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Theoretical Applications of Intuitionistic Fuzzy $\widehat{g}^*$ Semi Closed Sets

S. Pious Missier[1], A. Peter Arokiaraj[2*], A. Nagarajan[3] &A. Martina Jency[4]
[1]Head & Associate Professor(Retd.), Department of Mathematics, Don Bosco College of Arts and Science, (Affiliated to ManonmaniamSundaranar University, Abishekabatti, Tirunelveli-672 012) KeelaEral, Thoothukudi-628 908, Tamil Nadu, India.
[2]Research Scholar (Reg.No-19222232091032),PG and Research Department of Mathematics, V.O.Chidambaram College, (Affiliated to ManonmaniamSundaranar University, Abishekabatti, Tirunelveli-672 012), Thoothukudi-628 008,TamilNadu, India.*
3Head &AssistantProfessor(Retd.), PG and Research Department of Mathematics, V. O. Chidambaram College,(Affiliated to ManonmaniamSundaranar University, Tirunelveli-672 012) Thoothukudi-628 008, Tamilnadu, India.
[4]Assistant Professor, PG and Research Department of Mathematics, V. O. Chidambaram College, (Affiliated to ManonmaniamSundaranar University, Tirunelveli-672 012) Thoothukudi-628 008, Tamilnadu, India.

Abstract:*The theoretical applications of Intuitionistic Fuzzy $\widehat{g}^*$ semi closed sets bring forth the emergence of some new spaces, which will be dealt in detail in this section. These new spaces are compared with some other spaces in intuitionistic fuzzy topology.*

Key Words:*Intuitionistic Fuzzy $\widehat{g}$-Open set ($\mathcal{IF}\widehat{g}\mathcal{O}$),Intuitionistic Fuzzy $\widehat{g}^*$ Semi Closed set($\mathcal{IF}\widehat{g}^*s\mathcal{C}$) andIntuitionistic fuzzy $\widehat{g}^*$ semi $T^*_{1/2}$space ($\mathcal{IF}\widehat{g}^*s\ T^*_{1/2}$space)*

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I. INTRODUCTION

In 1965, Prof. Zadeh[13] propagated the theory of fuzzy sets to explain the uncertainties in the mathematical world. Prof. Chang[3] with his illuminating mind proceeded further and introduced fuzzy topology in 1967. Many research studies on the notions of fuzzy sets and fuzzy topology continued to be highlight for several years. Adding to these investigations, there continued a new leap with the Intuitionistic Fuzzy Sets by Atanassov[1] to generalize the fuzzy sets. In 1997, Coker[5] built up the Intuitionistic Fuzzy Topological spaces and paved the way for further development. In recent past Pious Missieret. al introduced a new class of Intuitionistic Fuzzy $\widehat{g}^*$Semi Closed Sets in Intuitionistic Fuzzy Topological Spaces. This paper is an attempt to derive the theoretical applications of Intuitionistic Fuzzy $\widehat{g}^*$Semi Closed Sets by way of bringing out some new spaces and studying their characterizations.

II. PRELIMINARIES

Definition 2.1. [1] Let $\mathbb{X}$ be a universal set. Let $\mathcal{A}_{\text{if}}$ be an object having the following form, i.e., $\mathcal{A}_{\text{if}} = \{\langle x, \mu_{\mathcal{A}_{\text{if}}}(x), \upsilon_{\mathcal{A}_{\text{if}}}(x)\rangle : x \in \mathbb{X}\}$ is called as an intuitionistic fuzzy subset ($\mathcal{IFS}$ in short) $\mathcal{A}_{\text{if}}$ in $\mathbb{X}$. Here the functions $\mu_{\mathcal{A}_{\text{if}}}: \mathbb{X} \to [0,1]$ and $\upsilon_{\mathcal{A}_{\text{if}}}: \mathbb{X} \to [0,1]$ denote the degree of membership (namely $\mu_{\mathcal{A}_{\text{if}}}(x)$) and the degree of non-membership (namely $\upsilon_{\mathcal{A}_{\text{if}}}(x)$) of each element $x \in \mathbb{X}$ to the set $\mathcal{A}_{\text{if}}$ respectively, and $0 \le \mu_{\mathcal{A}_{\text{if}}}(x) + \upsilon_{\mathcal{A}_{\text{if}}}(x) \le 1$ for each $x \in \mathbb{X}$. The set of all intuitionistic fuzzy sets in $\mathbb{X}$ is denoted by $\mathcal{IFS}(\mathbb{X})$. For any two$\mathcal{IFS}s\mathcal{A}_{\text{if}}and\mathcal{B}_{\text{if}}$, $(\mathcal{A}_{\text{if}} \cup \mathcal{B}_{\text{if}})^C = \mathcal{A}_{\text{if}}^C \cap \mathcal{B}_{\text{if}}^C$; $(\mathcal{A}_{\text{if}} \cap \mathcal{B}_{\text{if}})^C = \mathcal{A}_{\text{if}}^C \cup \mathcal{B}_{\text{if}}^C$.

