*International Journal of Mathematics and Statistics Invention (IJMSI)*
*E-ISSN: 2321 – 4767 P-ISSN: 2321 - 4759*
*www.ijmsi.org Volume 12 Issue 2 || Mar.-Apr. 2024 || PP-38-47*

# Tri Sum Perfect Square Cordial Labeling of Graphs

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***ABSTRACT:****A graph $G = (p, q)$ with $p$ vertices and $q$ edges is said to have a Tri sum perfect square cordial labeling if there exists a function $f: V(G) \to \{-1,0,1\}$ such that for each edge $e = uv$ the induced map $f^*: E(G) \to \{0,1\}$ is defined by,*

$$f^*(uv) = f(u)^2 + 2f(u)f(v) + f(v)^2$$

*and $|e_f(0) - e_f(1)| \leq 1$ where , $e_f(0)$ =number of edges with zero label and $e_f(1)$ =number of edges with one label.*

*In this paper we obtain tri sum perfect square cordial labeling of path, comb, star,fan, $H -$ graph of a path $P_n$,$H \odot K_1$ graph of a path $P_n$, $K_{1,2} * K_{1,n}$, two star.*

***KEYWORDS:*** *Tri sum perfect square cordial labeling, path, comb,star, fan,$H -$ graph of a path $P_n$, $H \odot K_1$ graph of a path $P_n$, $K_{1,2} * K_{1,n}$, two star.*

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Date of Submission: 25-04-2024 Date of acceptance: 02-05-2024

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## I. INTRODUCTION

All the graphs in this paper are finite and undirected. The symbols $V(G)$& $E(G)$ denotes the vertex set and edge set of a graph $G$. An excellence reference on this subject is the survey by J. A. Gallian [1]. The definitions which are useful for the present investigation are below. We refer Gross and Yellen [2], for all kinds of definitions and notations. U. Vaghela and D.Parmar [6] has define a concept of new labeling which is Difference perfect square cordial labeling. S. G. Sonchhatra and G. V. Ghodasara has defined Sum Perfect Square labeling [4]. Motivated by this we have define a new concept which is tri sum perfect square cordial labeling defined as follows.

A graph $G = (p, q)$ with $p$ vertices and $q$ edges is said to have a tri sum perfect square cordial labeling if there exists a function $f: V(G) \to \{-1,0,1\}$ such that for each edge $e = uv$ the induced map $f^*: E(G) \to \{0,1\}$is defined by,

$$f^*(uv) = f(u)^2 + 2f(u)f(v) + f(v)^2$$

and $|e_f(0) - e_f(1)| \leq 1$ where , $e_f(0)$ =number of edges with zero label and $e_f(1)$ =number of edges with one label. A graph which attains a tri sum perfect square cordial labeling is called tri sum perfect square cordial graph.

**Definition 1:** An open walk in which no vertex appears more than once is called a **path**. [2]

**Definition 2:**The graph attained by attaching a single pendant edge to each vertex of a path is called **Comb**. It is denoted by$P_n \odot K_1$. [1]

**Definition 3:** $\boldsymbol{K_{1,2} * K_{1,n}}$ is the graph obtained from $K_{1,2}$ by attaching root of a star $K_{1,n}$ at each pendant vertex of $K_{1,2}$. [1]

**Definition 4:** A **Star** graph is the complete bipartite graph $K_{1,n}$. [2]

**Definition 5:** The **fan graph** is obtained by joining a vertex to all vertices of a path $P_n$ by an edge which is denoted by $F_n$ or $P_n + K_1$. [1]

**Definition 6:** The ***H* graph** of path $P_n$ is the graph obtained from two copies of $P_n$ with vertices $u_1, u_2, \dots, u_n$&$v_1, v_2, \dots, v_n$ by joining the vertices $u_{\frac{n+1}{2}}$&$v_{\frac{n+1}{2}}$ by an edge if $n$ is odd and the vertices $u_{\frac{n}{2}+1}$ &$v_{\frac{n}{2}}$ if $n$ is even. [1]

**Definition 7:** $\boldsymbol{H \odot K_1}$ graph of a path $P_n$ is a graph obtained from $H$ graph of a path $P_n$by attaching a pendant edge to each vertex of it. [1]

**Definition 8:** The **two star** is the disjoint union of $K_{1,m}$&$K_{1,n}$. It is denoted by $K_{1,m} \cup K_{1,n}$. [1]

## II. MAIN RESULTS

**Theorem 1** The path $P_n$ is a tri sum perfect square cordial graph.

**Proof:** Let $G = P_n$.

$V(G) = \{u_1, u_2, \dots, u_n\}$and $E(G) = \{(u_k u_{k+1}): 1 \leq k \leq n-1\}$.

