International Journal of Mathematics and Statistics Invention (IJMSI)
e-ISSN: 2321 – 4767 P-ISSN: 2321 - 4759
www.ijmsi.org Volume 13 Issue 3 || May - June 2025 || PP: 79-88

"A Theoretical Analysis of Bile Flow Dynamics in The Diseased Cystic Duct with Axial Varying Viscosity: Casson Model"

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ABSTRACT
The way bile flows through our bile system is important - when it gets blocked or flows poorly, it can cause diseases. This study discovers how overlapping stones in the cystic duct affect bile flow characteristics through the Casson model. The expression for the flow characteristics including resistance, wall shear stress, and throat stress has been analyzed. The result shows that as the size of stone and the core radius increases, the resistance to flow and the shear stress also increases. It is also observed that as the axial viscosity increases then the resistance to flow and the shear stress also increases.

***Keywords**:Bile flow, Non-Newtonian fluid, Gallstone, Shear Stress, Viscosity.*

Date of Submission: 15-06-2025 Date of acceptance: 30-06-2025

I. INTRODUCTION

The gallbladder is a small, pear-shaped organ that stores and releases bile. Bile is a fluid produced by the liver that helps digest food. Before a meal begins, the gallbladder is full of bile. When we start eating, the gallbladder receives signals to contract and pushes the stored bile through the biliary tract. The bile then flows to the largest bile duct, called the common bile duct. Bile passes through the common bile duct into the duodenum, which is the first part of the small intestine, where it mixes with food waiting to be digested. After we eat a meal, your gallbladder is empty and resembles a deflated balloon, waiting to be filled again.

Cholelithiasis, a condition characterized by the formation of gallstones, is one of the most frequently occurring biliary diseases in humans. Standard medical intervention for this condition usually requires cholecystectomy, surgical gallbladder removal, which has become the most commonly performed abdominal surgery in Western countries. Understanding the mechanical properties of the human biliary system is important for understanding the etiology of biliary pathologies. The human biliary system functions to produce, store, transport, and discharge bile into the duodenum to facilitate the digestion of fats. This system encompasses both organs and a ductal network. Bile, the circulating fluid within the biliary system, is a yellowish-brown secretion produced by the liver, consisting of three primary components: cholesterol, bile salts, and bilirubin. Gallstones develop when bile components become supersaturated due to gallbladder dysfunction, resulting in the precipitation of solid crystalline structures. The primary biliary pathologies include: Cholelithiasis, defined as gallstone formation, and Cholecystitis, which involves inflammatory processes affecting the gallbladder. Patients diagnosed with remaining gallstones and post-cholecystectomy syndrome (PCS) need surgery right away to ease symptoms and stop serious problems. If left untreated, they risk life-threatening conditions like Mirizzi syndrome, recurring inflammation of the bile duct, bile duct cysts, and cancer [9, 14].

