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Probabilistic Analysis in Investment Project Appraisal

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ABSTRACT: *The financial uncertainty in the results of the investment evaluation creates conditions of risk in the decision making of any strategy in the management of the industrial units. The recent crises (economic and pandemic) have only reinforced the need to explicitly introduce the factor of uncertainty in any new investment decision. The aim of the present paper is to develop a methodological approach contributing to the appraisal of an investment project under risk and uncertainty. Methods from probabilistic analysis, are used and combined. Finally, the implementation of probabilistic theory combined to estimate in a more relevant way the factor of uncertainty considering all perspectives.*

KEY WORDS: *investment, risk, uncertainty, investment project appraisal, Hillier's method, Bayesian analysis, stochastic-probabilistic methods.*

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I. INTRODUCTION AND LITERATURE REVIEW

Economic volatility, because of the ten-year economic crisis in Greece, leads companies to financial uncertainty. Then the financial uncertainty in the results of the investment evaluation creates conditions of risk in the decision making of any strategy in the management of the industrial units. The recent crises (economic and pandemic) have only reinforced the need to explicitly introduce the factor of uncertainty in any new investment decision. If risk and investment are two inseparable concepts (Pezet, 2000) so are uncertainty and investment. In such a context, this paper proposes a methodological approach for the appraisal of an investment project under risk and uncertainty. Methods from probabilistic analysis, are used and combined. Finally, the implementation of probabilistic theory combined to estimate in a more relevant way the factor of uncertainty considering all perspectives.

II. LITERATURE REVIEW

In cases where projects have a significant difference in project costs and different expected net cash flows, the standard deviation used in the previous case is not an indicative method for calculating risk. As the amount of investment differs significantly between the investment proposals, the importance of the standard deviation is altered. (Artikis, 2002). One method for limiting this alteration is to calculate the coefficient variation CV of each sentence. The coefficient of variation shows the amount of risk, as calculated from the standard deviation per unit of expected (NPV). The investment with the lowest rate of volatility has the lowest relative risk. Obviously, that investment proposal will be chosen from among a few mutually excluded investment proposals, which will have the highest expected average (NPV) and the lowest risk (Polyzos, 2018).
For investment proposals that last more than one period and their return varies from period to period, then both the number of expected net cash flows and the number of standard deviations will be the same as the years of application of the investment. The existence of the above precludes a general conclusion. To overcome the difficulty, Hillier proposes the calculation of the expected Net Present Value of the investment plans (ENPV) and then the standard deviation of the ENPV. This calculation is made from the expected net cash flows and standard deviations from the probability distributions of the net cash flows over the life of the investment. Bayes Theorem states that if an event F is known to have occurred and is also known to be associated with one of a set of mutually exclusive events: $E_1, E_2, \ldots, E_n$, then for a particular event, E_j can calculated the values of $\mathrm{P}(F/\mathrm{E_j})$ which called prior probabilities. The probability is $\mathrm{P}(\mathrm{E_j}/\mathrm{F})$, calculated after the outcome F is known, is called a posterior probability (Adams, 1975). Karnavas C., (2024) presents an industrial unit investment appraisal methodology under risk and uncertainty.

HILLIER MODEL

For investment proposals that last more than one period and the performance of which varies from period to period, then both the number of expected net cash flows and the number of standard deviations will be as many years as the implementation of the investment. The existence of the above precludes a general conclusion. To overcome the difficulty, Hillier suggests calculating the expected Net Present Value (ENPV) of investment projects and then the standard deviation of the ENPV. This calculation is made from the expected net cash flows and standard deviations from the probability distributions of the net cash flows throughout the investment period. Of the alternative investment plans under consideration, the one with the smallest standard deviation is preferable. The mathematical formula of ENPV is given by the relation:

$$ENPV = \sum_{t=1}^{n} \frac{E(NCF_i)}{(1+r)^t} - C_0 \tag{1}$$

$E(NCF_i) =$ $\sum_{i=1}^{n} P_i \cdot NCF_i$, the expected period cash flow t
r: the cost of business capital
C_0: the investment cost