So, $|V(G)| = n$ and $|E(G)| = n-1$.
Define $f: V(G) \to \{-1,0,1\}$ as follows.
**Case:1 $n$ even.**
**Sub Case:1 $n = 4k-2$, $k \in \mathbb{N}$.**

$$f(u_{2k-1}) = 1 \qquad 1 \le k \le \frac{n+2}{4}$$
$$f(u_{2k}) = -1 \qquad 1 \le k \le \frac{n}{2}$$

$f\left(u_{\frac{n}{2}+2k}\right) = 0 \qquad 1 \le k \le \frac{n-2}{4}$.
**Sub Case:2 $n = 4k$, $k \in \mathbb{N}$.**

$$f(u_{2k-1}) = 1 \qquad 1 \le k \le \frac{n}{2}$$
$$f(u_{2k}) = -1 \qquad 1 \le k \le \frac{n}{4}$$

$f\left(u_{\frac{n}{2}+2k}\right) = 0 \qquad 1 \le k \le \frac{n}{4}$.
For this we define edge function $f^*: E(G) \to \{0,1\}$ as follows.

$$f^*(u_k u_{k+1}) = 0 \qquad 1 \le k \le \frac{n}{2}$$

$f^*(u_k u_{k+1}) = 1 \qquad \frac{n+2}{2} \le k \le n-1$.
**Case:2 $n$ odd.**
**Sub Case:1 $n = 4k-1$, $k \in \mathbb{N}$.**

$$f(u_{2k-1}) = 1 \qquad 1 \le k \le \frac{n+1}{4}$$
$$f(u_{2k}) = -1 \qquad 1 \le k \le \frac{n-1}{2}$$

$f\left(u_{\frac{n-1}{2}+2k}\right) = 0 \qquad 1 \le k \le \frac{n+1}{4}$.
**Sub Case:2 $n = 4k+1$, $k \in \mathbb{N}$.**

$$f(u_{2k-1}) = 1 \qquad 1 \le k \le \frac{n+1}{2}$$
$$f(u_{2k}) = -1 \qquad 1 \le k \le \frac{n-1}{4}$$

$f\left(u_{\frac{n-1}{2}+2k}\right) = 0 \qquad 1 \le k \le \frac{n-1}{4}$.
For this we define edge function $f^*: E(G) \to \{0,1\}$ as follows.

$$f^*(u_k u_{k+1}) = 0 \qquad 1 \le k \le \frac{n-1}{2}$$

$f^*(u_k u_{k+1}) = 1 \qquad \frac{n+1}{2} \le k \le n-1$.

**Table:1**

| $n$ | ***Edge condition*** |
|---|---|
| Even | $e_f(0) = \frac{n}{2}$, $e_f(1) = \frac{n-2}{2}$ |
| Odd | $e_f(0) = \frac{n-1}{2}$, $e_f(1) = \frac{n-1}{2}$ |

In both of the cases we have $\left|e_f(0) - e_f(1)\right| \le 1$.
Hence path is a tri sum perfect square cordial graph.
**Illustration:** A tri sum perfect square cordial labeling of $P_4$ and $P_5$ is shown in Figure-1(a) and Figure-1(b) respectively.

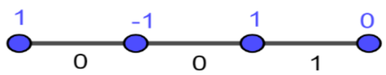


**Figure : 1(a) $P_4$**

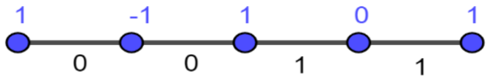


**Figure : 1(b) $P_5$**

**Theorem 2** The comb is a tri sum perfect square cordial graph.
**Proof:** Let $G = P_n \odot K_1$.
$V(G) = \{u_k, v_k : 1 \le k \le n\}$ and
$E(G) = \{(u_k u_{k+1}): 1 \le k \le n-1\} \cup \{(u_k v_k): 1 \le k \le n\}$.
So, $|V(G)| = 2n$ & $|E(G)| = 2n - 1$.
Define $f: V(G) \to \{-1,0,1\}$ as follows.

