl(A)\right) \subseteq F$. Thus $f\left(\widehat{S_P}^* Cl(A)\right) \subseteq Cl(f(A))$

(iv) → (v) Let $B \subseteq Y$ and $A = f^{-1}(B) \subset X$. By (iv) $f\left(\widehat{S_P}^* Cl(A)\right) \subseteq Cl(f(A)) \subseteq Cl(B)$ which implies that $\widehat{S_P}^* Cl(A) \subseteq f^{-1}(Cl(B))$. Hence $\widehat{S_P}^* \left(Cl(f^{-1}(B))\right) \subseteq f^{-1}(Cl(B))$ for every subset B of Y.

(v) → (vii) Let F be a closed set in Y. Then $Cl(F) = F$. Therefore, (v) implies $\widehat{S_P}^* Cl(f^{-1}(F)) \subseteq f^{-1}(Cl(F)) = f^{-1}(F)$. Since $f^{-1}(F) \subseteq \widehat{S_P}^* Cl(f^{-1}(F))$, we have $\widehat{S_P}^* Cl(f^{-1}(F)) = f^{-1}(F)$. Hence by Theorem 2.7 (ii), $f^{-1}(F)$ is $\widehat{S_P}^*$-closed. Therefore, by Theorem 2.6, (iv),$PInt^*\left(Cl(f^{-1}(F))\right) = PInt^*(f^{-1}(F))$ for every closed set F in Y.

(v) → (vi) The equivalence of (v) and (vi) can be proved by replacing B by $(X \backslash B)$ and taking complement on both sides and using topological results

$$\widehat{S_P}^* Cl(f^{-1}(X \backslash B)) \subseteq f^{-1}(Cl(X \backslash B))$$
$$\widehat{S_P}^* Cl(Y - f^{-1}(B)) \subseteq Cl(Y - f^{-1}(B))$$
$$\widehat{S_P}^* Int(f^{-1}(B)) \supseteq f^{-1}(Int(B))$$

Which implies $f^{-1}(Int(B)) \subseteq \widehat{S_P}^* Int(f^{-1}(B))$ for every closed set B in Y.

(iii) → (vii) Let F be a closed set in Y. Then by (iii), $f^{-1}(F)$ is $\widehat{S_P}^*$-closed and hence by Theorem 2.6, (iv), $PInt^*\left(Cl(f^{-1}(F))\right) = PInt^*(f^{-1}(F))$

(vii) → (viii) Let V be an open set in Y. Take $F = (X \backslash V)$. Then F is closed in Y. By assumption $PInt^*\left(Cl(f^{-1}(X \backslash V))\right) = PInt^*(f^{-1}(X \backslash V))$. Taking complements on both sides and using topological results we get $PCl^*\left(Int(f^{-1}(V))\right) = PCl^*(f^{-1}(V))$

(viii) → (i) Let V be any open set in Y. Then by assumption $PCl^*\left(Int(f^{-1}(V))\right) = PCl^*(f^{-1}(V))$. By definition, we get $f^{-1}(V)$ is $\widehat{S_P}^*$-open for every open set V in Y. Therefore f is $\widehat{S_P}^*$-continuous.

Theorem 3.4 Every $\widehat{S_P}^*$-continuous function in a topological space (X, τ) is $semi^*$-pre continuous.
Proof: Let $f: X \to Y$ be $\widehat{S_P}^*$-continuous. If V is an open set in Y, then $f^{-1}(V)$ is $\widehat{S_P}^*$-open in X. Since every $\widehat{S_P}^*$-open set is $semi^*$-pre-open set, $f^{-1}(V)$ is $semi^*$-pre-open in X. Therefore, f is $semi^*$-pre- continuous.

Remark 3.5 Converse of the above theorem need not be true as seen from the following example.