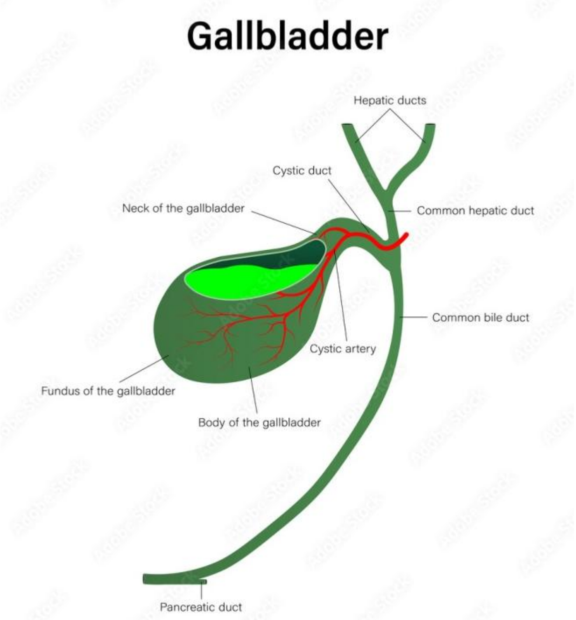


Fig.1 Gallbladder and Cystic Duct

Aloraini et al. [27] reported a challenging case in which a patient developed obstructive jaundice after laparoscopic cholecystectomy. MRCP imaging revealed that the patient had a long cystic duct with a low insertion variant and common hepatic duct compression due to an impacted stone. After multiple ERCP attempts, the stones were finally cleared successfully using spyglass-guided electrohydraulic lithotripsy and balloon sweep technique. Kuchumov et al. [22] developed an analytical model for describing pathological bile flow in the major duodenal papilla duct with a calculus. Agarwal et al. [23] studied the flow of bile in the elastic cystic duct during the emptying phase, finding that with increased viscosity of gallbladder bile, the flow of bile in the duct decreases. Agarwal et al. [24] presented the effect of an overlapping stone in the cystic duct on the flow characteristics of bile. The study presented a mathematical model considering a two-fluid model of bile, consisting of a core region of suspended bile salts, cholesterol, and bilirubin, and a peripheral layer of mucosa. It was observed that as the peripheral viscosity decreases, both the resistance to flow and the shear stress also decrease. Li et al. [21] proposed a fully nonlinear approach to estimate the mechanical properties of human gallbladder wall muscles from in vivo ultrasound images. This provides further understanding of the nonlinear characteristics of the human gallbladder. Yusupovna et al. [29] provided an overview of the mechanisms, experimental findings, and future directions for using corn silk in gallstone disease management. Agarwal et al. [18] described the effect of plug flow in the cystic duct on the flow characteristics of bile through the casson fluid. It was observed that as the size of the stone and the core radius increase, both the resistance to flow and the shear stress also increase. Jungst et al. [3], cholesterol supersaturation and accelerated nucleation processes within bile constitute fundamental mechanisms in the development of cholesterol gallstones. Ooi et al. [5] performed computational analysis investigating steady-state flow patterns in human cystic duct configurations. Their research used simplified models incorporating offset barriers within the channels to simulate Heister's valve and luminal structures. Al-Atabi et al. [7] developed mathematical frameworks incorporating geometric effects of the barrier arrangement, enabling pressure drop predictions in circular ducts equipped with segmental barriers. Bird et al. [8] documented geometric variations within the cystic ducts through acrylic resin casting techniques applied to the neck region and initial cystic duct segment from a gallbladder removed due to gallstone pathology. Li et al. [10] employed dual one-dimensional modeling approaches to calculate pressure differences in the healthy human biliary system when the Reynolds number reaches 20. Increased pressure gradients during bile discharge and refilling processes can cause incomplete bile evacuation, resulting in bile stasis and subsequent gallstone development. Their investigation examined how the structure of the biliary system, elasticity of the cystic duct, and viscosity of bile affect pressure variations. The results indicated that the maximum pressure drop occurs during postprandial biliary emptying and is significantly affected by bile viscosity and geometric characteristics of the cystic duct. Luo et al. [11] investigated physiological and pathological biliary system functions, proposing that mechanical elements contribute significantly to gallstone formation and biliary disease mechanisms. Li's et al. [12] established mechanical models for human biliary systems during emptying phases using clinical assessments measuring modifications in gallbladder volume in response to standardized stimuli and documented pain patterns. Agrawal et al. [17] performed theoretical analysis of bile flow characteristics through stone-containing cystic ducts, modeling bile as Herschel-Bulkley fluid. Amaral et al. [4] investigated to determine whether muscular defects could be responsible for gallbladder dysfunction, noting that the mechanisms behind gallbladder emptying abnormalities in patients with chronic acalculous gallbladder disease remain unclear. Avril et al. [15] presented a new approach for the bi-axial characterization of in vitro human arteries and proved its feasibility through an example. The specificity of the approach is that it can handle heterogeneous strain and stress distributions in arterial segments. The obtained results are promising, and the approach can effectively provide relevant values for the anisotropic hyperelastic properties of the tested sample. Al-Atabi et al. [19] worked on the coupling of secondary flow effects and bile rheology to provide flow resistance. The convoluted nature of the studied cystic ducts resulted in strong secondary flow that contributed towards a dimensionless pressure drop that is four times higher than that of a straight circular tube of equivalent length and average diameter. Behar et al. [2] studied the effect of cholecystokinin (CCK) and the octapeptide of cholecystokinin (OP-CCK) on the feline gallbladder and sphincter of Oddi. Tao peng et al. [25] study the influence of stricture on bile flow dynamics using numerical methods and compared the influence of stricture severity, stricture length, eccentricity, and bile flow property on the bile flow dynamics. Kuchumov et al. [20] presents an experimental study of diseased human bile taken from the gallbladder and bile ducts. It also presented results on hysteretic bile behavior in loading–unloading tests, which proved that pathological bile is a non-Newtonian thixotropic fluid. M. Baghaei et al. [26] studied about the Ascertaining bile secretion against a total CBD obstruction. It also determines the effect of changes in each parameter on the pressure rise. This model is the first numerical step toward understanding the pathogenesis of a completely disrupted CBD and the complications resulting from deformation of the CBD in general. Young [1] purpose of this study is to investigate how time-dependent arterial narrowing (stenosis) affects blood flow patterns and mechanical forces within blood vessels. The research aims to understand the complex interaction between growing atherosclerotic plaques or abnormal tissue growths and flowing blood, particularly focusing on how even small blockages can trigger important biological responses such as cell proliferation and vessel diameter changes. The study uses mathematical analysis using simplified Navier-Stokes equations to investigate how these growths alter the pressure distribution and wall shear stress in the tubes, ultimately attempting to provide insights into the biological implications of stenosis development in the cardiovascular system. Peng et al. [28] reviewed bile dynamics and microfluidic detection techniques, highlighting non-Newtonian (Casson-like) bile viscosity's role in cholesterol gallstone nucleation. Li et al. [13] extended their previous study of the human biliary system to include two new factors: the non-Newtonian properties of bile and elastic deformation of the cystic duct. A one-dimensional (1D) model was analyzed and compared with three-dimensional fluid-structure interaction simulations. It was found that non-Newtonian bile increases resistance to bile flow, which can be increased enormously by the elastic deformation (collapse) of the cystic duct.

