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# Mathematical Model for inventory management for perishable items with shortage and selling price dependent demand

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***Abstract**: In the present time, the selling price of inventory is most important for inventory management in the business world, which decides the direction of profit and loss. Every product has its own life, but if they are not preserved safely, they can get destroyed before the stipulated time. Along with this, if the selling price becomes favorable to the consumer, then the demand may increase due to which shortage is natural. Therefore, in this paper an inventory system for perishable goods has been developed using shortage. The objective of this function is to minimize the total costs. We have illustrated our model by numerical example with graphical illustration and finally sensitivity analysis has been illustrated to understand this model based on real-life scenarios.*
***Keywords:** shortage, demand depend on price, perishable items.*
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## I. Introduction:

Supply chain management is of utmost importance to the business world, it is a simple process of managing the flow of finished goods from raw materials through production to the consumer. The Supply chain works like an organization, in which producers, suppliers, and retailers are involved, and some arrange the raw material, some prepare the product in the production plant and some do the work of delivering the inventory to the consumer. Goyal (1977) was the first who present an integrated inventory model consisting of single-supplier and single-customer. In this integrated inventory model. Lee and Rosenblatt (1986) presented an inventory model to address the joint problem of ordering and offering discounts by a supplier to his sole buyer. In this model, they tried to show that the supplier aims to reduce his setup cost and inventory holding cost also motivating the buyer to change the quantity of the order, so that the annual profit of the supplier is to be maximum. Hill (1999) proposed an integrated inventory model consisting of a single vendor and a single supplier. The vendor manufactures the product and ships it to the buyer, where the buyer consumes that product at a fixed rate. Wu and Wee (2001) presented a supply chain inventory model for deteriorating items. Padiyar et al. (2021) developed an integrated inventory model with probabilistic demand. Again Padiyar et al. (2022) introduced green inventory model with reliability. This model considered buyer-seller combined cost with multiple lot size deliveries. This model also tried to achieve minimum combined cost for both seller and buyer.

**Structure of this study**: The current study sections are arranged as follows: Section 1 provides an introduction, while Section 2 presents a literature review to get the motivation for the current work. Section 3 contains notations and basic assumptions required for modelling purposes. The mathematical formulation of the current study is presented in Section 4, and the Solution methodology and numerical example are given in Section 5. Sensitivity analysis and Discussion are presented in Section 6 which helps to validate the model. Final concluding remark is presented in Section 7.

1. **Related Literature Review:** In this section, we have to discussed the literature review related in different direction (1) Inventory models based on deterioration (2) Inventory models based on shortage (3) Inventory models with inflation (4) Inventory models with controllable deterioration (5) Inventory models based on fuzzy learning

(1) **Inventory models based on deterioration:** Every type of Inventory is an important part for human life, which is helpful in running any business efficiently. There are different types of inventory and each inventory has its own life span, after which all these products depreciate at a certain rate. Medicines, chemicals, electric appliances, etc. can deteriorate during the period of storage. The deterioration of any substance after a certain time period is a natural property, so researchers considered it necessary to develop an inventory model keeping this factor in mind. In recent years, many researchers have developed properties related to deterioration of objects in

inventory modelling. Padiyar et al. (2021) developed inventory system with price dependent consumption. Padiyar et al.(2021) developed production inventory model with storage problem. Rathod and Bhathawala (2016) discussed the inventory model for deteriorating item where demand is of triangular type. Padiyar et al. (2022) developed an integrated model with preservation facilities. Rani et al. (2017) introduced the inventory model for deteriorating items through green supply chain with variable demand in inflationary environment. Padiyar et al. (2023) developed three echelon supply chain model. Padiyar et al. (2023) introduced MESCM with exponential demand

**(2) Inventory models based on shortage:** A Shortage occurs when, an incomplete quantity of inventory is responsible for causing disruption in the fulfillment of the customer's wishes. And this might also be possible when the demand for a commodity is greater than its supply. A company may face the following situations. (a)Shortage can be filled by prioritizing and (b) Shortage cannot be filled at all. Kuraie et al. (2021) discussed a model for multivariable demand. Dye et al. (2006) proposed an inventory model for deteriorating items having partially backlogged shortage and time varying demand by adding the lost sale and non-constant purchase cost in their model.

