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On Delta Harmonic and Multiplicative Delta Harmonic Indices of Some Nanostructures

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***ABSTRACT**: We introduce the delta harmonic index and its polynomial of a graph. Also we propose the multiplicative delta harmonic index of a graph. In this work, we compute these newly defined delta harmonic indices for certain nanostructures.*
***KEYWORDS**: delta harmonic index, multiplicative delta harmonic index, nanotube, network.*

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I. Introduction

Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u. Let $\delta(G)$ denote the minimum degree among the vertices of G. We refer [1] for undefined notations and terminologies.

Graph indices have their applications in various disciplines of Science and Technology.

The δ vertex degree was defined in [2] as

$$\delta_u = d_G(u) - \delta(G) + 1.$$

We introduce the delta harmonic index of a graph and it is defined as

$$\delta H(G) = \sum_{uv \in E(G)} \frac{2}{\delta_u + \delta_v}.$$

Considering the delta harmonic index, we define the delta harmonic polynomial of a graph G as

$$\delta H(G, x) = \sum_{uv \in E(G)} x^{\frac{2}{\delta_u + \delta_v}}.$$

Recently, some delta Banhatti indices were studied in [3, 4, 5, 6].

We define the multiplicative delta harmonic index of a graph G as

$$\delta NII(G) = \prod_{uv \in E(G)} \frac{2}{\delta_u + \delta_v}.$$

Recently, some harmonic indices were studied in [7, 8, 9, 10, 11, 12, 13, 14].

In this work, we determine the delta harmonic index and its corresponding polynomial of some nanostructures. Also we compute the multiplicative delta harmonic index of some nanostructures.

II. Results for $HC_5C_7[p,q]$ Nanotubes

We focus on the family of nanotubes, denoted by $HC_5C_7[p,q]$, in which p is the number of heptagons in the first row and q rows of pentagons repeated alternately. Let G be the graph of a nanotube $HC_5C_7[p,q]$.

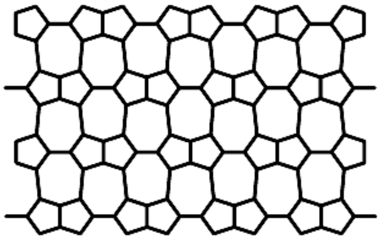

Figure 1: 2-*D* lattice ofnanotubeHC_5C_7 [8, 4]

The 2-D lattice of nanotube $HC_5C_7[p, q]$ is shown in Figure 1.We obtain that G has $4pq$ vertices and $6pq - p$ edges. The graph G has two types of edges based on the degree of end vertices of each edge as follows:

$E_1 = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$, $|E_1| = 4p$.
$E_2 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_2| = 6pq - 5p$.

Clearly $\delta(G)$=2. Therefore $\delta_u = d_G(u) - \delta(G) + 1 = d_G(u) - 1$. Thus there are two types of δ-edges as given in Table 1.

$\delta_u, \delta_v \backslash\ uv \in E(G)$	Number of edges
(1, 1)	$4p$
(2, 2)	$6pq - 5p$

Table 1: δ**-edge partition of** $HC_5C_7[p, q]$

Theorem 1. Let G be the graph of a nanotube $HC_5C_7[p, q]$.Then

(i) $\delta H(G) = 3pq + \frac{1}{6}p.$

(ii) $\delta H(G, x) = 4px^{\frac{2}{3}} + (6pq - 5p)x^{\frac{1}{2}}$

Proof: From definitions and by using Table 1, we deduce

(i)
$$\delta H(G) = \sum_{uv \in E(G)} \frac{2}{\delta_u + \delta_v}$$
$$= 4p\frac{2}{1+2} + (6pq - 5p)\frac{2}{2+2}$$
$$= 3pq + \frac{1}{6}p.$$

(ii)
$$\delta H(G, x) = \sum_{uv \in E(G)} x^{\frac{2}{\delta_u + \delta_v}}$$
$$= 4px^{\frac{2}{1+2}} + (6pq - 5p)x^{\frac{2}{2+2}}$$
$$= = 4px^{\frac{2}{3}} + (6pq - 5p)x^{\frac{1}{2}}.$$