$$f(u_{2k-1}) = 1 \qquad 1 \le k \le \left\lceil \frac{n}{2} \right\rceil$$
$$f(u_{2k}) = -1 \qquad 1 \le k \le \left\lfloor \frac{n}{2} \right\rfloor$$

$f(v_k) = 0 \qquad 1 \le k \le n.$
For this we define edge function $f^*: E(G) \to \{0,1\}$ as follows.

$$f^*(u_k u_{k+1}) = 0 \qquad 1 \le k \le n-1$$

$f^*(u_k v_k) = 1 \qquad 1 \le k \le n.$

**Table:2**

| $n$ | *Edge condition* |
|---|---|
| All | $e_f(0) = n-1, \quad e_f(1) = n$ |

We have $|e_f(0) - e_f(1)| \le 1$. Hence comb is a tri sum perfect square cordial graph.
**Illustration:** A tri sum perfect square cordial labeling of $P_6 \odot K_1$ is shown in Figure-2.

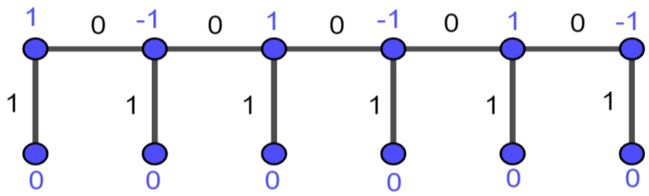


**Figure : 2 $P_6 \odot K_1$**

**Theorem 3** The graph $K_{1,2} * K_{1,n}$ is a tri sum perfect square cordial graph.
**Proof:** Let $G = K_{1,2} * K_{1,n}$.
$V(G) = \{u, v, w, u_k, v_k : 1 \le k \le n\}$ and
$E(G) = \{uw, wv, uu_k, vv_k : 1 \le k \le n\}$.
Hence $|V(G)| = 2n + 3$ & $|E(G)| = 2n + 2$.
Define $f: V(G) \to \{-1,0,1\}$ as follows.

$$f(u) = 1$$
$$f(v) = 0$$
$$f(w) = -1$$
$$f(u_k) = -1 \qquad 1 \le k \le n$$

$f(v_k) = 1 \qquad 1 \le k \le n.$
For this we define edge function $f^*: E(G) \to \{0,1\}$ as follows.

$$f^*(uw) = 0$$
$$f^*(wv) = 1$$
$$f^*(uu_k) = 0 \qquad 1 \le k \le n$$

$f^*(vv_k) = 1 \qquad 1 \le k \le n.$

**Table:3**

| $n$ | *Edge condition* |
|---|---|
| All | $e_f(0) = n+1, \quad e_f(1) = n+1$ |

We have $|e_f(0) - e_f(1)| \le 1$. Hence $K_{1,2} * K_{1,n}$ is a tri sum perfect square cordial graph.

**Illustration:** A tri sum perfect square cordial labeling of $K_{1,2} * K_{1,5}$ is shown in Figure-3.

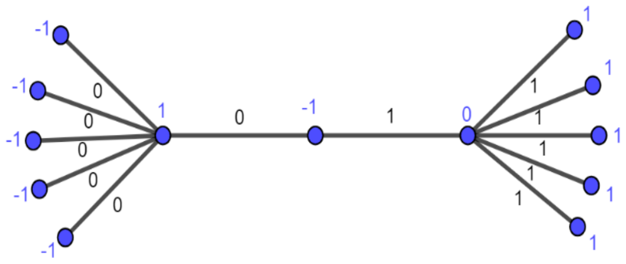

**Figure : 3 $K_{1,2} * K_{1,5}$**

**Theorem 4** The star is a tri sum perfect square cordial graph.
**Proof:** Let $G = K_{1,n}$.
$V(G) = \{u, u_k : 1 \leq k \leq n\}$ and $E(G) = \{(uu_k): 1 \leq k \leq n\}$.
So,$|V(G)| = n + 1$ &$|E(G)| = n$.
Define $f: V(G) \to \{-1,0,1\}$ as follows.

$$f(u) = 1$$

$$f(u_{2k-1}) = -1 \qquad 1 \leq k \leq \left\lceil \frac{n}{2} \right\rceil$$

$$f(u_{2k}) = 0 \qquad 1 \leq k \leq \left\lceil \frac{n-1}{2} \right\rceil.$$

For this we define edge function $f^*: E(G) \to \{0,1\}$ as follows.