## 3. Assumption and Notations

3.1 **Assumption:** (See Appendix A)

**3.2 Notation:** (See Appendix B)

## 4. Mathematical formulation:

Looking at the beginning of the inventory cycle, at t=0, ℵ (t) there is W units of inventory supply. But over time, the supply of inventory to the customer is affected by two important things, which are decline and demand for inventory, and both of them decrease during this time period. If the demand becomes favorable to the customer due to the selling price then there is a high possibility of shortage which is happening in the interval $[0,T_1]$. To understand this inventory model mathematically, it can be understood by the following equations.

### 4.1. Crisp Model:

$$\frac{d\aleph(t)}{dt} = -D(x) - \tau\aleph(t), \quad 0 \le t \le T_1 \tag{1}$$

$$\frac{d\aleph(t)}{dt} = -D(x), \quad T_1 \le t \le T \tag{2}$$

With the boundary condition ℵ (o) =W and $\aleph(T_1) = 0$

Solution of above equation are

$$\aleph(t) = We^{-\tau t} + \frac{ax^{-b}}{\tau}(e^{-\tau t} - 1); \quad 0 \le t \le T_1 \tag{3}$$

$$\aleph(t) = ax^{-b}(T_1 - t); \quad T1 \le t \le T \tag{4}$$

Since $\aleph(T_1) = 0 \Rightarrow W = -\frac{ax^{-b}}{\tau}(1 - e^{\tau T_1})$

$$\text{So } \aleph(t) = \left(e^{\tau(T_1 - t)} - 1\right)\frac{ax^{-b}}{\tau}; \quad 0 \le t \le T_1 \tag{5}$$

Total average cost of the model (TSC) depends on the following cost:

(a). Holding cost:

$$H_{DM} = h_{DM}\left[\int_0^{T_1} \aleph(t)dt\right]$$

$$H_{DM} = h_{DM}ax^{-b}\left[\frac{{T_1}^2}{2} + \frac{{T_1}^3\tau}{6}\right] \tag{6}$$

(b) Deteriorating cost: This cost is that is borne by the company when the material is damaged or broken or stolen. During inventory management, the biggest challenge in front of the supply manager is to monitor the deteriorating items and try to control them. Deterioration, breakage, and vaporization of goods are their natural properties. In addition, items may also deteriorate during storage or due to seasonal factors. Therefore, it is also necessary to take into account the factor of deterioration of items while developing an inventory model

$$D_{DM} = Z_{DM}\left[\aleph(0) - \int_0^{T_1} Ddt\right]$$

$$D_{DM} = Z_{DM}\frac{ax^{-b}}{2}\tau{T_1}^2 \tag{7}$$

(c) Shortage cost: shortage cost occurs when the demand of a commodity is greater than its supply, and the shortage cost for the entire model is

$$S_{DM} = R_{DM}\left[-\int_{T_1}^{T} \aleph(t)dt\right] = R_{DM}\frac{ax^{-b}}{2}(T_1 - T)^2 \tag{8}$$

(d) Ordering cost:

$O_{DM} = y_{DM}$

Total cost (TSC) for the model is:

$$TSC = \frac{1}{T}[O_{DM} + H_{DM} + D_{DM} + S_{DM}] \tag{9}$$

TSC=

$$\frac{1}{T}\begin{bmatrix} y_{DM} + R_{DM}\frac{ax^{-b}}{2}(T_1 - T)^2 \\ +Z_{DM}\frac{ax^{-b}}{2}\tau {T_1}^2 + h_{DM}ax^{-b}\left[\frac{{T_1}^2}{2} + \frac{{T_1}^3\tau}{6}\right] \end{bmatrix} \quad (10)$$

**5. Numerical example and Solution methodology:**

**5.1.: Solution Methodology**: Differentiate both sides of equation (10) with respect to T, and the necessary condition to obtain the optimal value is

$\frac{dTC(T)}{dT}$ **=0**

Again diffrentiate the equation (10) wrt. T

and if $\frac{d^2TC(T)}{dT^2} > 0$**,** Then the total cost of this model mínimum at t= T

**5.2.: Numerical Example:**

To understand the above model for real life, the optimal value has been calculated using Mathematica 11.3 software by giving numerical values of the parameters taken in the model.