If the probability distributions of E(NCF) (Average Net Cash Flows) follow the normal distribution and are independent throughout the duration of the investment plan, then the standard deviation is given by the relation:

$$\sigma_{ENPV}^{2t} = \sqrt{\sum_{t=1}^{n} \frac{\sigma_t^2}{(1+r)^{2t}}} \tag{2}$$

with σ_t^2 the variation in net cash flow period t

If the probability distributions of E(NCF) follow the normal distribution and are fully correlated throughout the duration of the investment plan, then the standard deviation is given by the relation (Artikis 2002; Fotis, 2015 Damigos, 2006):

$$\sigma_{t\ ENPV} = \sum_{t=1}^{n} \frac{\sigma_t}{(1+r)^t} \tag{3}$$

If the probability distributions of E(NCF) follow the normal distribution and are partially correlated, i.e. the probability of achieving a future performance depends partially on the probabilities of achieving past returns throughout the investment plan (Alexandridis, 2005; Fotis, 2015), then the following mathematical formulas apply:

$$NPV_t = \sum_{t=1}^{n} \left[\frac{E(NCF_i)}{(1+r)^t}\right] - C_0 \tag{4}$$

and

$$ENPV = \sum_{i=1}^{n} P_{i,t} \cdot NPV_t \tag{5}$$

where $P_{i,t}$ is the probability of realizing partially correlated returns for $t,\ i = 1, \ldots . n$ The standard deviation of an investment proposition with partially correlated returns is:

$$\sigma = \sqrt{\sum_{t=1}^{n} (NPV_t - ENPV)^2 \cdot P_{i,t}} \tag{6}$$

Among mutually exclusive investment plans, the one with the largest ENPV and smallest standard deviation is preferred. If the investment plans present the same ENPV, then the one with the smallest standard deviation is preferred, while if they have the same standard deviation, the one with the largest ENPV is preferred. Calculating the standard deviations of the NPV of investment projects with the Hillier method is relatively easy. (Fotis, 2015)

BAYES MODEL

For analysis using Bayesian statistics, a probability model is selected for the data (DeGroot & Schervish, 2002; Hirshleifer & Riley, 2002). Then an initial distribution - normal - is defined for the probabilistic parameters, which represents the current knowledge about the probabilistic quantities and the uncertainty about the evolution of their values. (Tasios, 2016)

In the first stage, the company operates in an environment with a strong lack of information and therefore the evolution of the probabilistic quantities shows great uncertainty. This situation is simulated with normal distributions that we assume have a large value of standard deviation.

If y_i, i = 1,2...,.are the n requested values of the probabilistic parameters which are assumed to follow a normal distribution with mean x and standard deviation σ, and θ is the vector of unknown parameters which follows a normal distribution with mean M and standard deviation τ, then the initial distribution p(θ) is given by the formula:

$$p(\theta) = \frac{1}{\sqrt{2\pi\tau^2}} exp\left\{-\frac{(\theta - M)^2}{2\tau^2}\right\} \tag{7}$$

In the second stage, when the data are known, the likelihood function is constructed. The probability function L(θ|y) is the complex probability function which is a function of the parameters and considers as fixed quantities the data that became known.

$$L(y/\theta) = p(y_1, y_2, \dots y_n/\theta) = \prod_1^n \frac{1}{\sqrt{2\pi\sigma^2}} exp\left\{-\frac{(x_i - M)^2}{2\sigma^2}\right\} \tag{8}$$

Then by combining the likelihood function with the initial distribution, the final distribution p(θ|y) is determined, which quantifies the uncertainty in relation to the values of the stochastic parameters of the model after the data are known. So, the evolution of the probability distributions of the stochastic parameters changes from the initial estimate considering the new information. Finally, based on the final distribution, the requested sizes are calculated. (Tasios, 2016)

$$p(\theta/y) = \frac{p(\theta)p(y/\theta)}{\int p(\theta)p(y/\theta)\, d\theta} = \frac{p(\theta)L(y/\theta)}{p(y)} \propto p(\theta)L(y/\theta) \tag{9}$$

he main goal of Bayesian analysis is to determine the final distribution of the stochastic parameters of the problem. The final distribution can be described as a weighted average between the knowledge of the values of the stochastic parameters in an uncertain environment (initial distribution) and the information obtained after the uncertainty is removed and the values of these parameters are known (likelihood function).

When the initial distribution and the likelihood function are normal distributions, it follows that the form of the final distribution is also normal (Lynch, 2007). The standard deviation of the final distribution is less than the standard deviation of the original distribution as well as less than the standard deviation of the likelihood function. Therefore, the combination of these two functions gives us more accurate information compared to the initial estimates for the values of the probabilistic quantities. The statistic is based on the likelihood f(y/θ), where θ is a vector of parameters, there are unknown quantities to be estimated, the estimation is done through estimators with some good properties (e.g. bias) and the estimators are found by maximizing the likelihood. Bayesian statistics considers the parameters as random variables, defines prior distributions f(θ) and is based on the posterior distribution $f(\theta/y)$.