$$f^*(uu_{2k-1}) = 0 \qquad 1 \leq k \leq \left\lceil \frac{n}{2} \right\rceil$$

$$f^*(uu_{2k}) = 1 \qquad 1 \leq k \leq \left\lceil \frac{n-1}{2} \right\rceil.$$

**Table:4**

| $n$ | ***Edge condition*** |
|---|---|
| All | $e_f(0) = \left\lceil \frac{n}{2} \right\rceil, \quad e_f(1) = \left\lceil \frac{n-1}{2} \right\rceil$ |

We have $|e_f(0) - e_f(1)| \leq 1$.
Hence star is a tri sum perfect square cordial graph.
**Illustration:** A tri sum perfect square cordial labeling of $K_{1,7}$ is shown in Figure-4.

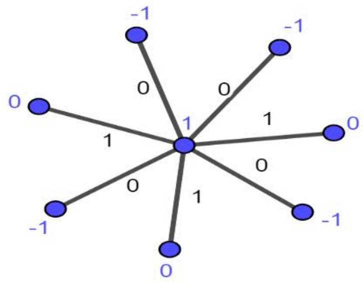

**Figure : 4 $K_{1,7}$**

**Theorem 5** The fan is a tri sum perfect square cordial graph.
**Proof:** Let $G = F_n$.
$V(G) = \{ u, u_k : 1 \leq k \leq n\}$ and
$E(G) = \{(uu_k): 1 \leq k \leq n\} \cup \{(u_k u_{k+1}): 1 \leq k \leq n-1\}$.
So,$|V(G)| = n + 1$&$|E(G)| = 2n - 1$.
Define $f: V(G) \to \{-1,0,1\}$ as follows.

$$f(u) = 0$$

$$f(u_{2k-1}) = 1 \qquad 1 \leq k \leq \left\lceil \frac{n}{2} \right\rceil$$

$$f(u_{2k}) = -1 \qquad 1 \leq k \leq \left\lceil \frac{n-1}{2} \right\rceil.$$

We define edge function $f^*: E(G) \to \{0,1\}$ as follows.

$$f^*(uu_k) = 1 \qquad 1 \le k \le n$$

$f^*(u_k u_{k+1}) = 0 \qquad 1 \le k \le n-1.$

**Table:5**

| $n$ | Edge condition |
|---|---|
| All | $e_f(0) = n-1, \quad e_f(1) = n$ |

We have $|e_f(0) - e_f(1)| \le 1$.
Hence fan is a tri sum perfect square cordial graph.
**Illustration:** A tri sum perfect square cordial labeling of $F_8$ is shown in Figure-5.

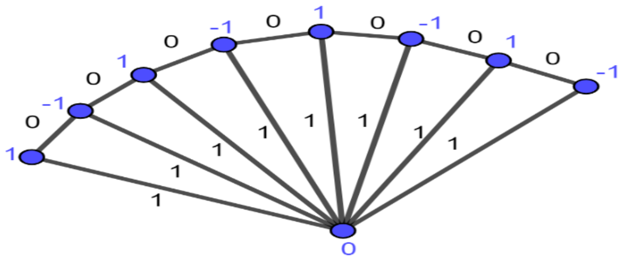


**Figure : 5 $F_8$**

**Theorem 6** The $H-$graph of a path $P_n$ is a tri sum perfect square cordial graph.
**Proof:**Let$G = H-$graph of a path $P_n$.
$V(G) = \{u_k, v_k : 1 \le k \le n\}$ and
$E(G) = \{(u_k u_{k+1}) : 1 \le k \le n-1\} \cup \{(v_k v_{k+1}) : 1 \le k \le n-1\} \cup \{\left(u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}\right) : n \text{ is odd}\}$ OR
$E(G) = \{(u_k u_{k+1}) : 1 \le k \le n-1\} \cup \{(v_k v_{k+1}) : 1 \le k \le n-1\} \cup \{\left(u_{\frac{n}{2}+1} v_{\frac{n}{2}}\right) : n \text{ is even}\}$.
So,$|V(G)| = 2n \& |E(G)| = 2n-1$.
Define $f: V(G) \to \{-1,0,1\}$ as follows.