Definition2.2.[1] If $\mathcal{A}_{\text{if}} = \{\langle x, \mu_{\mathcal{A}_{\text{if}}}(x), \upsilon_{\mathcal{A}_{\text{if}}}(x)\rangle : x \in \mathbb{X}\}$ and $\mathcal{B}_{\text{if}} = \{\langle x, \mu_{\mathcal{B}_{\text{if}}}(x), \upsilon_{\mathcal{B}_{\text{if}}}(x)\rangle : x \in \mathbb{X}\}$ be two $\mathcal{IFS}(\mathbb{X})$, then

(a) $\mathcal{A}_{\text{if}} \subseteq \mathcal{B}_{\text{if}}$ if and only if $\mu_{\mathcal{A}_{\text{if}}} \le \mu_{\mathcal{B}_{\text{if}}}$and $\upsilon_{\mathcal{A}_{\text{if}}}(x) \ge \upsilon_{\mathcal{B}_{\text{if}}}(x)$ for all $x \in \mathbb{X}$,

(b) $\mathcal{A}_{\text{if}} = \mathcal{B}_{\text{if}}$ if and only if $\mathcal{A}_{\text{if}} \subseteq \mathcal{B}_{\text{if}}$ and $\mathcal{A}_{\text{if}} \supseteq \mathcal{B}_{\text{if}}$,

(c) $\mathcal{A}_{\text{if}}^{C}$ = {⟨ x, $\upsilon_{\mathcal{A}_{\text{if}}}(x)$, $\mu_{\mathcal{A}_{\text{if}}}(x)$⟩: $x \in \mathbb{X}$} (complement of $\mathcal{A}_{\text{if}}$),
(d) $\mathcal{A}_{\text{if}} \cup \mathcal{B}_{\text{if}}$ = {⟨ x, $\mu_{\mathcal{A}_{\text{if}}}(x) \vee \mu_{\mathcal{B}_{\text{if}}}(x)$, $\upsilon_{\mathcal{A}_{\text{if}}}(x) \wedge \upsilon_{\mathcal{B}_{\text{if}}}(x)$⟩: $x \in \mathbb{X}$ },
(e) $\mathcal{A}_{\text{if}} \cap \mathcal{B}_{\text{if}}$ = {⟨ x, $\mu_{\mathcal{A}_{\text{if}}}(x) \wedge \mu_{\mathcal{B}_{\text{if}}}(x)$, $\upsilon_{\mathcal{A}_{\text{if}}}(x) \vee \upsilon_{\mathcal{B}_{\text{if}}}(x)$⟩: $x \in \mathbb{X}$ },
(f) $(\mathcal{A}_{\text{if}} \cup \mathcal{B}_{\text{if}})^{C} = \mathcal{A}_{\text{if}}^{C} \cap \mathcal{B}_{\text{if}}^{C}$ and $(\mathcal{A}_{\text{if}} \cap \mathcal{B}_{\text{if}})^{C} = \mathcal{A}_{\text{if}}^{C} \cup \mathcal{B}_{\text{if}}^{C}$.
(h) $\tilde{\mathbf{0}}$ = ⟨x, 0, 1⟩(empty set) and $\tilde{\mathbf{1}}$ = ⟨x, 1, 0 ⟩ (whole set).

Definition 2.3. [3] An intuitionistic fuzzy topology ($\mathcal{IFT}$ in short) on $\mathbb{X}$ is a family of $\mathcal{IFS}s$in $\mathbb{X}$, satisfying the following axioms.

1. $\tilde{0}$, $\tilde{1} \in \tau_{\text{if}}$
2. $\mathcal{A}_{\text{if}} \cap \mathcal{B}_{\text{if}} \in \tau_{\text{if}}$ for any $\mathcal{A}_{\text{if}}, \mathcal{B}_{\text{if}} \in \tau_{\text{if}}$
3. $\cup \mathcal{A}_{\text{if}_i} \in \tau_{\text{if}}$ for any family $\{\mathcal{A}_{\text{if}_i} / i \in \mathcal{J}\} \subseteq \tau_{\text{if}}$.

The pair ($\mathbb{X}$, τ_{if}) is called an intuitionistic fuzzy topological space ($\mathcal{IFTS}$in short) and any $\mathcal{IFS}$ inτ_{if} is known as an $\mathcal{IF}$open set ($\mathcal{IFOS}$ in short) in $\mathbb{X}$. The complement ($\mathcal{A}_{\text{if}}^{C}$) of an $\mathcal{IFOS}\mathcal{A}_{\text{if}}$ in an $\mathcal{IFTS}$($\mathbb{X}$, τ_{if}) is called an intuitionistic fuzzy closed set($\mathcal{IFCS}$ in short) in $\mathbb{X}$. In this paper, $\mathcal{IF}$ interior is denoted by $int_{\text{if}}$and $\mathcal{IF}$ closure is denoted by cl_{if}.