It has been demonstrated that cholecystokinin (CCK) stimulation activates coordinated contractions in the biliary system involving the gallbladder, cystic duct, and common bile duct. This peristaltic action propels bile from the gallbladder reservoir through the cystic duct pathway and subsequently into the common bile duct. Notably, CCK sensitivity exhibits a decreasing gradient from the gallbladder to the common bile duct, which affects regional flow dynamics. The present study investigates axial viscosity effects on bile transport within the cystic duct, specifically examining how viscous properties affect flow resistance and wall shear stress distribution. This investigation specifically addresses bile flow behavior in diseased cystic ducts, where bile exhibits non-Newtonian characteristics accurately modeled by the Casson fluid framework.

Formulations and solution of the problem

The mathematical analysis of bile flow through the rigid cystic duct is described by the following governing equation is [30] -

$$-\frac{dP}{dz} = \frac{1}{r}\frac{d(r\tau)}{dr} \qquad (1)$$

For modeling bile as a Casson fluid, the constitutive relationship is expressed as

$$-\frac{du_z}{dr} = \frac{1}{\mu(z)}\left(\tau^{\frac{1}{2}} - \tau_0^{\frac{1}{2}}\right)^2 \qquad : \tau \geq \tau_0$$

$$\frac{du_z}{dr} = 0 \qquad : \tau \leq \tau_0 \qquad (2)$$

In equations (1) and (2) z is the coordinate along the axis of the cystic duct in the flow direction, r is the coordinate in the radial direction and perpendicular to the fluid flow, τ represents the shear stress of the bile which is considered as Casson fluid, P is the pressure at any point, (-dP/dz) is the pressure gradient, u_z stands for the axial velocity of the bile, τ_0 is the yield stress. The axial viscosity is given by

$\mu(z) = \mu_1 \left(\frac{R(z)}{R_0}\right)^m$; $L_1 \leq z \leq L_1+L_0$

$= \mu_1$; otherwise (3)

We are assuming that the viscosity variation along the radius direction is laminar, steady and axially symmetric, where $\mu_1$is constant viscosity. Also, the viscosity $\mu(z)$ is the function of $R(z)/R_0$, where R_0 is the radius of the cystic duct and m is any arbitrary constant parameter which is the index of viscosity variation, where m = 0,1,2,3…..

2A. Boundary conditions

a) τ is finite at r=0 (4)
b) u_z=0 at r=R(z) (5)
c) $P = P_0$ at $z = 0$ and $P = P_l$ at $z = L$ (6)

Where the equation (4) and (5) are known as regularity and no-slip condition respectively.

2B. Geometry used

The overlapping geometry of the cystic duct in Fig.1 is given by –

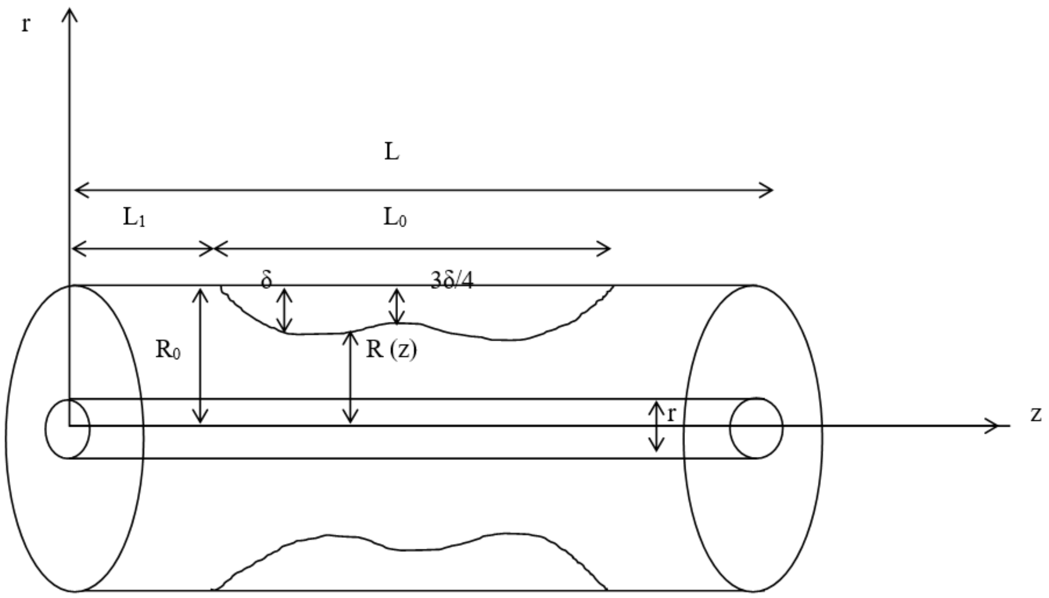


Fig.2 the overlapping geometry

$$\frac{R(z)}{R_0} = 1 - \frac{3\delta}{2R_0L_0^4}[11(z-L_1)L_0^3 - 47(z-L_1)^2L_0^2 + 72(z-L_1)^3L_0 - 36(z-L_1)^4]$$

; $L_1 \leq z \leq L_1+L_0$

=1 ;otherwise (7)

Where R (z) is the radius of the cystic duct in the obstructed region, R_0 is the radius of cystic duct in the normal region, L_1 is the length of cystic duct till the onset of stone, L_0 is the length of stone and L is the length of cystic duct, δ is the maximum height of the stone and bulging at two locations i.e. at $z = L_1 + \frac{L_0}{6}$ and $z = L_1 + \frac{5L_0}{6}$. (V.P.Srivastav et al.) [16].