**Case:1 $n$ even.**

$$f(u_{2k-1}) = 1 \qquad 1 \le k \le \frac{n}{2}$$
$$f(u_{2k}) = -1 \qquad 1 \le k \le \frac{n}{2}$$
$$f(v_{2k-1}) = 1 \qquad 1 \le k \le \frac{n}{2}$$

$f(v_{2k}) = 0 \qquad 1 \le k \le \frac{n}{2}.$
We define edge function $f^*: E(G) \to \{0,1\}$ as follows.

$$f^*(u_k u_{k+1}) = 0 \qquad 1 \le k \le n-1$$
$$f^*(v_k v_{k+1}) = 1 \qquad 1 \le k \le n-1$$
$$f^*\left(u_{\frac{n}{2}+1} v_{\frac{n}{2}}\right) = 0 \qquad n = 4k+2, \ k \in \mathbb{N}$$

$f^*\left(u_{\frac{n}{2}+1} v_{\frac{n}{2}}\right) = 1 \qquad n = 4k, \ k \in \mathbb{N}.$

**Case:2 $n$ odd.**

$$f(u_{2k-1}) = -1 \qquad 1 \le k \le \frac{n+1}{2}$$
$$f(u_{2k}) = 1 \qquad 1 \le k \le \frac{n-1}{2}$$
$$f(v_{2k-1}) = 1 \qquad 1 \le k \le \frac{n+1}{2}$$

$f(v_{2k}) = 0 \qquad 1 \le k \le \frac{n-1}{2}.$
We define edge function $f^*: E(G) \to \{0,1\}$ as follows.

$$f^*(u_k u_{k+1}) = 0 \qquad 1 \le k \le n-1$$
$$f^*(v_k v_{k+1}) = 1 \qquad 1 \le k \le n-1$$
$$f^*\left(u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}\right) = 0 \qquad n = 4k+1, \ k \in \mathbb{N}$$

$f^*\left(u_{\frac{n+1}{2}}v_{\frac{n+1}{2}}\right) = 1 \qquad n = 4k-1,\ k \in \mathbb{N}.$

**Table:6**

| $n$ | $Edge\ condition$ |
|---|---|
| $n = 4k+2,\ k \in \mathbb{N}$ | $e_f(0) = n, \quad e_f(1) = n-1$ |
| $n = 4k, \quad k \in \mathbb{N}$ | $e_f(0) = n-1, \quad e_f(1) = n$ |
| $n = 4k+1,\ k \in \mathbb{N}$ | $e_f(0) = n, \quad e_f(1) = n-1$ |
| $n = 4k-1,\ k \in \mathbb{N}$ | $e_f(0) = n-1,\ e_f(1) = n$ |

In all cases we have $|e_f(0) - e_f(1)| \le 1$.
Hence $H$ graph of a path $P_n$ is a tri sum perfect square cordial graph.

**Illustration:** A tri sum perfect square cordial labeling of $H$ graph of a path $P_4$ and $H$ graph of a path $P_5$ is shown in figure 6(a) and figure 6(b) respectively.

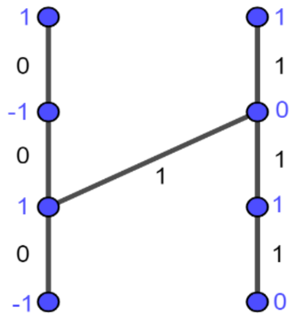

**Figure : 6(a) $H$ graph of a path $P_4$**

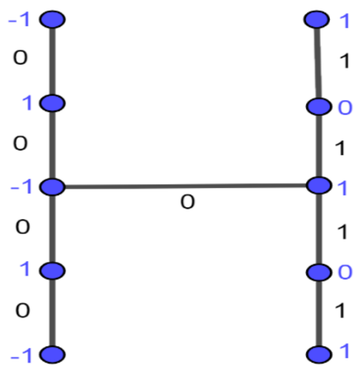

**Figure : 6(b) $H$ graph of a path $P_5$**

**Theorem 7** $H \odot K_1$ graph of a path $P_n$ is a tri sum perfect square cordial graph.
**Proof:** Let $G = H \odot K_1$ graph of a path $P_n$.
$V(G) = \{u_k, v_k, s_k, t_k : 1 \le k \le n\}$ and
$E(G) = \{(u_k u_{k+1}) : 1 \le k \le n-1\} \cup \{(v_k v_{k+1}) : 1 \le k \le n-1\} \cup \{(u_k t_k) : 1 \le k \le n\} \cup \{(v_k s_k) : 1 \le k \le n\} \cup \left\{\left(u_{\frac{n+1}{2}}v_{\frac{n+1}{2}}\right) : n \text{ is odd}\right\}$. OR

$$E(G) = \{(u_k u_{k+1}) : 1 \le k \le n-1\} \cup \{(v_k v_{k+1}) : 1 \le k \le n-1\} \cup \{(u_k t_k) : 1 \le k \le n\} \cup \{(v_k s_k) : 1 \le k \le n\} \cup \left\{\left(u_{\frac{n}{2}+1}v_{\frac{n}{2}}\right) : n \text{ is even}\right\}.$$