Theorem 2. Let G be the graph of a nanotube $HC_5C_7[p, q]$.Then

$$\delta HII(G) = \left(\frac{2}{3}\right)^{4p} \times \left(\frac{1}{2}\right)^{(6pq - 5p)}.$$

Proof: From definition and by using Table 1, we deduce

$$\delta HII(G)=\prod_{uv\in E(G)}\frac{2}{\delta_u+\delta_v}$$
$$=\left(\frac{2}{1+2}\right)^{4p}\times\left(\frac{2}{2+2}\right)^{(6pq-5p)}$$
$$=\left(\frac{2}{3}\right)^{4p}\times\left(\frac{1}{2}\right)^{(6pq-5p)}.$$

III. Results for $SC_5C_7[p,q]$ Nanotubes

We focus on the family of nanotubes, denoted by $SC_5C_7[p,q]$, in which p is the number of heptagons in the first row and q rows of vertices and edges are repeated alternately. The 2-D lattice of nanotube $SC_5C_7[p,q]$ is presented in Figure 2.

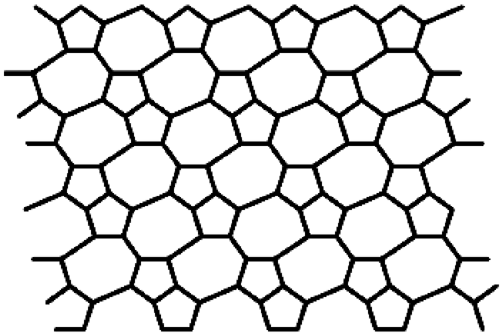

Figure 2: 2-*D* lattice of nanotube $SC_5C_7[p,q]$

Let G be the graph of $SC_5C_7[p,q]$. We obtain that G has $4pq$ vertices and $6pq - p$ edges. Also by calculation, we get that G has three types of edges based on the degree of end vertices of each edge as follows:

$E_1 = \{uv\in E(G) \mid d_G(u) = d_G(v) = 2\}$, $|E_1| = q$.
$E_2 = \{uv\in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$, $|E_2| = 6q$.
$E_2 = \{uv\in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_3| = 6pq - p - 7q$.

Clearly $\delta(G)$=2. Thus $\delta_u = d_G(u) - \delta(G) + 1 = d_G(u) - 1$. There are three types of δ-edges as given in Table 2.

$\delta_u,\delta_v \backslash uv \in E(G)$	Number of edges
(1, 1)	q
(1, 2)	$6q$
(2, 2)	$6pq - p - 7q$

Table 2: δ-edge partition of $SC_5C_7[p, q]$

Theorem 3. Let G be the graph of a nanotube $SC_5C_7[p, q]$. Then

(i) $\delta H(G)=3pq-\frac{1}{2}p+\frac{3}{2}q.$

(ii) $\delta H(G,x)=qx^{1}+6qx^{\frac{2}{3}}+(6pq-p-7q)x^{\frac{1}{2}}.$

Proof: From definitions and by using Table 2, we deduce

(i) $\delta H(G)=\sum_{uv\in E(G)}\frac{2}{\delta_u+\delta_v}$

$$= q\frac{2}{1+1} + 6q\frac{2}{1+2} + (6pq - p - 7q)\frac{2}{2+2}$$
$$= 3pq - \frac{1}{2}p + \frac{3}{2}q.$$

(ii) $\delta H(G,x) = \sum_{uv \in E(G)} x^{\frac{2}{\delta_u + \delta_v}}$

$$= qx^{\frac{2}{1+1}} + 6qx^{\frac{2}{1+2}} + (6pq - p - 7q)x^{\frac{2}{2+2}}$$
$$= qx^1 + 6qx^{\frac{2}{3}} + (6pq - p - 7q)x^{\frac{1}{2}}.$$