$h_{DM}$= 3 $/unit, a =100 unit, b=0.15, $\tau$ = 0.02 rs, x = 20$/unit, $Z_{DM}$ =5$ /unit, $R_{DM}$ = 10$/unit, $Y_{DM}$ =250&, $T_1$ = 10days

The optimum solution is $T = 20.days$ and TSC = 5245 $

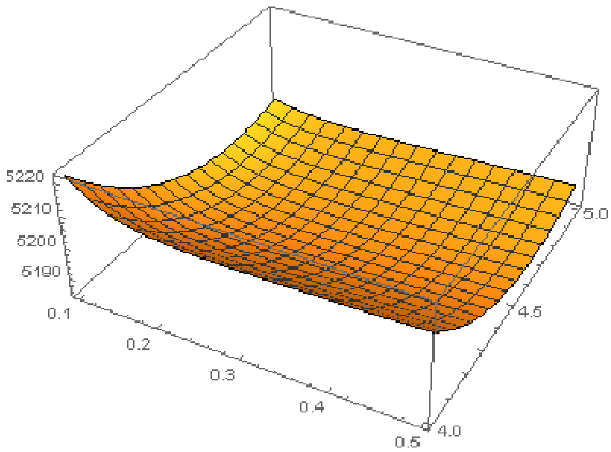


**Figure1:** The convexity graph of TSC for the above model

**6. Sensitivity analysis and Discussion:** Sensitivity analysis for some influencing parameters has been done in Table 1 to Table 4,

**Table 1.**

If the value of deteriorating cost is reduced by 20% from its original value, then it is seen that both the total cost and inventory cycle length are decreasing. Also, if the value of deteriorating cost is increased by 20% from its original value. While total costs and inventory cycle lengths are continuously increasing

| % variation in $Z_{DM}$ | TSC | T |
|---|---|---|
| -20% | 4024.50 | 20.004 |
| -10% | 4029.35 | 20.051 |
| 10% | 5246.25 | 21.240 |
| 20% | 5246.95 | 22.251 |

**Table.2**. If the value of shortage cost is reduced by 20% from its original value, then it is seen that the total cost is increasing but the inventory cycle length is decreasing, along with this if the value of shortage cost is reduced by 20% from its original value. %, the total cost is decreasing but the inventory cycle length is continuously increasing.

| % variation in $R_{DM}$ | TSC | T |
|---|---|---|
| -20% | 5245.50 | 19.5 |
| -10% | 5245.30 | 19.51 |
| 10% | 5244.25 | 20.20 |
| 20% | 5243.25 | 20.245 |

**Table.3.**
If the value of deterioration rate is reduced by 20% from its original value, then it is seen that both the total cost and inventory cycle length are decreasing. Also, if the value of deteriorating cost is reduced by 20% from its original value. When increased, total costs and inventory cycle lengths continue to increase.

| % variation in $\tau$ | TSC | T |
|---|---|---|
| -20% | 4024.50 | 20.004 |
| -10% | 4029.35 | 20.051 |
| 10% | 5246.25 | 21.240 |
| 20% | 5246.95 | 22.251 |

**Table.4.**
If the value of ordering cost is reduced by 20% from its original value, then it is seen that both the total cost and inventory cycle length are decreasing. Also, if the value of deteriorating cost is reduced by 20% from its original value. When increased, total costs and inventory cycle lengths continue to increase.

| % variation in $y_{DM}$ | TSC | T |
|---|---|---|
| -20% | 4024.50 | 20.004 |
| -10% | 4029.35 | 20.051 |
| 10% | 5246.25 | 21.240 |
| 20% | 5246.95 | 22.251 |

## 7. Conclusions

In this paper, an inventory model for items that are perishable after a certain period of time is considered. Shortage has been given importance in the model and also sensitivity analysis has been done by taking important numerical values to understand the model in real life, after sensitivity it is observed that. If the deterioration rate decreases then the total cost is also decreasing. And this indicates that more attention is needed to keep the product safe. This research can be extended in smart production inventory model, multi echelon supply chain model, different carbon reduction policies (such as carbon cap and trade, carbon caps, etc.), shortage, advance payment discount facility, muti-items, smart production with control on deteriorated items etc.

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## Appendix A:

(1) Demand is depended on selling price i.e. $D(x) = ax^{-b}$ where a ,b > 0 and x is the selling price of an item.
(2) Deterioration rate is constant.
(3) Shortages are allowed**.**

**Appendix B:**

$y_{DM}$ Ordering cost.
$\tau$ Deterioration rate, $0< \tau << 1$
$h_{DM}$ Holding cost per unit
$R_{DM}$ Shortage cost per unit.
$Z_{DM}$ Deteriorating cost per unit
x Selling price of an item
T Cycle length
W Ordering Quantity per unit
TSC Total supply cost