So, $|V(G)| = 4n \,\&\, |E(G)| = 4n-1$.
Define a function $f : V(G) \to \{-1,0,1\}$ as follows.
**Case:1 $n$ even.**

$$f(u_{2k-1}) = 1 \qquad 1 \le k \le \frac{n}{2}$$
$$f(u_{2k}) = -1 \qquad 1 \le k \le \frac{n}{2}$$

$$f(t_{2k-1}) = -1 \qquad 1 \le k \le \frac{n}{2}$$
$$f(t_{2k}) = 1 \qquad 1 \le k \le \frac{n}{2}$$
$$f(v_{2k-1}) = 1 \qquad 1 \le k \le \frac{n}{2}$$
$$f(v_{2k}) = 0 \qquad 1 \le k \le \frac{n}{2}$$
$$f(s_{2k-1}) = 0 \qquad 1 \le k \le \frac{n}{2}$$

$f(s_{2k}) = 1 \qquad 1 \le k \le \frac{n}{2}$.

We define edge function $f^*: E(G) \to \{0,1\}$ as follows.

$$f^*(u_k u_{k+1}) = 0 \qquad 1 \le k \le n-1$$
$$f^*(u_k t_k) = 0 \qquad 1 \le k \le n$$
$$f^*(v_k v_{k+1}) = 1 \qquad 1 \le k \le n-1$$
$$f^*(v_k s_k) = 1 \qquad 1 \le k \le n$$
$$f^*\left(u_{\frac{n}{2}+1} v_{\frac{n}{2}}\right) = 0 \qquad n = 4k+2,\ k \in \mathbb{N}$$

$f^*\left(u_{\frac{n}{2}+1} v_{\frac{n}{2}}\right) = 1 \qquad n = 4k,\ k \in \mathbb{N}$.

**Case:2 $n$ odd.**

$$f(u_{2k-1}) = -1 \qquad 1 \le k \le \frac{n+1}{2}$$
$$f(u_{2k}) = 1 \qquad 1 \le k \le \frac{n-1}{2}$$
$$f(t_{2k-1}) = 1 \qquad 1 \le k \le \frac{n+1}{2}$$
$$f(t_{2k}) = -1 \qquad 1 \le k \le \frac{n-1}{2}$$
$$f(v_{2k-1}) = 1 \qquad 1 \le k \le \frac{n+1}{2}$$
$$f(v_{2k}) = 0 \qquad 1 \le k \le \frac{n-1}{2}$$
$$f(s_{2k-1}) = 0 \qquad 1 \le k \le \frac{n+1}{2}$$

$f(s_{2k}) = 1 \qquad 1 \le k \le \frac{n-1}{2}$.

We define edge function $f^*: E(G) \to \{0,1\}$ as follows.

$$f^*(u_k u_{k+1}) = 0 \qquad 1 \le k \le n-1$$
$$f^*(u_k t_k) = 0 \qquad 1 \le k \le n$$
$$f^*(v_k v_{k+1}) = 1 \qquad 1 \le k \le n-1$$
$$f^*(v_k s_k) = 1 \qquad 1 \le k \le n$$
$$f^*\left(u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}\right) = 0 \qquad n = 4k+1,\ k \in \mathbb{N}$$

$f^*\left(u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}\right) = 1 \qquad n = 4k-1,\ k \in \mathbb{N}$.

**Table:7**

| $\boldsymbol{n}$ | ***Edge condition*** |
|---|---|
| $n = 4k+2,\ k \in \mathbb{N}$ | $e_f(0) = 2n, \quad e_f(1) = 2n-1$ |
| $n = 4k, \quad k \in \mathbb{N}$ | $e_f(0) = 2n-1, \quad e_f(1) = 2n$ |
| $n = 4k+1,\ k \in \mathbb{N}$ | $e_f(0) = 2n, \quad e_f(1) = 2n-1$ |
| $n = 4k-1,\ k \in \mathbb{N}$ | $e_f(0) = 2n-1,\ e_f(1) = 2n$ |

In all cases we have $|e_f(0) - e_f(1)| \le 1$.