Theorem 4. Let G be the graph of a nanotube $SC_5C_7[p, q]$.Then

$$\delta HII(G) = 1 \times \left(\frac{2}{3}\right)^{6q} \times \left(\frac{1}{2}\right)^{6pq-p-7q}.$$

Proof: From definition and by using Table 2, we deduce

$$\delta HII(G) = \prod_{uv \in E(G)} \frac{2}{\delta_u + \delta_v}$$
$$= \left(\frac{2}{1+1}\right)^{q} \times \left(\frac{2}{1+2}\right)^{6q} \times \left(\frac{2}{2+2}\right)^{6pq-p-7q}.$$
$$= 1 \times \left(\frac{2}{3}\right)^{6q} \times \left(\frac{1}{2}\right)^{6pq-p-7q}.$$

IV. Results for Honeycomb Networks

If we recursively use hexagonal tiling in particular pattern, honeycomb networks are formed. These networks are very useful in Chemistry and also in Computer Graphics. A honeycomb network of dimension n is denoted by HC_n. A honeycomb network of dimension four is shown in Figure 3.

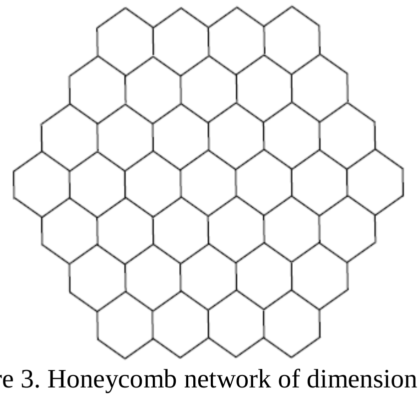

Figure 3. Honeycomb network of dimension four

Let G be the graph of a honeycomb network HC_n. We obtain that G has $6n^2$ vertices and $9n^2 - 3n$ edges.

In G, there are three types of edges based on degrees of end vertices of each edge as follows:

$E_1 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\}$, $|E_1| = 6$.
$E_2 = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$, $|E_2| = 12n - 12$.
$E_3 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_3| = 9n^2 - 15n+6$.

We have $\delta(G)$=3 and hence $\delta_u = d_G(u) - \delta(G) + 1 = d_G(u) - 2$.

Hence there are 3 types of δ-edges as given in Table 3.

$\delta_u, \delta_v \setminus uv \in E(G)$	(1, 1)	(1, 2)	(2, 2)
Number of edges	6	$12n - 12$	$9n^2 - 15n + 6$

Table 3. δ-edge partition of HC_n

Theorem 5. Let G be the graph of a nanotube HC_n. Then

(i) $\delta H(G) = \frac{9}{2}n^2 + \frac{1}{2}n + 1.$

(ii) $\delta H(G, x) = 6x^1 + (12n - 12)x^{\frac{2}{3}} + (9n^2 - 15n + 6)x^{\frac{1}{2}}.$

Proof: From definitions and by using Table 3, we deduce

(i) $$\delta H(G) = \sum_{uv \in E(G)} \frac{2}{\delta_u + \delta_v}$$

$$= 6\frac{2}{1+1} + (12n - 12)\frac{2}{1+2} + (9n^2 - 15n + 6)\frac{2}{2+2}$$

$$= \frac{9}{2}n^2 + \frac{1}{2}n + 1.$$

(ii) $$\delta H(G, x) = \sum_{uv \in E(G)} x^{\frac{2}{\delta_u + \delta_v}}$$

$$= 6x^{\frac{2}{1+1}} + (12n - 12)x^{\frac{2}{1+2}} + (9n^2 - 15n + 6)x^{\frac{2}{2+2}}$$

$$= 6x^1 + (12n - 12)x^{\frac{2}{3}} + (9n^2 - 15n + 6)x^{\frac{1}{2}}.$$

Theorem 6. Let G be the graph of a nanotube HC_n. Then

$$\delta HII(G) = 1 \times \left(\frac{2}{3}\right)^{12n-12} \times \left(\frac{1}{2}\right)^{9n^2-15n+6}.$$