Example 3.6 Let $X = \{a, b, c\} = Y$ and $\tau = \{X, \phi, \{a\}, \{b, c\}$ and $\sigma = \{Y, \phi, \{a\}, \{b, c\}$. Let $f: (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = b$, $f(b) = c$, $f(c) = a$. Clearly f is not $\widehat{S_P}^*$-continuous, since {b, c} is open in (Y, σ) but $f^{-1}(\{b, c\}) = \{a, b\}$ which is not $\widehat{S_P}^*$-open set in (X, τ), however f is $semi^*$-pre-continuous.

Theorem 3.7 Every $\widehat{S_P}^*$-continuous function in a topological space is semi-pre-continuous.
Proof: follows from theorem 3.4, since every $semi^*$-pre-continuous is semi-pre-continuous.

Remark 3.8 Converse of the above theorem need not be true as can be seen from the following example.

Example 3.9 Let $X = \{a, b, c\} = Y$ and $\tau = \{X, \phi, \{a\}, \{b, c\}$ and $\sigma = \{Y, \phi, \{a\}, \{b, c\}$. Let $f: (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = b$, $f(b) = c$, $f(c) = a$. Clearly f is not $\widehat{S_P}^*$-continuous, since {b, c} is open in (Y, σ) but $f^{-1}(\{b, c\}) = \{a, b\}$ which is not $\widehat{S_P}^*$-open set in (X, τ), however f is semi-pre-continuous.

Theorem 3.10 Every continuous function in a topological space is $\widehat{S_P}^*$-continuous.
Proof: Let $f: X \to Y$ be continuous. Let V be an open set in Y. Then $f^{-1}(V)$ is open in X. But every open set is $\widehat{S_P}^*$-open. Therefore, $f^{-1}(V)$ is $\widehat{S_P}^*$-open in X. Hence f is $\widehat{S_P}^*$-continuous.

Remark 3.11 The converse of the above theorem need not be true as can be seen from the following example.

Example 3.12 Let $X = \{a, b, c, d\} = Y$ and $\tau = \{X, \phi, \{a, b\}, \{a, b, c\} = \sigma$. Let $f: (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = b$, $f(b) = b$, $f(c) = a$, $f(d) = c$. Clearly f is not continuous, since {a, b, c} is an open set of (Y, σ) but $f^{-1}(\{a, b, c\}) = \{d, a, b\}$ which is not open in (X, τ), however f is $\widehat{S_P}^*$-continuous.

Theorem 3.13 Every α-continuous functions in a topological space is $\widehat{S_P}^*$-continuous
Proof: Let $f: X \to Y$ be α-continuous. Let V be open set in Y. Then $f^{-1}(V)$ is α-open in X. But every α-open is $\widehat{S_P}^*$-open in X. Hence f is $\widehat{S_P}^*$-continuous.

Remark 3.14 The converse of the above theorem need not be true as can be seen from the following example

Example 3.15 Let $X = \{a, b, c\} = Y$ and $\tau = \{X, \phi, \{a\}, \{b\}\} = \sigma$. Let $f: (X, \tau) \to (Y, \sigma)$. Clearly f is not α-continuous, since {a} is an open set of (Y, σ) but $f^{-1}(a) = \{b, c\}$ which is not in α-continuous. However, f is $\widehat{S_P}^*$-continuous.

Theorem 3.16 Every semi-α-continuous function in a topological space is a $\widehat{S_P}^*$-continuous function.
Proof: Let $f: X \to Y$ be semi-α-continuous. Let V be open in Y. Then $f^{-1}(V)$ is semi-α-open in X. Then by theorem 2.5. (iii),$f^{-1}(V)$ is $\widehat{S_P}^*$-open in X. Therefore, f is $\widehat{S_P}^*$-continuous
Theorem 3.17 Every $semi^*g$-continuous function in a topological space is a $\widehat{S_P}^*$- continuous function.
Proof: Let $f: X \to Y$ be $semi^*g$-continuous. Let V be open in Y. Then $f^{-1}(V)$ is $\widehat{S_P}^*$-open in X. Therefore, f is $\widehat{S_P}^*$-continuous.