Definition 2.4.[3] Let ($\mathbb{X}$, τ_{if}) be an $\mathcal{IFTS}$ and $\mathcal{A}_{\text{if}}$ = {⟨x, $\mu_{\mathcal{A}_{\text{if}}}(x)$, $\upsilon_{\mathcal{A}_{\text{if}}}(x)$⟩: $x \in \mathbb{X}$} be an $\mathcal{IFS}$ in $\mathbb{X}$. Then the interior and closure of the above $\mathcal{IFS}$ are defined as follows,

(i) $int_{\text{if}}(\mathcal{A}_{\text{if}}) = \cup \{\mathcal{G}_{\text{if}} \mid \mathcal{G}_{\text{if}}$ is an $\mathcal{IFOS}$ in $\mathbb{X}$ and $\mathcal{G}_{\text{if}} \subseteq \mathcal{A}_{\text{if}}\}$,
(ii)$cl_{\text{if}}(\mathcal{A}_{\text{if}}) = \cap \{\mathcal{K}_{\text{if}} \mid \mathcal{K}_{\text{if}}$ is an $\mathcal{IFCS}$ in $\mathbb{X}$ and $\mathcal{A}_{\text{if}} \subseteq \mathcal{K}_{\text{if}}\}$

Definition 2.5. An intuitionistic fuzzy set$\mathcal{A}_{\text{if}}$ of an intuitionistic fuzzy topological space ($\mathbb{X}$, $\boldsymbol{\tau_{\text{if}}}$) is called:

a. $\mathcal{IFgCS}$ [8] if$cl_{\text{if}}(\mathcal{A}_{\text{if}}) \subseteq \mathcal{O}_{\text{if}}$ whenever $\mathcal{A}_{\text{if}} \subseteq \mathcal{O}_{\text{if}}$ and $\mathcal{O}_{\text{if}}$ is intuitionistic fuzzy open.
b. $\mathcal{IFsgCS}$ [9] if$scl_{\text{if}}(\mathcal{A}_{\text{if}}) \subseteq \mathcal{O}_{\text{if}}$ whenever $\mathcal{A}_{\text{if}} \subseteq \mathcal{O}_{\text{if}}$ and $\mathcal{O}_{\text{if}}$ is intuitionistic fuzzy semi open.
c. $\mathcal{IFgsCS}$[11] if$scl_{\text{if}}(\mathcal{A}_{\text{if}}) \subseteq \mathcal{O}_{\text{if}}$ whenever $\mathcal{A}_{\text{if}} \subseteq \mathcal{O}_{\text{if}}$ and $\mathcal{O}_{\text{if}}$ is intuitionistic fuzzy open.
d. $\mathcal{IFg}^{*}\mathcal{CS}$ [4] if$cl_{\text{if}}(\mathcal{A}_{\text{if}}) \subseteq \mathcal{O}_{\text{if}}$ whenever $\mathcal{A}_{\text{if}} \subseteq \mathcal{O}_{\text{if}}$ and $\mathcal{O}_{\text{if}}$ is intuitionistic fuzzy g-open.
e. $\mathcal{IFwCS}$ [10] or Intuitionistic fuzzy $\widehat{g}$-closed ($\mathcal{IF}\widehat{g}\mathcal{CS}$ in short) if $cl_{\text{if}}(\mathcal{A}_{\text{if}}) \subseteq \mathcal{O}$ whenever $\mathcal{A}_{\text{if}} \subseteq \mathcal{O}_{\text{if}}$ and $\mathcal{O}_{\text{if}}$ is intuitionistic fuzzy semi open.
f. $\mathcal{IFg}^{*}s\mathcal{CS}$ [7] if $scl_{\text{if}}(\mathcal{A}_{\text{if}}) \subseteq \mathcal{O}_{\text{if}}$ whenever $\mathcal{A}_{\text{if}} \subseteq \mathcal{O}_{\text{if}}$ and $\mathcal{O}_{\text{if}}$ is intuitionistic fuzzy g-open
g. $\mathcal{IF}\Psi\mathcal{CS}$ [6] if $scl_{\text{if}}(\mathcal{A}_{\text{if}}) \subseteq \mathcal{O}_{\text{if}}$ whenever $\mathcal{A}_{\text{if}} \subseteq \mathcal{O}_{\text{if}}$ and $\mathcal{O}_{\text{if}}$ is intuitionistic fuzzy sg-open

Definition 2.6. [8] An $\mathcal{IFS}\mathcal{A}_{\text{if}}$ of an $\mathcal{IFTS}$($\mathbb{X}$, τ_{if})is called an intuitionistic fuzzy $\widehat{g}^{*}$ semi closed set (in short $\mathcal{IF}\widehat{g}^{*}s\mathcal{CS}$),if $scl_{\text{if}}(\mathcal{A}_{\text{if}}) \subseteq \mathcal{O}_{\text{if}}$ whenever $\mathcal{A}_{\text{if}} \subseteq \mathcal{O}_{\text{if}}$ and $\mathcal{O}_{\text{if}}$ is any intuitionistic fuzzy $\widehat{g}$-open in ($\mathbb{X}$, τ_{if}).

Remark 2.7.