2C. Analysis of Model and Method of Solution

The model (1) and (2) is solved using boundary condition (4) and (5). On integrating (1) and using (4) the stress component is given by-

$\tau = -\frac{r}{2}\frac{dP}{dz}$ (8)

We can write, $\tau = \frac{pr}{2}$ where $p = -\frac{dP}{dz}$ (9)

Since, in the core region, $0 \leq r \leq r_0$, the yield stress exists and is given by J.N. Mazumdar [6]

$\tau_0 = \frac{pr_0}{2}$, where r_0 is the radius of the core region. (10)

The presence of this yield stress induces the plug flow in the region. Hence, the velocity gradient within the core becomes negligible, i.e. $du_z/dr = 0$.

$\Rightarrow u_z =$ constant

Let $u_z = u_0$ be the velocity of the bile in the core region which shows that it has straight velocity profile. The equation (2) is rearranged as

$$\sqrt{\mu(z)\dot{\gamma}} = \sqrt{\tau} - \sqrt{\tau_0} \tag{11}$$

Where $\dot{\gamma}$= shear strain rate

Also $$\dot{\gamma} = -\frac{du_z}{dr} \tag{12}$$

Squaring in equation (11) and using (12) we get

$$\dot{\gamma} = -\frac{du_z}{dr} = \frac{1}{\mu(z)}\left(\sqrt{\tau} - \sqrt{\tau_0}\right)^2 \tag{13}$$

Now, using equation (9), (10) and (13) we get

$$\frac{du_z}{dr} = \frac{\mathrm{p}}{2\mu(\mathrm{z})}\left(2\sqrt{rr_0} - r - r_0\right) \tag{14}$$

On integration equation (14) we get

$$u_z = \frac{p}{2\mu(z)}\left[\frac{2r^{\frac{3}{2}}*2\sqrt{r_0}}{3} - \frac{r^2}{2} - r_0 r\right] + c \tag{15}$$

On using boundary condition of equation (5), the above equation is given by-

$$u_z = \frac{p}{4\mu(z)}\left[\left((R(z))^2 - r^2\right) - \frac{8}{3}\sqrt{r_0}\left(\sqrt{(R(z))^3} - \sqrt{r^3}\right) + 2\sqrt{r_0}(R(z) - r)\right] \tag{16}$$

Where r represents $r_0 \le r \le R(z)$ and $R=R(z)$

At the core region, $u_z=u_0$ and $r=r_0$, therefore equation (16) can be written as-

$$u_0 = \frac{p}{4\mu(z)}\left[(R(z))^2 + 2R(z)r_0 - \frac{1}{3}r_0^2 - \frac{8}{3}\sqrt{r_0(R(z))^3}\right] \tag{17}$$

Equation (17) describes the velocity distribution across the entire range from the center (r=0) to the wall ($r=r_0$). Equation (16) can be rewritten as

$$u_z = u_0 - \frac{p}{4\mu(z)}\left[r^2 + 2rr_0 - \frac{1}{3}r_0^2 - \frac{8}{3}\sqrt{r_0 r^3}\right] \tag{18}$$

The flux Q is given by

Q = flux in core region + flux outside of core region

$$Q = \pi(R(z))^2 u_0 - \frac{p\pi(R(z))^4}{\mu(z)}\left[\frac{1}{8}\left(1 - \left(\frac{r_0}{R(z)}\right)^4\right) + \frac{r_0}{3R(z)}\left(1 - \left(\frac{r_0}{R(z)}\right)^3\right) - \frac{8}{21}\sqrt{\frac{r_0}{R(z)}}\left(1 - \left(\frac{r_0}{R(z)}\right)^{\frac{7}{2}}\right) - \frac{1}{12}\left(\frac{r_0^2}{(R(z))^2}\right)\left(1 - \left(\frac{r_0^2}{(R(z))^2}\right)\right)\right]$$

Let $\frac{r_0}{R(z)} = a_0$

$$Q = \frac{p\pi(R(z))^4}{8\mu(z)}\left[1 + \frac{4}{3}a_0 - \frac{16}{7}a_0^{\frac{1}{2}} - \frac{1}{21}a_0^4\right] \tag{19}$$

Therefore, $$Q = \frac{p\pi(R(z))^4}{8\mu(z)}f(a_0) \tag{20}$$

Where, $$f(a_0) = f(a_0(z)) = \left[1 + \frac{4}{3}a_0 - \frac{16}{7}a_0^{\frac{1}{2}} - \frac{1}{21}a_0^4\right] \tag{21}$$