III. RESEARCH METHODOLOGY

The proposed methodological approach to the risk analysis and uncertainty of an industrial investment plan is based on probabilities, followed by a detailed description through a series of stages. (Karnavas, 2024)

ESTIMATION UNCERTAINTY OF NPV USING PROBABILITIES.

The variance of NPV is given by the equation:

$$(\sigma^2_{\mathrm{NPV_k}}) = \sum_{i=1}^{3}\sum_{j=1}^{4}[\mathrm{P(A_i)} \cap \mathrm{P(B_j)}]\left[\overline{\mathrm{NPV}}_k - \mathrm{NPV_{kij}}\right]^2 \tag{10}$$

where:

$$\overline{\mathrm{NPV}}_k = \sum_{i=1}^{3}\sum_{j=1}^{4}[\mathrm{P(A_i)} \cap \mathrm{P(B_j)}]\mathrm{NPV_{kij}} \tag{11}$$

and

$$\mathrm{NPV_{kij}} = -\mathrm{C_{ik}} + (\mathrm{DCF_{kj}})_{\mathrm{MC}} \qquad \mathrm{i} = 1,2,3 \quad \mathrm{j} = 1,2,3,4 \tag{12}$$

with:

$\mathrm{k} = 1$ perspective of national economy

$\mathrm{k} = 2$ perspective of investors

$k = 3$ perspective of shareholders
$i = 1$ basic scenario of construction period
$i = 2$ optimistic scenario of construction period
$i = 3$ pessimistic scenario of construction period
$j = 1$ basic scenario of operation period
$j = 2$ optimistic scenario of operation period
$j = 3$ pessimistic scenario of operation period
$j = 4$ super-pessimistic scenario of operation period
C_{ik}: cost of project for each scenario i and perspective k
$(DCF_{kj})_{MC}$: Discount Cash Flow which calculated by MC simulation for each scenario j and perspective k

The standard deviation σ_{NPV_k} is:

$$\sigma_{NPV_k} = \sqrt{\sigma^2_{NPV_k}} \tag{13}$$

and the coefficient of variation which shows the risk of project is:

$$CV_k = \frac{\sigma_{NPV_k}}{\overline{NPV}_k} \tag{14}$$

After the Expected Net Present Value for each perspective k is calculated:

$$E(NPV)_k = -E(C_{ki}) + E(DCF)_{kj}$$
$$E(NPV)_k = -\sum_{i=1}^{3} P(A_i)C_{ki} + \sum_{j=1}^{4} P(B_j)(DCF_{kj})_{MC} \tag{15}$$

where:

$P(A_i)$: Probabilities of financial conditions in construction period
$P(B_j)$: Probabilities of financial conditions in operation period
$(DCF_{kj})_{MC}$: Discount Cash Flow which calculated by MC simulation for each scenario j and perspective k

$$(DCF_{kj})_{MC} = \sum_{n=1}^{10} \frac{ENCF_{kj}}{(1 + r_{kj})^n}, \qquad j = 1,.,,4 \qquad n = 1,.,10 \qquad r = \text{discount rate}$$

with:
$ENCF_{kj}$: Expected Net Cash Flow which calculated by MC simulation for each scenario j and perspective k.

A sensitivity analysis is performed on the changes of the probability percentages of the realization of the financial conditions. The cases are described:
1. Increase the probability percentage of stagnation conditions by 0.05 and 0.10 points and at the same time decrease (equal cumulatively) the probability percentage of growth and recession conditions, respectively.
2. Reduction of the probability percentage of stagnation conditions by 0.05 and 0.10 points and simultaneous increase (equal cumulatively) of the probability percentage of growth and recession conditions, respectively.

ESTIMATION UNCERTAINTY OF NPV USING BAYES PROBABILITIES.