(i) Every $\mathcal{IF}s\mathcal{CS}$ is $\mathcal{IFg}^{*}s\mathcal{CS}$ [12]
(ii) Every $\mathcal{IFCS}$ is $\mathcal{IF}\Psi\mathcal{CS}$ [11]
(iii) Every $\mathcal{IF}\alpha\mathcal{CS}$ is $\mathcal{IF}s\mathcal{CS}$ [10]
(iv) Every $\mathcal{IFCS}$ is $\mathcal{IF}s\mathcal{CS}$ [11]
(v) Every $\mathcal{IFCS}$ is $\mathcal{IF}\alpha\mathcal{CS}$ [11]
(vi) Every$\mathcal{IF}\widehat{g}\mathcal{OS}$ is $\mathcal{IFsgOS}$ [12]
(vii) Every$\mathcal{IF}\widehat{g}\mathcal{OS}$ is $\mathcal{IFgOS}$ [12]

III. THEORETICAL APPLICATIONS OF INTUITIONISTIC FUZZY $\widehat{g}^{*}$SEMI CLOSED SETS

Definition 3.1. An $\mathcal{IFTS}$ ($\mathbb{U}$,τ_{if}) is called Intuitionistic fuzzy $\widehat{g}^{*}$ semi $T^{*}_{1/2}$space ($\mathcal{IF}\widehat{g}^{*}s$ $T^{*}_{1/2}$space) if every $\mathcal{IF}\widehat{g}^{*}s\mathcal{CS}$ is $\mathcal{IFCS}$ in ($\mathbb{U}$,τ_{if}).

Definition 3.2 An $\mathcal{IFTS}$ ($\mathbb{X}$,τ_{if}) is called

(i) An $\mathcal{IF}\widehat{g}^{*}s$ $T^{*}_{1/2}$space if every $\mathcal{IF}\widehat{g}^{*}s\mathcal{CS}$ is $\mathcal{IFCS}$ in ($\mathbb{X}$,τ_{if}).
(ii) An $s\mathcal{IF}\widehat{g}^{*}s$ $T^{*}_{1/2}$space if every $\mathcal{IF}\widehat{g}^{*}s\mathcal{CS}$ is $\mathcal{IF}s\mathcal{CS}$ in ($\mathbb{X}$,τ_{if}).
(iii) An $\alpha\mathcal{IF}\widehat{g}^{*}s$ $T^{*}_{1/2}$space if every $\mathcal{IF}\widehat{g}^{*}s\mathcal{CS}$ is $\mathcal{IF}\alpha\mathcal{CS}$ in ($\mathbb{X}$,τ_{if}).

(iv) An $\Psi\mathcal{IF}\widehat{\mathcal{G}}^{*}s\ T^{*}_{1/2}$space if every $\mathcal{IF}\widehat{\mathcal{G}}^{*}s\mathcal{CS}$ is $\mathcal{IF}\Psi\mathcal{CS}$ in $(\mathbb{X},\tau_{if})$.

(v) An $\mathcal{G}^{*}s\mathcal{IF}\widehat{\mathcal{G}}^{*}s\ T^{*}_{1/2}$space if every $\mathcal{IF}\widehat{\mathcal{G}}^{*}s\mathcal{CS}$ is $\mathcal{IF}\mathcal{G}^{*}s\mathcal{CS}$ in $(\mathbb{X},\tau_{if})$.

Example 3.3. Let $\mathbb{X}$ = {𝕖, 𝕗, 𝕘} and τ_{if}= {$\tilde{0}$,$\mathcal{A}_{if}$, $\tilde{1}$} where $\mathcal{A}_{if}$= {<𝕖, 0.7, 0.3>, <𝕗, 0.8, 0.2>}, <𝕘, 0.75, 0.25>}. Then$(\mathbb{X},\tau_{if})$ is an $\mathcal{IFTS}$ and $\mathcal{IF}\widehat{\mathcal{G}}^{*}s\mathcal{C}(\mathbb{X}, \tau_{if})$ = {$\tilde{0}$, $\mathcal{P}_{if}$, $\tilde{1}$ /$\mathcal{P}_{if}$ ≤{<𝕖, 0.3, 0.7>, <𝕗, 0.2, 0.8>, <𝕘, 0.25, 0.75>} and $\mathcal{IFC}(\mathbb{X}, \tau_{if})$ = {$\tilde{0}$, $\mathcal{P}_{if}$, $\tilde{1}$ /$\mathcal{P}_{if}$ ≤{<𝕖, 0.3, 0.7>, <𝕗, 0.2, 0.8>}, <𝕘, 0.25, 0.75>}. Thereforeevery $\mathcal{IF}\widehat{\mathcal{G}}^{*}s\mathcal{CS}$ is$\mathcal{IFCS}$ in $(\mathbb{X},\tau_{if})$. Hence$(\mathbb{X},\tau_{if})$ is $\mathcal{IF}\widehat{\mathcal{G}}^{*}s\ T^{*}_{1/2}$space.

Example 3.4. Let $\mathbb{X}$ = {𝕖, 𝕗, 𝕘} and τ_{if}= {$\tilde{0}$,$\mathcal{A}_{if}$, $\tilde{1}$} where $\mathcal{A}_{if}$= {<𝕖, 0.82, 0.18>, <𝕗, 0.77, 0.23>}, <𝕘, 0.69, 0.25>}. Then $(\mathbb{X},\tau_{if})$ is an $\mathcal{IFTS}$. Now $\mathcal{IF}\widehat{\mathcal{G}}^{*}s\mathcal{C}(\mathbb{X}, \tau_{if})$ = {$\tilde{0}$, $\mathcal{P}_{if}$, $\tilde{1}$ /$\mathcal{P}_{if}$ ≤{<𝕖, 0.18, 0.82>, <𝕗, 0.23, 0.77>}, <𝕘, 0.25, 0.69>} and $\mathcal{IF}s\mathcal{C}(\mathbb{X}, \tau_{if})$ = {$\tilde{0}$, $\mathcal{P}_{if}$, $\tilde{1}$ /$\mathcal{P}_{if}$ ≤{<𝕖, 0.18, 0.82>, <𝕗, 0.23, 0.77>}, <𝕘, 0.25, 0.69>}. Therefore,every $\mathcal{IF}\widehat{\mathcal{G}}^{*}s\mathcal{CS}$ is$\mathcal{IFCS}$ in $(\mathbb{X},\tau_{if})$. Hence$(\mathbb{X},\tau_{if})$ is $s\mathcal{IF}\widehat{\mathcal{G}}^{*}s\ T^{*}_{1/2}$space.