Case study: when $\tau_0 = 0$ (i.e. there is no yield stress), then the Casson equation (11) reduces to $\tau = \mu(z)\dot{\gamma}$, which is the Newtonian case.
Therefore, τ_0=0 i.e a_0=0 into equation (20) we get

$Q = Q_0 = \frac{p\pi}{8\mu(z)}(R(z))^4$, which corresponds with poiseullie equation which says $Q = \frac{(p_1-p_2)}{L}\frac{\pi}{8\mu(z)}(R(z))^4$, where Q_0 is flux at center (i.e. r=0). Thus we can take Poiseullie model as a particular case of Casson's model.
On integrate equation (19) and using equation (5) & (3) we get

$$p_0 - p_L = \frac{8Q\mu_1 L}{\pi R_0^4 f_0}\left[1 - \frac{L_0}{L} + \frac{f_0}{L}\int_{L_1}^{L_1+L_0}\frac{\left(\frac{R(z)}{R_0}\right)^m}{f(a_0)\left(\frac{R(z)}{R_0}\right)^4}dz\right], \tag{22}$$

Where $f_0 = \left[1 + \frac{4}{3}\left(\frac{r_0}{R_0}\right) - \frac{16}{7}\left(\frac{r_0}{R_0}\right)^{\frac{1}{2}} - \frac{1}{21}\left(\frac{r_0}{R_0}\right)^4\right]$

The resistance to flow

$$\lambda = \frac{p_0 - p_L}{Q} = \frac{8\mu_1 L}{\pi R_0^4 f_0}\left[1 - \frac{L_0}{L} + \frac{f_0}{L}\int_{L_1}^{L_1+L_0}\frac{1}{f(a_0)}\left(\frac{R(z)}{R_0}\right)^{m-4}dz\right] \tag{23}$$

In the absence of any stone (i.e. δ=0), the resistance to flow λ_N is given by

$$\lambda_N = \frac{8\mu_1 L}{\pi R_0^4 f_0} \tag{24}$$

The flow resistance due to the presence of stone in the cystic duct can be written as

$$\bar{\lambda} = \frac{\lambda}{\lambda_N} = \left[1 - \frac{L_0}{L} + \frac{f_0}{L}\int_{L_1}^{L_1+L_0}\frac{1}{f(a_0)}\left(\frac{R(z)}{R_0}\right)^{m-4}dz\right] \tag{25}$$

Wall shear stress,

Using (8) we get
$\tau = -\frac{r}{2}\frac{dP}{dz}$at r=R(z)=R
On substitute the value of $p = -\frac{dP}{dz}$ from equation (20) we get

$$\tau_R = \frac{4\mu(z)Q}{\pi(R(z))^3 f(a_0)} = \frac{4\mu_1 Q\left(R(z)\right)^{m-3}}{\pi R_0^m f(a_0)}$$

When there is no stone ($R(z)=R_0$ and $f(a_0)=f_0$) in the cystic duct

$$\Rightarrow \tau_N = \frac{4\mu_1 Q}{\pi R_0^3 f_0}$$

Now the non-dimensional shear stress is given by

$$\Rightarrow \tau_1 = \frac{\tau_R}{\tau_N} = \left(\frac{R(z)}{R_0}\right)^{(m-3)}\left(\frac{\left[1+\frac{4}{3}\left(\frac{r_0}{R_0}\right)-\frac{16}{7}\left(\frac{r_0}{R_0}\right)^{\frac{1}{2}}-\frac{1}{21}\left(\frac{r_0}{R_0}\right)^4\right]}{\left[1+\frac{4}{3}\left(\frac{r_0}{R(z)}\right)-\frac{16}{7}\sqrt{\left(\frac{r_0}{R(z)}\right)}-\frac{1}{21}\left(\frac{r_0}{R(z)}\right)^4\right]}\right) \tag{26}$$

τ_2 is the non-dimensional value of shear stress at the critical height of stone and is given by

$$\tau_2 = \frac{f_0}{f(c_0)}\left(\frac{R(z)}{R_0}\right)^{(m-3)} = \left(1 - \frac{3}{4}\frac{\delta}{R_0}\right)^{(m-3)}\left(\frac{\left[1+\frac{4}{3}\left(\frac{r_0}{R_0}\right)-\frac{16}{7}\left(\frac{r_0}{R_0}\right)^{\frac{1}{2}}-\frac{1}{21}\left(\frac{r_0}{R_0}\right)^4\right]}{\left[1+\frac{4}{3}\left(\frac{r_0}{R(z)}\right)-\frac{16}{7}\sqrt{\left(\frac{r_0}{R(z)}\right)}-\frac{1}{21}\left(\frac{r_0}{R(z)}\right)^4\right]}\right) \tag{27}$$

Similarly,

$$\tau_3 = \frac{f_0}{f(c_0)}\left(\frac{R(z)}{R_0}\right)^{(m-3)} = \left(1 - \frac{5}{4}\frac{\delta}{R_0}\right)^{(m-3)}\left(\frac{\left[1+\frac{4}{3}\left(\frac{r_0}{R_0}\right)-\frac{16}{7}\left(\frac{r_0}{R_0}\right)^{\frac{1}{2}}-\frac{1}{21}\left(\frac{r_0}{R_0}\right)^4\right]}{\left[1+\frac{4}{3}\left(\frac{r_0}{R(z)}\right)-\frac{16}{7}\sqrt{\left(\frac{r_0}{R(z)}\right)}-\frac{1}{21}\left(\frac{r_0}{R(z)}\right)^4\right]}\right) \tag{28}$$

$\tau_3$is shown the non-dimensional value of shear stress at stone throat.