Remark 3.18 The converse of the above theorem need not be true as can be seen in the following example.

Example 3.19 Let $X = \{a, b, c\} = Y$ and $\tau = \{X, \phi, \{a\}, \{b\}\} = \sigma$. Let $f: (X, \tau) \to (Y, \sigma)$. Clearly f is not $semi^*g$-continuous, since $\{a\}$ is an open set of (Y, σ) but $f^{-1}(a) = \{b, c\}$ which is not $semi^*g$-continuous. However, f is $\widehat{S_P}^*$-continuous.

Theorem 3.20 Let $f: X \to Y$ be $\widehat{S_P}^*$-continuous and $g: Y \to Z$ be continuous. Then the composite $g \circ f: X \to Z$ is $\widehat{S_P}^*$-continuous.
Proof: Let Y be an open set in Z. Since g is continuous, $g^{-1}(V)$ is open in Y. Let $g^{-1}(V)$ is open in Y. Since f is $\widehat{S_P}^*$-continuous, $f^{-1}(V)$ is $\widehat{S_P}^*$ in X. Now $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V)) = f^{-1}(V)$ which is $\widehat{S_P}^*$-open in X. Hence $g \circ f$ is $\widehat{S_P}^*$-continuous.

Remark 3.21 The composition of two $\widehat{S_P}^*$-continuous functions need not be $\widehat{S_P}^*$-continuous.
Let $X = Y = Z = \{a, b, c\}$, $\tau = \sigma = \eta = \{X, \phi, \{a, b\}\}$ and let $f: (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = b$, $f(b) = c$, $f(c) = c$, $g: (Y, \sigma) \to (Z, \eta)$ be defined by $g(a) = a$, $g(b) = b$, $g(c) = c$. Here f and g are $\widehat{S_P}^*$-continuous.

IV. $\widehat{S_P}^*$ - Irresolute Functions

In this section, we define $\widehat{S_P}^*$-irresolute function give characterization for this function and study their properties.

Definition 4.1 A function $f: X \to Y$ is said to be $\widehat{S_P}^*$-irresolute at a point $x \in X$ if for each $\widehat{S_P}^*$-open set V in Y containing $f(x)$, there is a $\widehat{S_P}^*$-open set U of X such that $x \in U$ and $f(U) \subseteq V$.

Definition 4.2 A function $f: X \to Y$ is said to be $\widehat{S_P}^*$-irresolute if $f^{-1}(V)$ is $\widehat{S_P}^*$-open in X for every $\widehat{S_P}^*$-open set V in Y.

Definition 4.3 A function $f: X \to Y$ is said to be contra $\widehat{S_P}^*$-irresolute if $f^{-1}(V)$ is $\widehat{S_P}^*$- closed in X for every $\widehat{S_P}^*$-open V in Y.

Definition 4.4 A function $f: X \to Y$ is said to be strongly $\widehat{S_P}^*$-irresolute if $f^{-1}(V)$ is open in X for every $\widehat{S_P}^*$-open set V in Y.

Definition 4.5 A function $f: X \to Y$ is said to be contra strongly $\widehat{S_P}^*$-irresolute if $f^{-1}(V)$ is closed in X for every $\widehat{S_P}^*$-open set V in Y.

Theorem 4.6

(i) Every $\widehat{S_P}^*$-irresolute function is $\widehat{S_P}^*$-continuous.

(ii) Every contra $\widehat{S_P}^*$-irresolute function is contra $\widehat{S_P}^*$-continuous

(iii) Every strongly $\widehat{S_P}^*$-irresolute function is $\widehat{S_P}^*$-iresolute and hence $\widehat{S_P}^*$-continuous.