The case of the sample survey Δ is introduced, according to which in a total of n companies in the sector, x has a market share of p_1 which is desirable for the investment to be examined. In case of optimistic economic conditions, the share amounts to p_2, in pessimistic conditions to p_3, while in very pessimistic conditions it reaches p_4. The priori probabilities Bayes $P(^{\Delta}/_{B_i})$, i.e., the result for Δ to occur since the economic conditions are B_i, $i = 1,2,3,4$ is made using the binomial distribution,

$$P(X = x) = \frac{n!}{x!\,(n - x)!} p^x (1 - p)^{n-x} \tag{16}$$

p: market share in the respective economic conditions
The posterior probabilities $P(^{B_i}/_{\Delta})$ from Bayes' theorem are then calculated:

$$\mathrm{P}\left(\mathrm{B_j}/_{\Delta}\right) = \frac{\mathrm{P}\left(\Delta/_{\mathrm{B_j}}\right)\mathrm{P}(\mathrm{B_j})}{\sum_{i=1}^{4}\mathrm{P}\left(\Delta/_{\mathrm{j}}\right)\mathrm{P}(\mathrm{B_j})} = \frac{\mathrm{P}(\Delta \cap \mathrm{B_j})}{\mathrm{P}(\Delta)} \tag{17}$$

Similarly for the construction period the priori $P(\Delta/_{A_i})$ and then the posterior probabilities Bayes $P\left(\Delta/_{A_i}\right)$ are calculated.

$$\mathrm{P}\left(\mathrm{A_i}/_{\Delta}\right) = \frac{\mathrm{P}\left(\Delta/_{\mathrm{A_i}}\right)\mathrm{P}(\mathrm{A_i})}{\sum_{i=1}^{3}\mathrm{P}\left(\Delta/_{\mathrm{A_i}}\right)\mathrm{P}(\mathrm{A_i})} = \frac{\mathrm{P}(\Delta \cap \mathrm{A_i})}{\mathrm{P}(\Delta)} \tag{18}$$

The variance of NPV is given by the equation:

$$\sigma^2_{\mathrm{NPV\ BAYES_k}} = \sum_{i=1}^{3}\sum_{j=1}^{4}[\mathrm{P}(^{\mathrm{A_i}}/_{\Delta}) \cap \mathrm{P}(^{\mathrm{B_j}}/_{\Delta})]\left[\overline{\mathrm{NPV}}_{\mathrm{BAYES_k}} - \mathrm{NPV_{kij}}\right]^2 \tag{19}$$

where:

$$\overline{\mathrm{NPV}}_{\mathrm{BAYES_k}} = \sum_{i=1}^{3}\sum_{j=1}^{4}[\mathrm{P}(^{\mathrm{A_i}}/_{\Delta}) \cap \mathrm{P}(^{\mathrm{B_j}}/_{\Delta})]\ \mathrm{NPV_{kij}} \tag{20}$$

And

$$\mathrm{NPV_{kij}} = -\mathrm{C_{ki}} + (\mathrm{DCF_{kj}})_{\mathrm{MC}} \quad \mathrm{k} = 1,2,3 \quad \mathrm{i} = 1,2,3 \quad \mathrm{j} = 1,2,3,4$$

where:

$\mathrm{C_{ki}}$: cost of project for each scenario i and perspective k

$(\mathrm{DCF_{kj}})_{\mathrm{MC}}$ Discount Cash Flow which calculated by MC simulation for each scenario j and perspective k

The standard deviation $\sigma_{\mathrm{NPV\ BAYES_k}}$ is:

$$\sigma_{\mathrm{NPV\ BAYES_k}} = \sqrt{\sigma^2_{\mathrm{NPV\ BAYES_k}}} \tag{21}$$

And the Coefficient of Variation (Bayes) which shows the risk of project is:

$$\mathrm{CV_{BAYES_k}} = \frac{\sigma_{\mathrm{NPV\ BAYES_k}}}{\overline{\mathrm{NPV}}_{\mathrm{BAYES_k}}} \tag{22}$$

After the Expected Net Present Value (Bayes) for each perspective k is calculated:

$$\mathrm{E(NPV)_{BAYES_k}} = -\mathrm{E(C_i)_{BAYES}} + \mathrm{E(DCF)_{BAYES\ kj}}$$

$$\mathrm{E(NPV)_{BAYES_k}} = -\sum_{i=1}^{3}\mathrm{P}\left(\mathrm{A_i}/_{\Delta}\right)\mathrm{C_{ki}} + \sum_{j=1}^{4}\mathrm{P}\left(\mathrm{B_j}/_{\Delta}\right)(\mathrm{DCF_{kj}})_{\mathrm{MC}} \tag{23}$$

where:

$\mathrm{P}\left(\mathrm{A_i}/_{\Delta}\right)$: Posteriori probabilities Bayes of financial conditions in construction period