Example 3.5. Let $\mathbb{X}$ = {𝕖, 𝕗, 𝕘} and τ_{if}= {$\tilde{0}$,$\mathcal{A}_{if}$, $\tilde{1}$} where $\mathcal{A}_{if}$= {<𝕖, 0.79, 0.2>, <𝕗, 0.8, 0.2>}, <𝕘, 0.9, 0.1>}. Then$(\mathbb{X},\tau_{if})$ is an $\mathcal{IFTS}$. Now $\mathcal{IF}\widehat{\mathcal{G}}^{*}s\mathcal{C}(\mathbb{X}, \tau_{if})$ = {$\tilde{0}$, $\mathcal{P}_{if}$, $\tilde{1}$ /$\mathcal{P}_{if}$ ≤{<𝕖, 0.2, 0.79>, <𝕗, 0.2, 0.8>}, <𝕘, 0.1, 0.9>} and $\mathcal{IF}\alpha\mathcal{C}(\mathbb{X}, \tau_{if})$ = {$\tilde{0}$, $\mathcal{P}_{if}$, $\tilde{1}$ /$\mathcal{P}_{if}$ ≤ {< 𝕖, 0.2, 0.79 >, < 𝕗, 0.2, 0.8 >}, < 𝕘, 0.1, 0.9 > }. Therefore every $\mathcal{IF}\widehat{\mathcal{G}}^{*}s\mathcal{CS}$ is $\mathcal{IF}\alpha\mathcal{CS}$ in $(\mathbb{X},\tau_{if})$. Hence$(\mathbb{X},\tau_{if})$ is $\alpha\mathcal{IF}\widehat{\mathcal{G}}^{*}s\ T^{*}_{1/2}$space.

Example 3.6. Let $\mathbb{X}$ = {𝕖, 𝕗, 𝕘} and τ_{if}= {$\tilde{0}$,$\mathcal{A}_{if}$, $\tilde{1}$} where $\mathcal{A}_{if}$= {<𝕖, 0.3, 0.66>, <𝕗, 0.4, 0.6>}, <𝕘, 0.25, 0.75>}. Then $(\mathbb{X},\tau_{if})$ is an $\mathcal{IFTS}$. Now $\mathcal{IF}\widehat{\mathcal{G}}^{*}s\mathcal{C}(\mathbb{X}, \tau_{if})$ = {$\tilde{0}$, $\mathcal{P}_{if}$, $\mathcal{Q}_{if}$, $\tilde{1}$ /$\mathcal{A}_{if} < \mathcal{P}_{if} < \mathcal{A}_{if}{}^{c}$& $\mathcal{Q}_{if} \nless \mathcal{A}_{if}$, $\mathcal{A}_{if} \nless \mathcal{Q}_{if}$, $\mathcal{Q}_{if} < \mathcal{A}_{if}{}^{c}$} and $\mathcal{IF}\Psi\mathcal{C}(\mathbb{X}, \tau_{if})$ ={$\tilde{0}$, $\mathcal{P}_{if}$, $\mathcal{Q}_{if}$, $\tilde{1}$ /$\mathcal{A}_{if} < \mathcal{P}_{if} < \mathcal{A}_{if}{}^{c}$, $\mathcal{Q}_{if} \nless \mathcal{A}_{if}$, $\mathcal{A}_{if} \nless \mathcal{Q}_{if}$, $\mathcal{Q}_{if} < \mathcal{A}_{if}{}^{c}$}. Thereforeevery $\mathcal{IF}\widehat{\mathcal{G}}^{*}s\mathcal{CS}$ is$\mathcal{IF}\Psi\mathcal{CS}$ in $(\mathbb{X},\tau_{if})$. Hence$(\mathbb{X},\tau_{if})$ is$\Psi\mathcal{IF}\widehat{\mathcal{G}}^{*}s\ T^{*}_{1/2}$space.