(iv) Every contra strongly $\widehat{S_P}^*$-irresolute function is contra $\widehat{S_P}^*$-irresolute

(v) Every constant function is $\widehat{S_P}^*$-irresolute

Proof: (i) Let $f: X \to Y$ be a $\widehat{S_P}^*$-irresolute function. Let V be open in Y. By Theorem 2.5(i), V is $\widehat{S_P}^*$-open. Since f is $\widehat{S_P}^*$-irresolute, $f^{-1}(V)$ is $\widehat{S_P}^*$-open in X. Therefore, f is $\widehat{S_P}^*$-continuous.

(ii) Let $f: X \to Y$ be a contra $\widehat{S_P}^*$-irresolute function. Let V be opn in Y. By Theorem 2.5(i),V is $\widehat{S_P}^*$-open. Since f is contra $\widehat{S_P}^*$-irresolute, $f^{-1}(V)$ is $\widehat{S_P}^*$-closed in X. Therefore, f is contra $\widehat{S_P}^*$-continuous.

(iii) Let $f: X \to Y$ be a strongly $\widehat{S_P}^*$-irresolute function. Let V be $\widehat{S_P}^*$-open in Y. Since f is strongly $\widehat{S_P}^*$-irresolute, $f^{-1}(V)$ is open in X. By Theorem 2.5(i), $f^{-1}(V)$ is open in X. Therefore, f is $\widehat{S_P}^*$-irresolute and by (i), f is $\widehat{S_P}^*$-continuous.

(iv) Let $f: X \to Y$ be a contra strongly $\widehat{S_P}^*$-irresolute function. Let V be $\widehat{S_P}^*$-open in Y. Since f is contra strongly $\widehat{S_P}^*$-irresolute, $f^{-1}(V)$ is closed in X. By Theorem 3.10,$f^{-1}(V)$ is $\widehat{S_P}^*$-closed. Therefore, f is contra $\widehat{S_P}^*$-irresolute.

(v) Let $f: X \to Y$ be a constant function defined by $f(x) = y_0$ for all x in X, where y_0 is a fixed point in Y. If V is a $\widehat{S_P}^*$-open set in Y. Then $f^{-1}(V) = X$ (or) ϕ according as $y_0 \in V$ or $y_0 \notin V$. Thus $f^{-1}(V)$ is $\widehat{S_P}^*$-open in X. Hence f is $\widehat{S_P}^*$-irresolute.

Remark 4.7 The converse of the statements of the above theorem need not be true.

Example 4.8 Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{a, b\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{a, b\}\}$. Define $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = a$, $f(b) = c$, and $f(c) = b$. Then f is $\widehat{S_P}^*$-continuous but not strongly $\widehat{S_P}^*$-continuous.

Theorem 4.9 Every contra $\widehat{S_P}^*$-irresolute function is contra $\widehat{S_P}^*$-continuous.

Proof: Let $f: X \to Y$ be a contra $\widehat{S_P}^*$-irresolute function. Let V be an open set in Y. Since f is contra $\widehat{S_P}^*$-irresolute, $f^{-1}(V)$ is closed in X. Hence f is contra $\widehat{S_P}^*$-irresolute, $f^{-1}(V)$ is closed in X. Hence, f is contra $\widehat{S_P}^*$-continuous.

Remark 4.10 The converse of the above theorem is not true as shown in the following example.

Example 4.11 Let $X = Y = \{a, b, c, d\}$ and $\tau = \sigma = \{X, \phi, \{a\}, \{a, b, c\}\}$.Let $f: (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = d$, $f(b) = c$, $f(c) = b$, and $f(d) = a$. Here f is contra $\widehat{S_P}^*$-continuous but not contra $\widehat{S_P}^*$-irresolute.

Theorem 4.12 For a function $f: X \to Y$ the following statements are equivalent.

(i) f is contra $\widehat{S_P}^*$-irresolute

(ii) The inverse image of each $\widehat{S_P}^*$-closed set in Y is $\widehat{S_P}^*$-open in X.

(iii) For each $x \in X$ and each $\widehat{S_P}^*$-closed set F in Y with $f(x) \in F$, there exists a $\widehat{S_P}^*$-open set U in X such that $x \in U$ and $f(U) \subseteq F$

(iv) $PCl^*\left(int\left(f^{-1}(F)\right)\right) = PCl^*\left(f^{-1}(F)\right)$ for every $\widehat{S_P}^*$-closed set F in Y.