$\mathrm{P}\left(\mathrm{B_j}/_{\Delta}\right)$: Posteriori probabilities Bayes of financial conditions in operation period

$(\mathrm{DCF_{kj}})_{\mathrm{MC}}$: Discount Cash Flow which calculated by MC simulation

$$(\mathrm{DCF_{kj}})_{\mathrm{MC}} = \sum_{n=1}^{10}\frac{\mathrm{ENCF_{kj}}}{(1 + \mathrm{r_{kj}})^{\mathrm{n}}}, \quad \mathrm{j} = 1,2,3,4 \quad \mathrm{n} = 1,2,\ldots,10 \quad \mathrm{r} = \text{discount rate}$$

with:

$ENCF_{kj}$: Expected Net Cash Flow which calculated by MC simulation for each scenario j and perspective k

Then a sensitivity analysis will be performed on the changes in the priori probabilities of the economic conditions of stagnation, growth, and recession. Finally, the uncertainty (probability) and the Bayesian probability of E(NPV)will be estimated to be between different values for all optical k (national economy, investors, shareholders) using the formula:

$$P(a \leq NPV_k \leq b) = P\left(\frac{a - E(NPV)_k}{\sigma_{NPV_k}} \leq \frac{NPV_k - E(NPV)_k}{\sigma_{NPV_k}} \leq \frac{b - E(NPV)_k}{\sigma_{NPV_k}}\right)$$
$$= \Phi\left(\frac{b - E(NPV)_k}{\sigma_{NPV_k}}\right) - \Phi\left(\frac{\alpha - E(NPV)_k}{\sigma_{NPV_k}}\right) \quad (24)$$

Then the application of the methodology to the construction of an industrial brewery unit will be presented.

DATA ANALYSIS

Next, the data that will be used in the application of the methodology are given.

Table 1: Net Cash Flows (NCF), Investment Costs (C) and Cost of Capital (Cost Cap) for the three scenarios under consideration

BASELINE SCENARIO	OPTIMISTIC SCENARIO	PESSIMISTIC SCENARIO	VERY PESSIMISTIC SCENARIO
$\mathbf{NCF_1 \cong 2.735.000}$ € $\mathbf{C_1 = 10.000.000}$ € $\mathbf{cost_{cap} = 4,4\%}$	$NCF_1 \cong 8.415.000$ € $C_1 = 8.000.000$€ $cost_{cap} = 4,2\%$	$NCF_1 \cong 182.000$ € $C_1 = 12.000.000$€ $cost_{cap} = 4,6\%$	$NCF_1 \cong 100.000$ € $cost_{cap} = 4,6\%$
$\mathbf{NCF_2 \cong 1.676.000}$ € $\mathbf{C_2 = 10.000.000}$€ $\mathbf{cost_{cap} = 4,4\%}$	$NCF_2 \cong 5.145.000$ € $C_2 = 8.000.000$€ $cost_{cap} = 4,2\%$	$NCF_2 \cong -43.500$ € $C_2 = 12.000.000$€ $cost_{cap} = 4,6\%$	$NCF_2 \cong -200.500$ € $cost_{cap} = 4,6\%$
$\mathbf{NCF_3 \cong 1.967.000}$ € $\mathbf{C_3 = 3.210.000}$ $\mathbf{cost_{cap} = 10\%}$	$NCF_3 \cong 7.063.000$ € $C_3 = 2.610.000$ $cost_{cap} = 10\%$	$NCF_3 \cong -686.000$ € $C_3 = 3.810.000$ $cost_{cap} = 10\%$	$NCF_3 \cong -2.000.000$ € $cost_{cap} = 10\%$

Table 2: Possibilities of financial conditions of construction and operating period of investment plan

POSSIBILITIES OF FINANCIAL CONDITIONS OF CONSTRUCTION PERIOD		POSSIBILITIES OF FINANCIAL CONDITIONS OF OPERATING PERIOD	
FINANCIAL CONDITIONS (A_i)	PROBABILITIES $P(A_i)$	FINANCIAL CONDITIONS (B_j)	PROBABILITIES $P(B_j)$
A1. STAGNATION (BASIC SCENARIO)	$P(A_1) = 0,5$	B1. STAGNATION (BASIC SCENARIO)	$P(B_1) = 0,40$
A2. GROWTH (OPTIMISTIC SCENARIO)	$P(A_2) = 0,25$	B2. GROWTH (OPTIMISTIC SCENARIO)	$P(B_2) = 0,30$
A3RECESSION (PESSIMISTIC SCENARIO)	$P(A_3) = 0,25$	B3 RECESSION (PESSIMISTIC SCENARIO)	$P(B_3) = 0,20$
		B4 HIGH RECESSION (VERY-PESSIMISTIC SCENARIO)	$P(B_4) = 0,10$