Example 3.7. Let $\mathbb{X}$ = {𝕖, 𝕗, 𝕘} and τ_{if}= {$\tilde{0}$,$\mathcal{A}_{if}$, $\tilde{1}$} where $\mathcal{A}_{if}$= {<𝕖, 0.27, 0.69>, <𝕗, 0.34, 0.66>}, <𝕘, 0.2, 0.78>}. Then $(\mathbb{X},\tau_{if})$ is an $\mathcal{IFTS}$. Now $\mathcal{IF}\widehat{\mathcal{G}}^{*}s\mathcal{C}(\mathbb{X}, \tau_{if})$ = {$\tilde{0}$,$\mathcal{P}_{if}$, $\mathcal{Q}_{if}$, $\tilde{1}$ / $\mathcal{A}_{if} < \mathcal{P}_{if} < \mathcal{A}_{if}{}^{c}$& $\mathcal{A}_{if}{}^{c} \le \mathcal{Q}_{if} < 1$} and $\mathcal{IF}\Psi\mathcal{C}(\mathbb{X}, \tau_{if})$ ={$\tilde{0}$, $\mathcal{P}_{if}$, $\mathcal{Q}_{if}$, $\tilde{1}$ /$\mathcal{A}_{if} < \mathcal{P}_{if} < \mathcal{A}_{if}{}^{c}$& $\mathcal{A}_{if}{}^{c} \le \mathcal{Q}_{if} < 1$ } Therefore every $\mathcal{IF}\widehat{\mathcal{G}}^{*}s\mathcal{CS}$ is $\mathcal{IF}\mathcal{G}^{*}s\mathcal{CS}$ in $(\mathbb{X},\tau_{if})$. Hence$(\mathbb{X},\tau_{if})$ is$\mathcal{G}^{*}s\mathcal{IF}\widehat{\mathcal{G}}^{*}s\ T^{*}_{1/2}$space.

Proposition 3.8. Every $\alpha\mathcal{IF}\widehat{\mathcal{G}}^{*}s\ T^{*}_{1/2}$space is $s\mathcal{IF}\widehat{\mathcal{G}}^{*}s\ T^{*}_{1/2}$space but not vice versa.

Proof: Since every $\mathcal{IF}\alpha\mathcal{CS}$ is$\mathcal{IF}s\mathcal{CS}$ by Remark 2.7 (iii), we can say that every $\alpha\mathcal{IF}\widehat{\mathcal{G}}^{*}s\ T^{*}_{1/2}$space is $s\mathcal{IF}\widehat{\mathcal{G}}^{*}s\ T^{*}_{1/2}$space.

Example 3.9. Let $\mathbb{X}$ = {𝕖, 𝕗, 𝕘} and τ_{if}= {$\tilde{0}$,$\mathcal{A}_{if}$, $\tilde{1}$} where $\mathcal{A}_{if}$= {<𝕖, 0.2, 0.79>, <𝕗, 0.4, 0.6>}, <𝕘, 0.2, 0.8>}. Then $(\mathbb{X},\tau_{if})$ is an $\mathcal{IFTS}$. Now $\mathcal{IF}\widehat{\mathcal{G}}^{*}s\mathcal{C}(\mathbb{X}, \tau_{if})$ = {$\tilde{0}$, $\mathcal{P}_{if}$, $\tilde{1}$ / $\mathcal{A}_{if} < \mathcal{P}_{if} < \mathcal{A}_{if}{}^{c}$}, and $\mathcal{IF}s\mathcal{C}(\mathbb{X}, \tau_{if})$ = {$\tilde{0}$, $\mathcal{P}_{if}$, $\tilde{1}$ / $\mathcal{A}_{if} < \mathcal{P}_{if} < \mathcal{A}_{if}{}^{c}$} and $\mathcal{IF}\alpha\mathcal{C}(\mathbb{X}, \tau_{if})$ = {$\tilde{0}$, $\tilde{1}$}. Therefore every $\mathcal{IF}\widehat{\mathcal{G}}^{*}s\mathcal{CS}$ is $\mathcal{IF}s\mathcal{CS}$ but not $\mathcal{IF}\alpha\mathcal{CS}$. Hence $(\mathbb{X},\tau_{if})$ is an $s\mathcal{IF}\widehat{\mathcal{G}}^{*}s\ T^{*}_{1/2}$space but not $\alpha\mathcal{IF}\widehat{\mathcal{G}}^{*}s\ T^{*}_{1/2}$space.

Proposition 3.10. Every $s\mathcal{IF}\widehat{\mathcal{G}}^{*}s\ T^{*}_{1/2}$space is $\mathcal{G}^{*}s\mathcal{IF}\widehat{\mathcal{G}}^{*}s\ T^{*}_{1/2}$space but not vice versa.

Proof: Since every $\mathcal{IF}s\mathcal{CS}$ is $\mathcal{IF}\mathcal{G}^{*}s\mathcal{CS}$ by Remark 2.7 (i), we can say that every $s\mathcal{IF}\widehat{\mathcal{G}}^{*}s\ T^{*}_{1/2}$space is $\mathcal{G}^{*}s\mathcal{IF}\widehat{\mathcal{G}}^{*}s\ T^{*}_{1/2}$space.

