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The Sum of Fibionacci Series

Najmuj Sahadat Sarkar*
Mahishadal Raj College, 6295454694, West Bengla, India.

Abstract:
In this paper i try to showed how to calculate the sum of Fibonacci digits or series, inspired by ramanujan infinite sum of natural numbers. I used so many mathematical operation based on numbers.
Keywords: *Fibonacci, series, operation,*

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I. Introduction:

Fibionacci series is one of greatest mathematical series discovered by mathematician fibionacci in 16th century. Our physical world deeply influence or moved by fibionacci numbers. The spirals in sunflower, spirals in our galaxy, ocean waves, even in stock market, the fibionacci series and its complicated structures takes an important role. Nature know math, and its totally mathematical. I just find a way to calculate the sum of fibionacci digits. Here is the brief proof.

Proof:

First considered the Fibonacci sequence which may define as f_n.

f_n=1,1,2,3,5,8,13,21,34,55,89,144,233,377,
610,987,1597,2584,.....

Another important series that known as Lucas series, can be defined as L_n.

L_n=2,1,3,4,7,11,18,29,47,76,123,199,322,
521,843,1364,.......

Now we substract f_n from L_n, so we shall get:

$N_n=L_n-f_n$
=(2,1,3,4,7,11,18,29,47,76,123,199,322,
521,843,1364,.......)−(1,1,2,3,5,8,13,21,34,55,89,144,233,377,610,987,1597,2584,.....)

N_n=1,0,1,1,2,3,5,8,13,21,34,55,89,144,233,
377,610,.......

Let $S(N_n)$ is the sum of new number series, which define as:

$S(N_n)$=1+0+1+1+2+3+5+8+13+21+34+55+89+144+233+377+610+.....

$S(N_n)=1+S(f_n)$

$S(N_n)-S(f_n)=1$ equation [A]

Let looked at $S(N_n)$, we looked at there's digit, which my define as two ways (let i choose for my calculations).

1+0=1
1+1=2
2+3=5
5+8=13
13+21=34
34+55=89
89+144=233

So, $S(N_n)$=1+2+5+13+34+89+233+610+......

Another way we can write $S(N_n)$ such as:

1=1
1+0=1
1+2=3
3+5=8
8+13=21
21+34=55
55+89=144

So, $S(N_n)$=1+1+3+8+21+55+144+... equation [B]

Here i try to construct another number series, which is known as $S(P_n)$.

$S(P_n)$=1+3+8+21+55+144+......

Put that value of $S(P_n)$ into the equation [B], we shall get:

$S(N_n)=1+S(P_n)$

$S(N_n)-S(P_n)=1$ equation [C]

Now, we substract $S(P_n)$ from $S(N_n)$, here is the arithmetic operations in different way:

$S(N_n)-S(P_n)$

=(1+1+3+8+21+55+144+377+987+...)−(1+3+8+21+55+144+377+987+.....)

=0−(2+5+13+34+89+233+610+....)

=1−(1+2+5+13+34+89+233+610+....)

The, substraction can we define as:

1−1=0
1−3=−2
3−8=−5
8−21=−13
21-55=−34
55-144=−89
144−377=−233
377−987=−610

So, $S(N_n)-S(P_n)$

=1−(1+2+5+13+34+89+233+610+....)

$S(N_n)-S(P_n)=1-S(N_n)$

$2S(N_n)=1+S(P_n)$

$S(N_n)=[1/2]+[1/2]\cdot S(P_n)$ equation [D]

Notice, that $S(N_n)$=1+2+5+13+...., $S(N_n)$=1+1+3+8+21+...., the both series are different numerical manifestation of the same thing "The Sum Of Fibonacci Series".

So if we add $S(N_n)$ and $S(N_n)$, we shall get:

$S(N_n)+S(N_n)$

$=(1+2+5+13+34+89+233+610+...)+1+(1+3+8+21+55+144+377+987+....)$

$2S(N_n)=1+(1+1+2+3+5+8+13+21+34+55+ 89+.......)$

$S(N_n)=1+S(f_n)$

$S(N_n)=[1/2]+[1/2]\cdot S(f_n)$ equation [E]

According to the equation[D], if we compare with equation [E], we shall get:

$S(N_n)=[1/2]+[1/2]\cdot S(P_n)$

$[1/2]+[1/2]\cdot S(f_n)=[1/2]+[1/2]\cdot S(P_n)$

So, $S(f_n)=S(P_n)$ equation[F]

So, $S(f_n)-S(P_n)=0$

According to the equation[A], we can say:

$S(f_n)=S(N_n)-1$

$S(P_n)=S(N_n)-1$ {As, $S(f_n)=S(P_n)$}

$S(N_n)=S(P_n)+1$ equation[G]

According to equation[D],

$S(N_n)=[1/2]+[1/2]\cdot S(P_n)$

$S(P_n)+1=[1/2]+[1/2]\cdot S(P_n)$

$2S(P_n)+2=1+S(P_n)$

$S(P_n)=-1$ equation[H]

Put that equation[H] value, into the equation[F], we get:

$S(P_n)=S(f_n)=-1$ (proved)

So, the final result, "The Sum Of Fibonacci" digits is tends to (-1).

Conclusion:

At the first place, its looked like very odd, but remember Ramanujan sum of natural numbers, like $[1+2+3+4+5+.... \rightarrow (-1/12)]$. So my consideration on Fibonacci series is also clear and logistic. Its helps us a lot in higher dimenssional theory such as "string or M-theory". So, the final result, "The Sum Of Fibonacci" digits is tends to (-1). $S(f_n)$: $1+1+2+3+5+8+13+21+.... \rightarrow (-1)$.