(v) $PInt^*\left(Cl\left(f^{-1}(V)\right)\right) = PInt^*\left(f^{-1}(V)\right)$ for every $\widehat{S_P}^*$-open set V in Y.

Proof: (i) → (ii) Let F be a $\widehat{S_P}^*$-closed in Y. Then $Y \backslash F$ is $\widehat{S_P}^*$-open in Y. Since f is contra $\widehat{S_P}^*$-irresolute, $f^{-1}(Y \backslash F) = X \backslash f^{-1}(F)$ is $\widehat{S_P}^*$-open in Y. Hence $f^{-1}(F)$ is $\widehat{S_P}^*$-open in X. This proves (ii).

(ii) → (iii) Let F be a $\widehat{S_P}^*$-closed in X. Then $U = f^{-1}(F)$ is a $\widehat{S_P}^*$-open set in X. Then there exist $\widehat{S_P}^*$-open U in X such that $x \in U$ and $f(U) \subseteq F$. This proves (iii)

(iii) → (iv) Let F be a $\widehat{S_P}^*$-closed set in Y and $x \in f^{-1}(F)$, then $f(x) \in F$. By our assumption there exists a $\widehat{S_P}^*$-open set U_x in X containing x such that $f(x) \in f(U_x) \subseteq F$ which implies that $x \in U_x \subseteq f^{-1}(F)$. This follows that $f^{-1}(F) = \cup \{U_x : x \in f^{-1}(F)\}$. By Theorem 2.3,$f^{-1}(F)$ is $\widehat{S_P}^*$-open in X. By Theorem 2.6 (iv),$PCl^*(Int\left(f^{-1}(F)\right) = PCl^*\left(f^{-1}(F)\right)$. This proves (iv)

(iv) → (v) Let V be a $\widehat{S_P}^*$-open set in Y. Then $Y \backslash V$ is $\widehat{S_P}^*$-closed in Y. By Assumption, $PCl^*\left(Int\left(f^{-1}(Y \backslash V)\right)\right) = PCl^*\left(f^{-1}(Y \backslash V)\right)$. Taking the complement, we get $PInt^*\left(Cl\left(f^{-1}(V)\right)\right) = PInt^*\left(f^{-1}(V)\right)$. This proves (v).

(v) → (i) Let V be a $\widehat{S_P}^*$-open set in Y. Then by assumption, $PInt^*\left(Cl\left(f^{-1}(V)\right)\right) = PInt^*\left(f^{-1}(V)\right)$. By Theorem 2.9,$f^{-1}(V)$ is $\widehat{S_P}^*$-closed in X. Hence f is contra $\widehat{S_P}^*$-irresolute. This proves (i)

Theorem 4.13 Let $f: X \to Y$ be a function. Then the following statements are equivalent.

(i) f is strongly $\widehat{S_P}^*$-irresolute

(ii) $f^{-1}(F)$ is closed in X for every $\widehat{S_P}^*$-closed set F in Y.

(iii) $f\left(Cl(A)\right) \subseteq \widehat{S_P}^* Cl\left(f(A)\right)$ for every subset A of X.

(iv) $Cl\left(f^{-1}(B)\right) \subseteq f^{-1}\left(\widehat{S_P}^* Cl(B)\right)$ for every subset B of Y.

(v) $f^{-1}\left(\widehat{S_P}^* Int(B)\right) \subseteq Int\left(f^{-1}(B)\right)$ for every subset B of Y.

Proof: (i) → (ii) Let F be a $\widehat{S_P}^*$-closed set in Y. Then $V = Y\backslash F$ is $\widehat{S_P}^*$-open in Y. Then by our assumption, $f^{-1}(V)$ is open in X. Hence, $f^{-1}(F) = X\backslash f^{-1}(V)$ is closed in X. This proves (ii).