Example 3.11. Let $\mathbb{X}$ = {𝕖, 𝕗, 𝕘} and τ_{if}= {$\tilde{0}$,$\mathcal{A}_{if}$, $\tilde{1}$} where $\mathcal{A}_{if}$= {<𝕖, 0.2, 0.79>, <𝕗, 0.4, 0.6>}, <𝕘, 0.2, 0.8>}. Then$(\mathbb{X},\tau_{if})$ is an $\mathcal{IFTS}$. Now $\mathcal{IF}\widehat{\mathcal{G}}^{*}s\mathcal{C}(\mathbb{X}, \tau_{if})$ = {$\tilde{0}$, $\mathcal{P}_{if}$, $\mathcal{Q}_{if}$, $\tilde{1}$ / $\mathcal{A}_{if} < \mathcal{P}_{if} < \mathcal{A}_{if}{}^{c}$& $\mathcal{A}_{if}{}^{c} \le \mathcal{Q}_{if} < 1$},$\mathcal{IF}\mathcal{G}^{*}s\mathcal{C}(\mathbb{X}, \tau_{if})$ = {$\tilde{0}$, $\mathcal{P}_{if}$, $\mathcal{Q}_{if}$, $\tilde{1}$ / $\mathcal{A}_{if} < \mathcal{P}_{if} < \mathcal{A}_{if}{}^{c}$& $\mathcal{A}_{if}{}^{c} \le \mathcal{Q}_{if} < 1$} and$\mathcal{IF}s\mathcal{C}(\mathbb{X}, \tau_{if})$ = {$\tilde{0}$, $\mathcal{P}_{if}$, $\tilde{1}$ / $\mathcal{A}_{if} < \mathcal{P}_{if} < \mathcal{A}_{if}{}^{c}$}. Therefore every $\mathcal{IF}\widehat{\mathcal{G}}^{*}s\mathcal{CS}$ is $\mathcal{IF}\mathcal{G}^{*}s\mathcal{CS}$ but not $\mathcal{IF}s\mathcal{CS}$. Hence $(\mathbb{X},\tau_{if})$ is an $\mathcal{G}^{*}s\mathcal{IF}\widehat{\mathcal{G}}^{*}s T^{*}_{1/2}$space but not $s\mathcal{IF}\widehat{\mathcal{G}}^{*}s\ T^{*}_{1/2}$space.

Proposition 3.12. Every $\mathcal{IF}\widehat{\mathcal{G}}^{*}s\ T^{*}_{1/2}$space is $s\mathcal{IF}\widehat{\mathcal{G}}^{*}s\ T^{*}_{1/2}$space but not vice versa.

Proof: Since every $\mathcal{IFCS}$ is $\mathcal{IF}s\mathcal{CS}$ by Remark 2.7 (iv), we can say every $\mathcal{IF}\widehat{\mathcal{G}}^{*}s\ T^{*}_{1/2}$space is $s\mathcal{IF}\widehat{\mathcal{G}}^{*}s\ T^{*}_{1/2}$space.

Example 3.13. Let $\mathbb{X} = \{p, q, r\}$ and $\tau_{if} = \{\tilde{0}, \mathcal{A}_{if}, \tilde{1}\}$ where $\mathcal{A}_{if} = \{<p, 0.2, 0.8>, <q, 0.27, 0.73>\}, <r, 0.19, 0.81>\}$. Then $(\mathbb{X}, \tau_{if})$ is an $\mathcal{IFTS}$. Now $\mathcal{IF}\widehat{g}^*s\mathcal{C}(\mathbb{X}, \tau_{if}) = \{\tilde{0}, \mathcal{P}_{if}, \tilde{1} / \mathcal{A}_{if} < P_{if} < \mathcal{A}_{if}{}^{c}\}$ and $\mathcal{IF}s\mathcal{C}(\mathbb{X}, \tau_{if}) = \{\tilde{0}, \mathcal{P}_{if}, \tilde{1} / \mathcal{A}_{if} < P_{if} < \mathcal{A}_{if}{}^{c}\}$. But $\mathcal{IFC}(\mathbb{X}, \tau_{if}) = \{\tilde{0}, \mathcal{A}_{if}{}^{c}, \tilde{1}\}$. Therefore $(\mathbb{X}, \tau_{if})$ is $s\mathcal{IF}\widehat{g}^*s\ T^*_{1/2}$ space but not $\mathcal{IF}\widehat{g}^*s\ T^*_{1/2}$ space.

Proposition 3.14. Every $\mathcal{IF}\widehat{g}^*s\ T^*_{1/2}$ space is $\alpha\mathcal{IF}\widehat{g}^*s\ T^*_{1/2}$ space but not vice versa.

Proof: Since every $\mathcal{IFCS}$ is $\mathcal{IF}\alpha\mathcal{CS}$ by Remark 2.7 (v), we can say every $\mathcal{IF}\widehat{g}^*sT^*_{1/2}$ space is $\alpha\mathcal{IF}\widehat{g}^*sT^*_{1/2}$ space.

Example 3.15. Let $\mathbb{X} = \{\ell, m, n\}$ and $\tau_{if} = \{\tilde{0}, \mathcal{A}_{if}, \tilde{1}\}$ where $\mathcal{A}_{if} = \{<\ell, 0.76, 0.24>, <m, 0.77, 0.23>\}, <n, 0.9, 0.1>\}$. Then $(\mathbb{X}, \tau_{if})$ is an $\mathcal{IFTS}$. Now, $\mathcal{IF}\widehat{g}^*s\mathcal{C}(\mathbb{X}, \tau_{if}) = \{\tilde{0}, \mathcal{P}_{if}, \tilde{1} / \tilde{0} < P_{if} < \mathcal{A}_{if}{}^{c}\}$ and $\mathcal{IF}\alpha\mathcal{C}(\mathbb{X}, \tau_{if}) = \{\tilde{0}, \mathcal{P}_{if}, \tilde{1} / \tilde{0} < P_{if} < \mathcal{A}_{if}{}^{c}\}$. But $\mathcal{IFC}(\mathbb{X}, \tau_{if}) = \{\tilde{0}, \mathcal{A}_{if}{}^{c}, \tilde{1}\}$. Therefore $(\mathbb{X}, \tau_{if})$ is $\alpha\mathcal{IF}\widehat{g}^*s\ T^*_{1/2}$ space but not $\mathcal{IF}\widehat{g}^*sT^*_{1/2}$ space.