Hence $H \odot K_1$ graph of a path $P_n$ is a tri sum perfect square cordial graph.

**Illustration:** A tri sum perfect square cordial labeling of $H \odot K_1$ graph of a path $P_3$ and $H$ graph of a path $P_4$ is shown in figure 7(a) and figure 7(b) respectively.

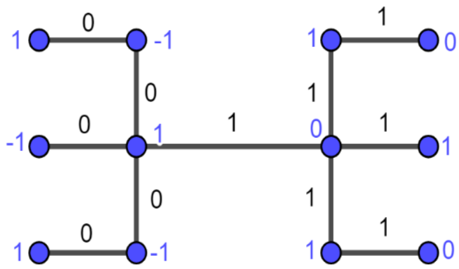

**Figure : 7(a)** $H \odot K_1$ **graph of a path** $P_3$

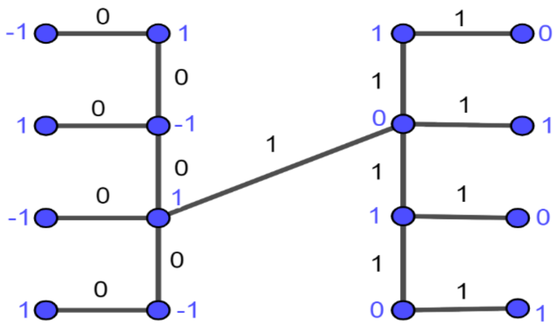

**Figure : 7(b)** $H \odot K_1$ **graph of a path** $P_4$

**Theorem 8** The two star is a tri sum perfect square cordial graph.
**Proof:** Let $G = K_{1,m} \cup K_{1,n}$.
**Case : 1**$n = m$.
We have $V(G) = \{u, v\} \cup \{u_k, v_k / 1 \leq k \leq m\}$ and
$E(G) = \{uu_k, vv_k / 1 \leq k \leq m\}$.
Hence $|V(G)| = 2m + 2$ &$|E(G)| = 2m$.
The required vertex labeling$f: V(G) \to \{-1,0,1\}$ is defined as follows.
**Sub Case: 1**$n = m$ even.

$$f(u) = 1$$
$$f(v) = 1$$
$$f(u_{2k-1}) = 0 1 \leq k \leq \frac{n}{2}$$
$$f(u_{2k}) = -1 \quad 1 \leq k \leq \frac{n}{2}$$
$$f(v_{2k-1}) = 0 1 \leq k \leq \frac{n}{2}$$

$f(v_{2k}) = -1 \quad 1 \leq k \leq \frac{n}{2}$.
We define edge function $f^*: E(G) \to \{0,1\}$ as follows.

$$f^*(uu_{2k-1}) = 1 \qquad 1 \leq k \leq \frac{n}{2}$$
$$f^*(uu_{2k}) = 0 \qquad 1 \leq k \leq \frac{n}{2}$$
$$f^*(vv_{2k-1}) = 1 \qquad 1 \leq k \leq \frac{n}{2}$$

$f^*(vv_{2k}) = 0 \qquad 1 \leq k \leq \frac{n}{2}$.
**Sub Case: 2** $n = m$ odd.

$$f(u) = 1$$
$$f(v) = 1$$
$$f(u_{2k-1}) = 0 1 \leq k \leq \frac{n+1}{2}$$
$$f(u_{2k}) = -1 \quad 1 \leq k \leq \frac{n-1}{2}$$
$$f(v_{2k-1}) = -1 1 \leq k \leq \frac{n+1}{2}$$

$f(v_{2k}) = 0 \quad 1 \leq k \leq \frac{n-1}{2}$.

We define edge function $f^*: E(G) \rightarrow \{0,1\}$ as follows.

$$f^*(uu_{2k-1}) = 1 \qquad 1 \leq k \leq \frac{n+1}{2}$$

$$f^*(uu_{2k}) = 0 \qquad 1 \leq k \leq \frac{n-1}{2}$$

$$f^*(vv_{2k-1}) = 0 \qquad 1 \leq k \leq \frac{n+1}{2}$$

$f^*(vv_{2k}) = 1 \qquad 1 \leq k \leq \frac{n-1}{2}$.