Table 3: Bayesian probabilities matrix of economic conditions for construction period

(A_i)	$P(A_i)$	$P(\Delta/A_i)$	$P(\Delta \cap A_i) = P(\Delta/A_1)P(A_1)$	$P(A_i/\Delta)$
0,035	0,25	0,2713833618	0,0678458405	0,2977635165
0,025	0,50	0,2271107333	0,1135553667	0,498374625
0,020	0,25	0,1858008572	0,0464502143	0,2038618587
SUM	1		0,2278514215	1

Table 4: Bayesian probabilities matrix of economic conditions for annual net operating flows

(B_i)	$P(B_i)$	$P(\Delta/B_i)$	$P(\Delta \cap B_i) = P(\Delta/B_1)P(B_1)$	$P(B_i/\Delta)$
0,035	0,30	0,2713833618	0,0814150085	0,3654788005
0,025	0,40	0,2271107333	0,0908442933	0,4078076507
0,020	0,20	0,1858008572	0,0371601714	0,1668151256
0,015	0,10	0,1334312857	0,0133431286	0,0598984232
SUM	1		0,2227626018	1

IV. RESULTS

By substituting the data into the equations of the methodology, the following results are obtained.

ESTIMATION UNCERTAINTY OF NPV USING PROBABILITIES.

Table 5: Valuation of risk (CV) and average expected net present value (NPV) of investment project E(NPV) under conditions of uncertainty for the national economy, investors, and shareholders

NATIONAL ECONOMY	INVESTORS	SHAREHOLDERS
$\mathbf{S_{NPV1} = 26.704.320}$ $\mathbf{CV_{NPV1} = 1,354}$ $\mathbf{E(NPV_1) = 19.722.339}$	$\mathbf{S_{NPV2} = 16.865.262}$ $\mathbf{CV_{NPV2} = 2,184}$ $\mathbf{E(NPV_2) = 7.721.972}$	$\mathbf{S_{NPV3} = 13.493.475}$ $\mathbf{CV_{NPV3} = 1,793}$ $\mathbf{E(NPV_3) = 7.526.330}$

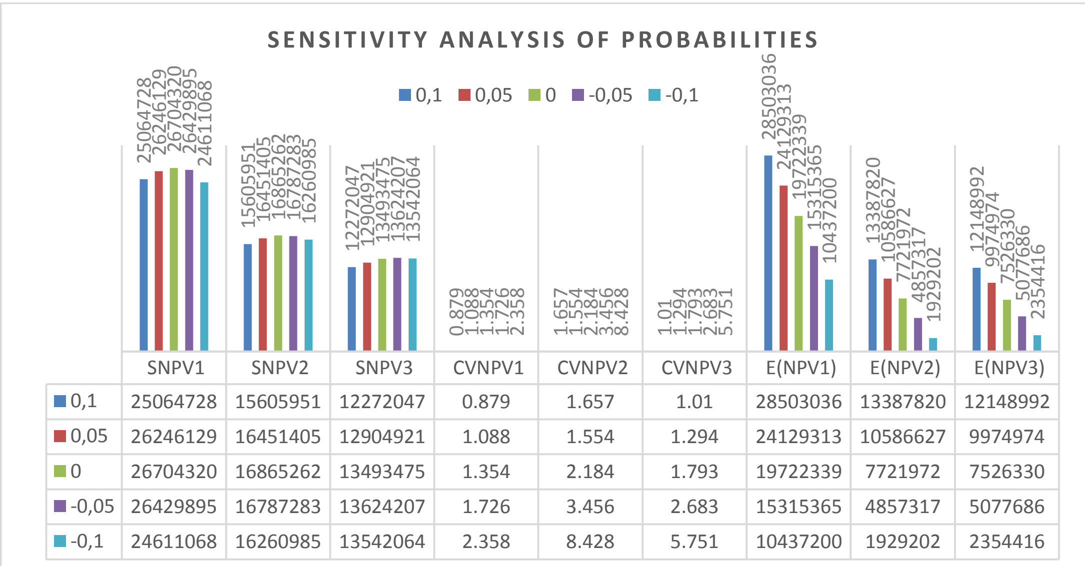


Figure 1: Change S_{NPV}, CV_{NPV}, E(NPV) for all perspectives on a change of financial statements in the interval [-10%, 10%]