II. RESULTS AND DISCUSSION

The mathematical equations (25), (26), (27) and (28) from the previous section were computed and the results are shown in several graphs.

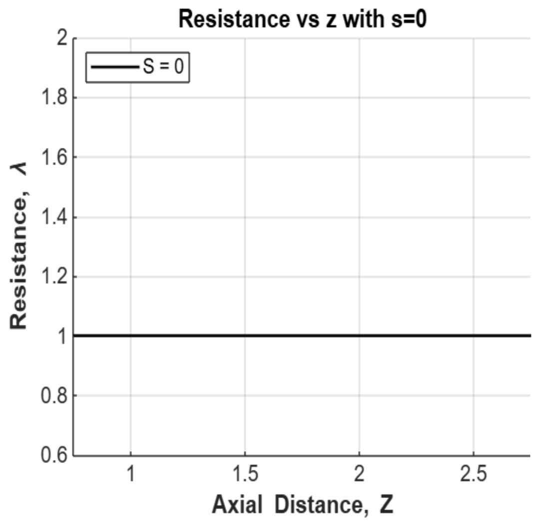
Figure 3 shows the variation in resistance to flow with the axial distance in the non-uniform region. When the stone height increases resistance to flow also increases. When there is no stone (i.e. δ=0) the resistance remains constant. It is also observed that when the core radius and axial viscosity (for arbitrary constant m which is the index of viscosity variation) increases the flow of resistance also increases.

Figure 4 depicts the variation in resistance to flow with the axial distance in the non-uniform region of the diseased cystic duct for different values of stone length. It is noticed that as the length of the stone increases the flow of resistance also increases.

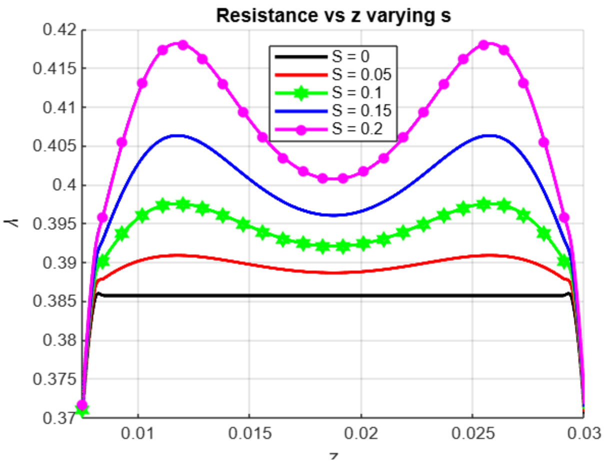
Figure 5 depicts the variation in the shear stress (the force on the duct walls) with the axial distance. When stone height and core radius increases then the shear stress also increases. When the axial viscosity (for arbitrary constant m which is the index of viscosity variation) increases then shear stress also increases.

Figure 6 depicts the variation in wall shear stress (τ_2) with stone height for different values of core radius and axial viscosity (for arbitrary constant m which is the index of viscosity variation). When stone height and the axial viscosity increases, the shear stress also increases.

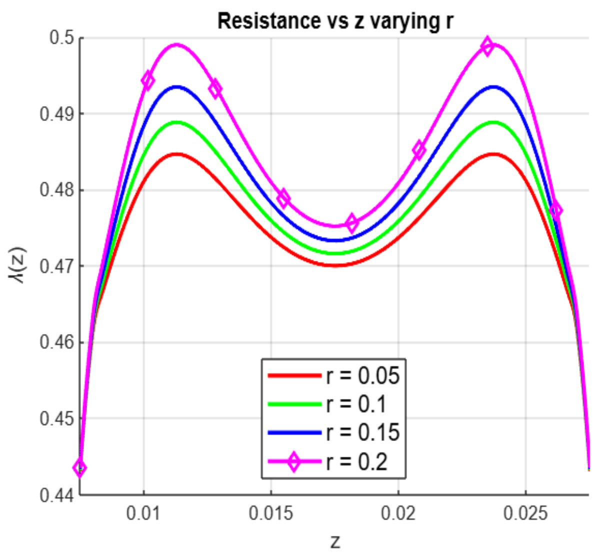
Figure 7 depicts the variation in wall shear stress (τ_3) with stone height for different values of core radius and axial viscosity (for arbitrary constant m which is the index of viscosity variation). When stone height and the axial viscosity increases there also the shear stress increases.