(ii) → (i) Let V be a $\widehat{S_P}^*$-open set in Y. Then $F = Y\backslash V$ is $\widehat{S_P}^*$-closed. By (ii) $f^{-1}(F)$ is closed. Hence $f^{-1}(V) = X\backslash f^{-1}(F)$ is open in X. Therefore, f is strongly $\widehat{S_P}^*$-irresolute.

(ii) → (iii) Let $A \subseteq X$. Let F be a $\widehat{S_P}^*$-closed set containing $f(A)$. Then by (ii), $f^{-1}(F)$ is a closed set containing A. This implies that $Cl(A) \subseteq f^{-1}(F)$ and hence $f\left(Cl(A)\right) \subseteq F$. Therefore $f\left(Cl(A)\right) \subseteq \widehat{S_P}^* Cl\left(f(A)\right)$.

(iii) → (iv) Let $B \subseteq Y$ and let $A = f^{-1}(B)$. By assumption, $f\left(Cl(A)\right) \subseteq \widehat{S_P}^* Cl\left(f(A)\right) \subseteq \widehat{S_P}^* Cl(B)$. This implies that $Cl(A) \subseteq f^{-1}\left(\widehat{S_P}^* Cl(B)\right)$. This proves (iv).

(iv) → (ii) Let F be $\widehat{S_P}^*$-closed in Y. Then by Theorem 3.2.2 (ii) $\widehat{S_P}^* Cl(F) = F$. Therefore, (iv) implies $Cl\left(f^{-1}(A)\right) \subseteq f^{-1}(F)$. Hence $Cl\left(f^{-1}(F)\right) = f^{-1}(F)$. Therefore $f^{-1}(F)$ is closed. This proves (ii)

(iv) → (v) The equivalence of (iv) and (v) follows from taking the complements and using results in topology.

Theorem 4.14 For a function $f: X \to Y$, the following statements are equivalent.

(i) f is contra strongly $\widehat{S_P}^*$-irresolute

(ii) The inverse image of each $\widehat{S_P}^*$-closed set in Y is open in X.

(iii) For each $x \in X$ and each $\widehat{S_P}^*$-closed set F in Y with $f(x) \in F$, there exists an open set U such that $x \in U$ and $f(U) \subseteq F$

Proof: (i) → (ii) Let F be a $\widehat{S_P}^*$-closed set in Y. Then $Y\backslash F$ is $\widehat{S_P}^*$-open in Y. Since f is contra strongly $\widehat{S_P}^*$-irresolute, $f^{-1}(F)$ is open in X. This proves (ii).

(ii) → (i) Let U be a $\widehat{S_P}^*$-open set in Y. Then $Y\backslash U$ is $\widehat{S_P}^*$-closed in Y. By assumption, $f^{-1}(Y\backslash U) = X\backslash f^{-1}(U)$ is open in X. Hence $f^{-1}(U)$ is closed in X.

(ii) → (iii) Let F be a $\widehat{S_P}^*$-closed set in Y containing $f(x)$. Then $U = f^{-1}(F)$ is open in X containing x such that $f(U) \subseteq F$

(iii) → (i) Let F be a $\widehat{S_P}^*$-closed set in Y. Let $x \in f^{-1}(F)$. Then $f(x) \in F$. By assumption, there is an open set U_x in X containing x such that $f(x) \in f(U_x) \subseteq F$ which implies that $x \in U_x \subseteq f^{-1}(F)$. Hence, $f^{-1}(F)$ is open in X.

Theorem 4.15 Let $f: X \to Y$ be a $\widehat{S_P}^*$-irresolute. Let $g: Y \to Z$ be $\widehat{S_P}^*$-continuous. Then $g \circ f: X \to Z$ is $\widehat{S_P}^*$-continuous.