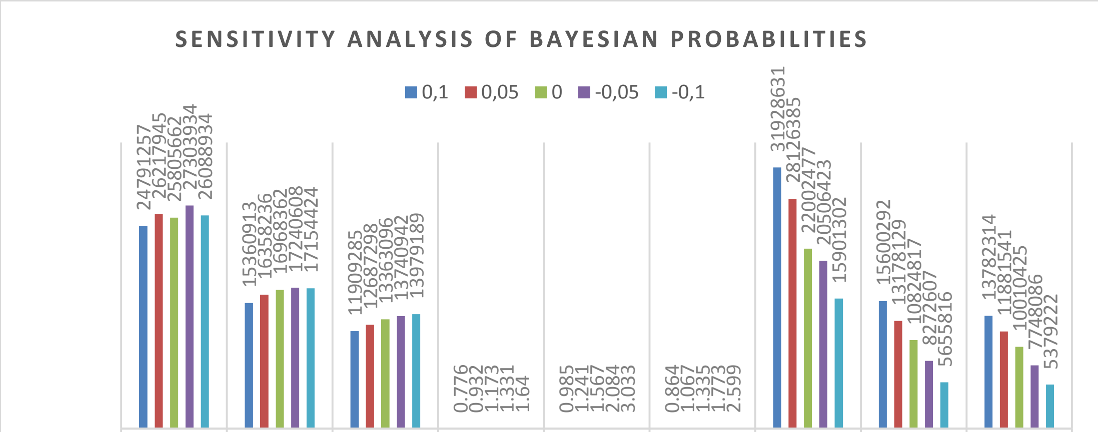
Table 6: Uncertainty estimates of 〖NPV〗_1 〖NPV〗_2, 〖NPV〗_3 for various intervals.

$P(NPV_1 > 0)$	76,73%
$P(15.000.000 < NPV_1 < 25.000.000)$ $P(0 < NPV_1 < 15.000.000)$	15.07% 19,35%
$P(NPV_1 > 15.000.000)$	57,4%
$P(NPV_1 > 25.000.000)$	42,07%
$P(NPV_2 > 0)$ $P(0 < NPV_2 < 5.000.000)$	67,72% 11,36%
$P(5.000.000 < NPV_2 < 15.000.000)$	23%
$P(NPV_2 > 5.000.000)$	56,36%
$P(NPV_2 > 15.000.000)$	33,36%
$P(NPV_3 > 0)$ $P(0 < NPV_3 < 5.000.000)$	71,23% 13,70%
$P(5.000.000 < NPV_3 < 15.000.000)$	28,41%
$P(NPV_3 > 5.000.000)$	57,53%
$P(NPV_3 > 15.000.000)$	29,12%

ESTIMATION UNCERTAINTY OF NPV USING BAYES PROBABILITIES.

Table 7: Valuation of risk (CV) and average expected net present value (NPV) of investment project E(NPV) under conditions of uncertainty for the national economy, investors, and shareholders (Bayes)

NATIONAL ECONOMY	INVESTORS	SHAREHOLDERS
$\mathbf{S_{NPV1BAYES} = 25.805.662}$ $\mathbf{CV_{NPV1\ BAYES} = 1,173}$ $\mathbf{E(NPV_1) = 22.002.477}$	$S_{NPV2BAYES} = 16.968.362$ $CV_{NPV2\ BAYES} = 1,567$ $E(NPV_2) = 10.824.816$	$S_{NPV3\ BAYES} = 13.363.096$ $CV_{NPV3\ BAYES} = 1,335$ $E(NPV_3) = 10.010.425$



	SNPV1	SNPV2	SNPV3	CVNPV1	CVNPV2	CVNPV3	E(NPV1)	E(NPV2)	E(NPV3)
0,1	24791257	15360913	11909285	0.776	0.985	0.864	31928631	15600292	13782314
0,05	26217945	16358236	12687298	0.932	1.241	1.067	28126385	13178129	11881541
0	25805662	16968362	13363096	1.173	1.567	1.335	22002477	10824817	10010425
-0,05	27303934	17240608	13740942	1.331	2.084	1.773	20506423	8272607	7748086
-0,1	26088934	17154424	13979189	1.64	3.033	2.599	15901302	5655816	5379222