Proposition 3.16. Every $\mathcal{IF}\widehat{g}^*sT^*_{1/2}$ space is $\Psi\mathcal{IF}\widehat{g}^*sT^*_{1/2}$ space but not vice versa.

Proof: Since every $\mathcal{IFCS}$ is $\mathcal{IF}\Psi\mathcal{CS}$ by Remark 2.7 (ii), we can say every $\mathcal{IF}\widehat{g}^*s\ T^*_{1/2}$ space is $\Psi\mathcal{IF}\widehat{g}^*s\ T^*_{1/2}$ space.

Example 3.17. Let $\mathbb{X} = \{r, s, t\}$ and τ_{if}□$= \{\tilde{0}, \mathcal{A}_{if}, \tilde{1}\}$ where $\mathcal{A}_{if} = \{<r, 0.27, 0.69>, <s, 0.34, 0.66>\}, <t, 0.2, 0.78>\}$. Then $(\mathbb{X}, \tau_{if})$ is an $\mathcal{IFTS}$. Now $\mathcal{IF}\widehat{g}^*s\mathcal{C}(\mathbb{X}, \tau_{if}) = \{\tilde{0}, \mathcal{P}_{if}, \mathcal{Q}_{if}, \tilde{1} / \mathcal{A}_{if} < \mathcal{P}_{if} < \mathcal{A}_{if}{}^{c} \& \mathcal{A}_{if}{}^{c} \leq \mathcal{Q}_{if} < 1\}$, $\mathcal{IF}\Psi\mathcal{C}(\mathbb{X}, \tau_{if}) = \{\tilde{0}, \mathcal{P}_{if}, \mathcal{Q}_{if}, \tilde{1} / \mathcal{A}_{if} < \mathcal{P}_{if} < \mathcal{A}_{if}{}^{c} \& \mathcal{A}_{if}{}^{c} \leq \mathcal{Q}_{if} < 1\}$. But $\mathcal{IFC}(\mathbb{X}, \tau_{if}) = \{\tilde{0}, \mathcal{A}_{if}{}^{c}, \tilde{1}\}$. Hence $(\mathbb{X}, \tau_{if})$ is $\Psi\mathcal{IF}\widehat{g}^*s\ T^*_{1/2}$ space but not $\mathcal{IF}\widehat{g}^*s\ T^*_{1/2}$ space.

The diagram below depicts the interrelationship of $\mathcal{IF}\widehat{g}^*s\ T^*_{1/2}$ space with some of other intuitionistic fuzzy $T^*_{1/2}$ spaces discussed above.

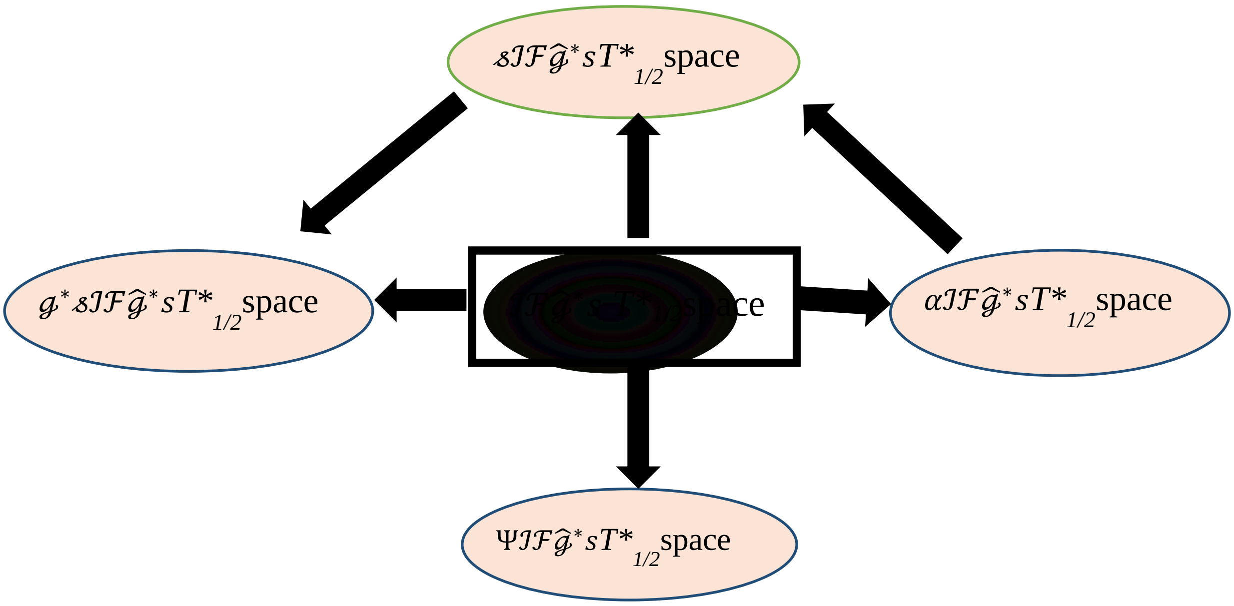

Figure 3.3

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