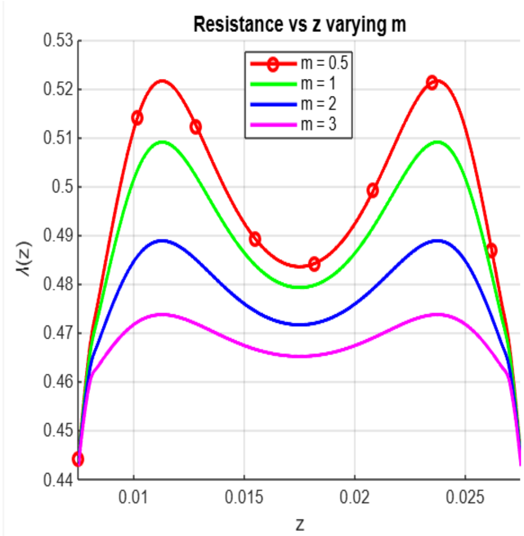


(a)



(b)





(c) (d)

Figure 3: The variation of resistance to flow with the axial distance in the non-uniform region of the cystic duct

a) For stone height S=0

b) For variable stone height (s)

c) For variable core radius (r)
d) For varying axial viscosity (arbitrary constant m)

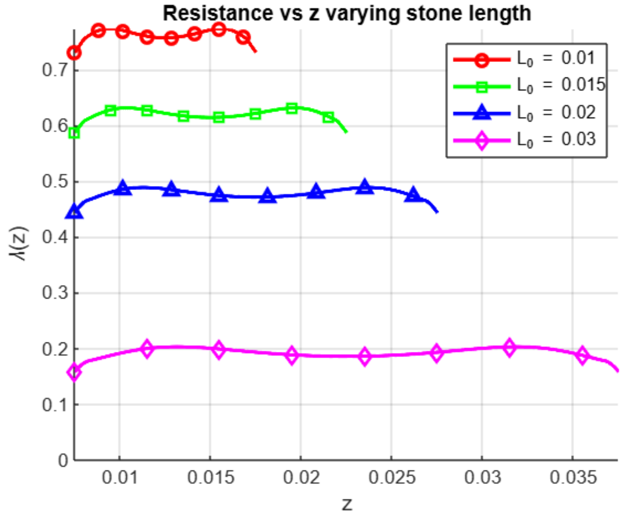
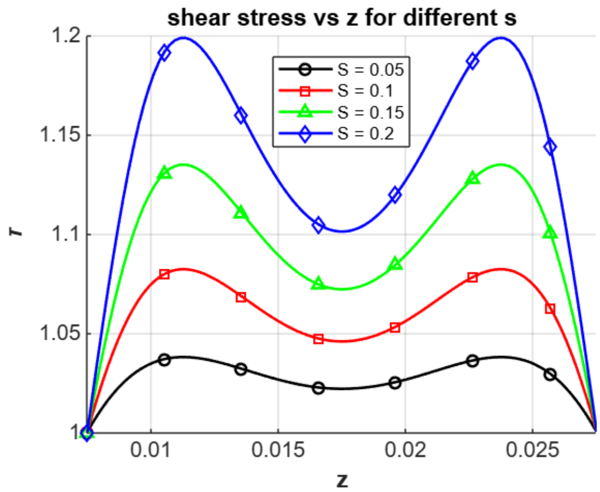
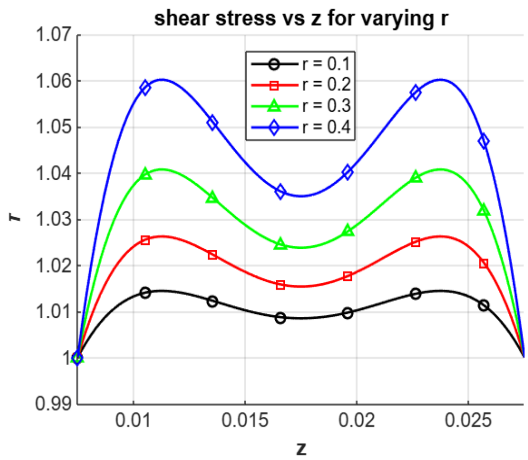


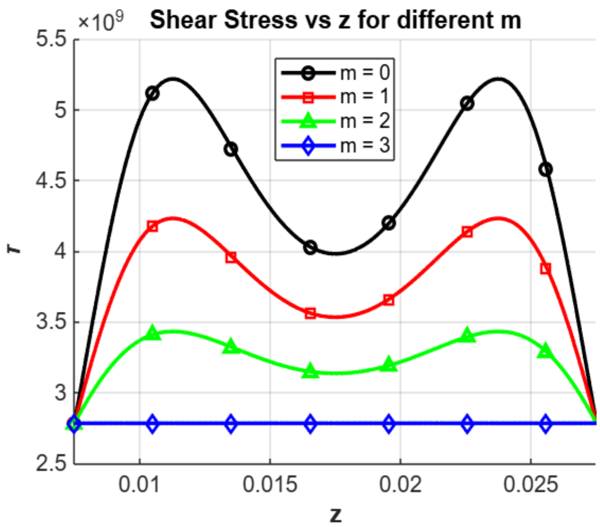
Figure 4: The variation of resistance to flow with the axial distance in the non-uniform region of the cystic duct for variable stone lengths (L_0)