Figure 2: Change S_{NPV}, CV_{NPV}, E(NPV) for all perspectives on a Bayesian change of financial statements in the interval [-10%, 10%]

Table 8: Uncertainty estimates (Bayes) of 〖NPV〗_1〖NPV〗_2, 〖NPV〗_3 for various intervals

$P(NPV_{1\ BAYES} > 0)$ $P(0 < NPV_{1\ BAYE\Sigma} < 15.000.000)$	80,23% 19,59%
$P(15.000.000 < NPV_{1\ BAYE\Sigma} < 25.000.000)$	15%
$P(NPV_{1\ BAYE\Sigma} > 15.000.000)$	60,64%
$P(NPV_{1\ BAYE\Sigma} > 25.000.000)$	45,62%
$P(NPV_{2\ BAYES} > 0)$ $P(0 < NPV_{2\ BAYE\Sigma} < 5.000.000)$	73,57% 10,26%
$P(5.000.000 < NPV_{2\ BAYES} < 15.000.000)$	23,18%
$P(NPV_{2\ BAYES} > 5.000.000)$	63,31%
$P(NPV_{2\ BAYES} > 15.000.000)$	40,13%
$P(NPV_{3\ BAYES} > 0)$ $P(0 < NPV_{3\ BAYE\Sigma} < 5.000.000)$	77,34% 12,91%
$P(5.000.000 < NPV_{3\ BAYES} < 15.000.000)$	28,86%
$P(NPV_{3\ BAYES} > 5.000.000)$	64,43%
$P(NPV_{3\ BAYES} > 15.000.000)$	35,57%

V. CONCLUSIONS

With the help of probabilities and the Hillier method, the risk (standard deviation S) and the degree of risk (coefficient of variability CV) of the investment plan were calculated for all perspectives. By using the expected average value (EMV) theorem, $E(NPV_1)$, $E(NPV_2)$, $E(NPV_3)$ were calculated knowing the probability rates for the various economic conditions (stagnation, growth, recession) that prevail both during the construction , as well as during the operational period. The changes of $E(NPV_1)$, $E(NPV_2)$, $E(NPV_3)$ at a change of [-0.10, 0.10] units in the probability rates of economic conditions were also studied (sensitivity analysis).

Finally, with the use of the Bayes theorem, the prior probabilities were calculated and then with the additional knowledge of the market share information of the respective companies with the investment under consideration, the posterior probabilities according to Bayes were calculated. $E(NPV_1)_{BAYES}$, $E(NPV_2)_{BAYES}$, $E(NPV_3)_{BAYES}$ were calculated with the same probability ratios for the various economic conditions. Also studied were the changes in $E(NPV_1)_{BAYES}$, $E(NPV_2)_{BAYES}$, $E(NPV_3)_{BAYES}$ to a [-0.10, 0.10] unit change in the odds ratios of economic conditions (sensitivity analysis). In addition, with the help of Bayes' theorem, the value of information was incorporated into the uncertainty assessment process using prior and posterior probabilities.

In the methodology, a clear separation of the uncertainty assessment using probabilities from that using Bayesian probabilities is made, where the concept of additional information after a sample survey is introduced. The result of a sample survey D is considered, according to which out of a total of 50 companies in the sector, 2 have a market share of 2.5%, which is the target for the investment under consideration. In case of optimistic economic conditions, the share amounts to 3.5%, in pessimistic conditions to 2%, while in very pessimistic conditions it reaches 1.5%.

The subjectivity of the selection of the probabilities of the financial statements is "mitigated" by performing the sensitivity analysis of the uncertainty in a change in the probabilities of the financial statements in an interval [-10% , 10%], both by the method of using probabilities, and when using Bayesian probabilities. An extensive analysis of the change in E(NPV) of the investment is carried out for corresponding changes in the probabilities, but also in the prior probabilities of the financial states (stagnation, growth, recession) for all its visual evaluation.

The methodology developed in this paper is an approach, which helps to formulate a framework for evaluating future industrial investments in conditions of uncertainty. It is therefore a useful guide in decision-making at the level of strategic planning of industrial investments and projects of local or national scope. It is nevertheless necessary to underline that investment planning should include a quantification of the interaction between production and the uncertainty of economic conditions. Therefore, those responsible for the selection and planning of investments must consider the parameter of uncertainty in their evaluation. The exploitation of the positive results brought by the development of investment plans, after their evaluation - contributing to economic growth - requires the strengthening of the efficiency of the economy, giving incentives that improve productivity and competitiveness.

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