Proof: Let W be an open set in Z. Then by $\widehat{S_P}^*$-continuity of g, $g^{-1}(W)$ is $\widehat{S_P}^*$-open in Y. Since f is $\widehat{S_P}^*$-irresolute, $(g \circ f)^{-1}(W) = f^{-1}\left(g^{-1}(W)\right)$ is $\widehat{S_P}^*$-open in X. Hence $g \circ f$ is $\widehat{S_P}^*$-continuous.

Theorem 4.16 Let $f: X \to Y$ and $g: Y \to Z$ be $\widehat{S_P}^*$-irresolute then $g \circ f: X \to Z$ is $\widehat{S_P}^*$-irresolute.

Proof: Let W be a $\widehat{S_P}^*$-open set in Z. Since g is $\widehat{S_P}^*$-irresolute $g^{-1}(W)$ is $\widehat{S_P}^*$-open in Y. Since f is $\widehat{S_P}^*$-irresolute, $(g \circ f)^{-1}(W) = f^{-1}\left(g^{-1}(W)\right)$ is $\widehat{S_P}^*$-open in X. Hence $g \circ f$ is $\widehat{S_P}^*$-irresolute.

Theorem 4.l7 Let $f: X \to Y$ be $\widehat{S_P}^*$-irresolute and $g: Y \to Z$ be contra $\widehat{S_P}^*$-continuous. Then $g \circ f: X \to Z$ is contra $\widehat{S_P}^*$-continuous.

Proof: Let W be an open set in Z. Since g is contra $\widehat{S_P}^*$-continuous, $g^{-1}(W)$ is $\widehat{S_P}^*$-closed in Y. Since f is $\widehat{S_P}^*$-irresolute, then by Theorem 4.3.6 (ii), $(g \circ f)^{-1}(W) = f^{-1}\left(g^{-1}(W)\right)$ is $\widehat{S_P}^*$-continuous

Theorem 4.18 Let $f: X \rightarrow Y$ be $\widehat{S_P}^*$-irresolute and $g: Y \rightarrow Z$ is contra $\widehat{S_P}^*$-irresolute. Then $g \circ f: X \rightarrow Z$ is contra $\widehat{S_P}^*$-irresolute.

Proof: Let V be a $\widehat{S_P}^*$-open set in Z. Since g is contra $\widehat{S_P}^*$-irresolute, $g^{-1}(V)$ is $\widehat{S_P}^*$-closed in Y. Since f is $\widehat{S_P}^*$-irresolute $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\widehat{S_P}^*$-closed in X. Hence, $g \circ f$ is contra $\widehat{S_P}^*$-irresolute.

Theorem 4.19 Let $f: X \rightarrow Y$ be contra $\widehat{S_P}^*$-irresolute and let $g: Y \rightarrow Z$ be $\widehat{S_P}^*$-irresolute. Then $g \circ f: X \rightarrow Z$ is contra $\widehat{S_P}^*$-irresolute.

Proof: Let V be $\widehat{S_P}^*$-open set in Z. Since g is $\widehat{S_P}^*$-irresolute, $g^{-1}(V)$ is $\widehat{S_P}^*$-open in Y. Since f is contra $\widehat{S_P}^*$-irresolute, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\widehat{S_P}^*$-closed in X. Hence $g \circ f$ is contra $\widehat{S_P}^*$-irresolute.

Theorem 4.20 Let $f: X \rightarrow Y$ be contra $\widehat{S_P}^*$-irresolute. Let $g: Y \rightarrow Z$ be contra $\widehat{S_P}^*$-irresolute then $g \circ f: X \rightarrow Z$ is $\widehat{S_P}^*$-irresolute.

Proof: Let V be $\widehat{S_P}^*$-open set in Z. Since g is contra $\widehat{S_P}^*$-irresolute, $g^{-1}(V)$ is $\widehat{S_P}^*$-closed in Y. Since f is contra $\widehat{S_P}^*$-irresolute, by Theorem 4.3.1 (ii), $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\widehat{S_P}^*$-open in X. Hence $g \circ f$ is $\widehat{S_P}^*$-irresolute.

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