(a)

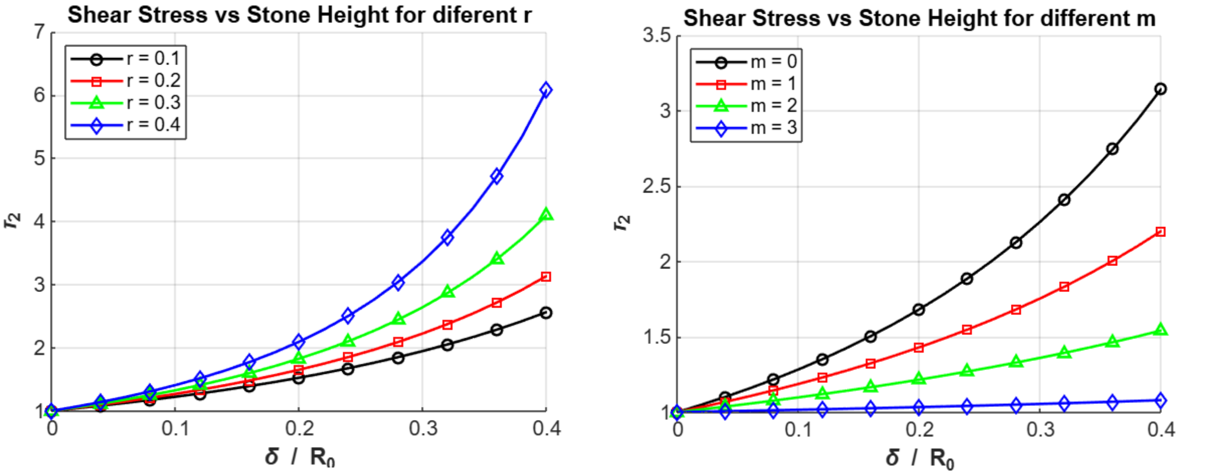


(b)



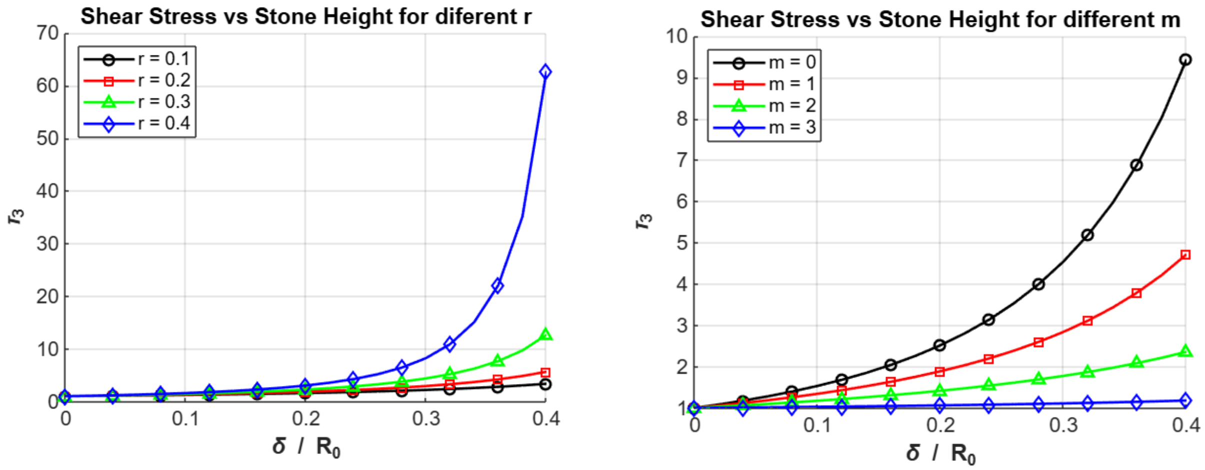
(c)

Figure 5: The variation of shear stress with the axial distance in the non-uniform region of the cystic duct.
a) For stone height(s)
b) For plug core radius(r)
c) For varying axial viscosity (arbitrary constant m)



(a) (b)

Figure 6: The variation of shear stress (τ_2) with the stone height
a) For different values of the core radius (r)
b) For varying axial viscosity (different values of m)



(a) (b)

Figure 7: The variation of shear stress (τ_3) with the stone height
a) For different values of the core radius (r)
b) For varying axial viscosity (different values of m)

III. CONCLUSIONS

A model has been developed to study bile flow pattern through the cystic duct in the presence of gallstones. This study investigates the characteristics of bile flow through a diseased cystic duct. The presence of yield stress causes plug flow in the center region, here bile behaves as a Casson fluid. Under specific conditions, shear stress and flow resistance have been calculated. The calculated results are shown in graphs for different parameters like m, r, δ, μ_1 and L_0, those stands respectively for arbitrary constant, plug core radius, stone height, dynamic viscosity and stone length. This model investigates how shape of the stone and core radius affect flow resistance and wall shear stress. It is observed that shear stress and resistance increases in the presence of varying axial viscosity. It is also observed that the resistance and shear stress increases as the size (height and length) of stone and radius increases.

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