**Case : 2**$n < morm < n$.

We have $V(G) = \{u, v\} \cup \{u_{j,}, v_k / 1 \leq j \leq m, 1 \leq k \leq n\}$ and
$E(G) = \{uu_j, vv_k / 1 \leq j \leq m, 1 \leq k \leq n\}$.
So, $|V(G)| = m + n + 2$ &$|E(G)| = m + n$.
$f: V(G) \rightarrow \{-1,0,1\}$ is defined as follows.

**Sub Case: 1** $n$ even, $m$ odd.

$$f(u) = 1$$

$$f(v) = 1$$

$$f(u_{2j-1}) = -11 \leq j \leq \frac{m+1}{2}$$

$$f(u_{2j}) = 0 \quad 1 \leq j \leq \frac{m-1}{2}$$

$$f(v_{2k-1}) = -11 \leq k \leq \frac{n}{2}$$

$f(v_{2k}) = 0 \quad 1 \leq k \leq \frac{n}{2}$.

We define edge function $f^*: E(G) \rightarrow \{0,1\}$ as follows.

$$f^*(uu_{2j-1}) = 0 \qquad 1 \leq j \leq \frac{m+1}{2}$$

$$f^*(uu_{2j}) = 1 \qquad 1 \leq j \leq \frac{m-1}{2}$$

$$f^*(vv_{2k-1}) = 0 \qquad 1 \leq k \leq \frac{n}{2}$$

$f^*(vv_{2k}) = 1 \qquad 1 \leq k \leq \frac{n}{2}$.

**Sub Case: 2** $n$ odd, $m$ even.

$$f(u) = 1$$

$$f(v) = 1$$

$$f(u_{2j-1}) = -11 \leq j \leq \frac{m}{2}$$

$$f(u_{2j}) = 0 \quad 1 \leq j \leq \frac{m}{2}$$

$$f(v_{2k-1}) = -11 \leq k \leq \frac{n+1}{2}$$

$f(v_{2k}) = 0 \quad 1 \leq k \leq \frac{n-1}{2}$.

We define edge function $f^*: E(G) \rightarrow \{0,1\}$ as follows.

$$f^*(uu_{2j-1}) = 0 \qquad 1 \leq j \leq \frac{m}{2}$$

$$f^*(uu_{2j}) = 1 \qquad 1 \leq j \leq \frac{m}{2}$$

$$f^*(vv_{2k-1}) = 0 \qquad 1 \leq k \leq \frac{n+1}{2}$$

$f^*(vv_{2k}) = 1 \qquad 1 \leq k \leq \frac{n-1}{2}$.

**Case : 3$n < m$**

Similar to case 2 by replacing $n$ &$m$.

**Table:8**

| $\boldsymbol{n}$ | $\boldsymbol{Edge\ condition}$ |
|---|---|
| $n = m$ | $e_f(0) = n, \quad e_f(1) = n$ |
| $m < n \;\; / \; m > n$ | $e_f(0) = \frac{m+n+1}{2}, \quad e_f(1) = \frac{m+n-1}{2}$ |

In all cases we have $|e_f(0) - e_f(1)| \leq 1$.

Hence two star is a tri sum perfect square cordial graph.
**Illustration:** A tri sum perfect square cordial labeling of $K_{1,5} \cup K_{1,6}$is shown in Figure 8.

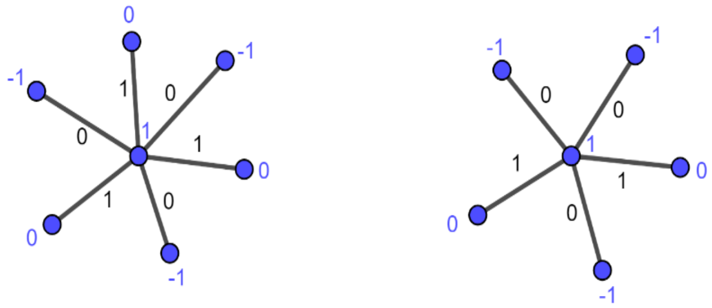


**Figure 8 $K_{1,5} \cup K_{1,6}$**

## III. CONCLUSION

In this paper we have proved results for path, comb, $K_{1,2} * K_{1,n}$, fan, star, $H$ graph of a path $P_n$, $H \odot K_1$ graph of a path $P_n$, two star. All the results in this paper are novel. For the better understanding of the proofs of the theorems, labeling pattern defined in each theorem is demonstrated by an illustration. We can discuss more similar results for various graphs. We have planned to investigate results for more new graphs in our next paper.

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