Proof: From definition and by using Table 3, we deduce

$$\delta HII(G) = \prod_{uv \in E(G)} \frac{2}{\delta_u + \delta_v}$$

$$= \left(\frac{2}{1+1}\right)^6 \times \left(\frac{2}{1+2}\right)^{12n-12} \times \left(\frac{2}{2+2}\right)^{9n^2-15n+6}$$

$$= 1 \times \left(\frac{2}{3}\right)^{12n-12} \times \left(\frac{1}{2}\right)^{9n^2-15n+6}.$$

V. Results for Oxide Networks

The oxide networks are of vital importance in the study of silicate networks. An oxide network of dimension n is denoted by of OX_n. An oxide network of dimension five is shown in Figure 4.

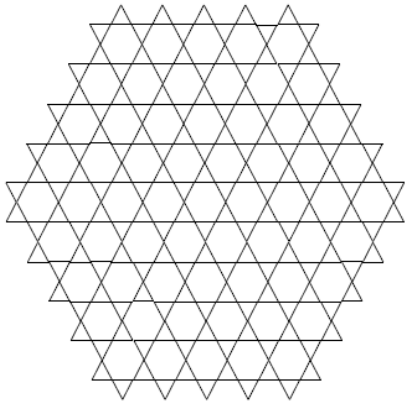

Figure 4. Oxide network of dimension 5

Let G be the graph of an oxide network OX_n. We find that G has $9n^2 + 3n$ vertices and $18n^2$edges.

In G, there are two types of edges based on degrees of end vertices of each edge as follows:

$E_1 = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 4\}$, $|E_1| = 12n$.
$E_2 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 4\}$, $|E_2| = 18n^2 - 12n$.

We have $\delta(G)=2$ and hence $\delta_u = d_G(u) - \delta(G) + 1 = d_G(u) - 1$. Thus there are two types of δ-vertices as given in Table 4.

$\delta_u, \delta_v \setminus uv \in E(G)$	(1, 3)	(3, 3)
Number of edges	$6n$	$9n^2 - 3n$

Table 4. δ-vertex partition of OX_n

Theorem 7. Let G be the graph of a nanotube OX_n. Then

(i) $\delta H(G) = 3n^2 + 2n.$

(ii) $\delta H(G,x) = 6nx^{\frac{1}{2}} + (9n^2 - 3n)x^{\frac{1}{3}}.$

Proof: From definitions and by using Table 4, we deduce

(i) $$\delta H(G) = \sum_{uv \in E(G)} \frac{2}{\delta_u + \delta_v}$$
$$= 6n\frac{2}{1+3} + (9n^2 - 3n)\frac{2}{3+3}$$
$$= 3n^2 + 2n.$$

(ii) $$\delta H(G,x) = \sum_{uv \in E(G)} x^{\frac{2}{\delta_u + \delta_v}}$$
$$= 6nx^{\frac{2}{1+3}} + (9n^2 - 3n)x^{\frac{2}{3+3}}$$
$$= 6nx^{\frac{1}{2}} + (9n^2 - 3n)x^{\frac{1}{3}}.$$

Theorem 8. Let G be the graph of a nanotube OX_n. Then

$$\delta HII(G) = \left(\frac{1}{2}\right)^{6n} \times \left(\frac{1}{3}\right)^{9n^2 - 3n}.$$

Proof: From definition and by using Table 4, we deduce

$$\delta HII(G) = \prod_{uv \in E(G)} \frac{2}{\delta_u + \delta_v}$$
$$= \left(\frac{2}{1+3}\right)^{6n} \times \left(\frac{2}{3+3}\right)^{9n^2-3n}$$
$$= \left(\frac{1}{2}\right)^{6n} \times \left(\frac{1}{3}\right)^{9n^2-3n}.$$

VI. Results for Hexagonal Networks

It is known that there exist three regular plane tilings with composition of some kind of regular polygons such as triangular, hexagonal and square. Triangular tiling is used in the construction of hexagonal networks. This network is denoted by HX_n. A hexagonal network of dimension six is shown in Figure 5.

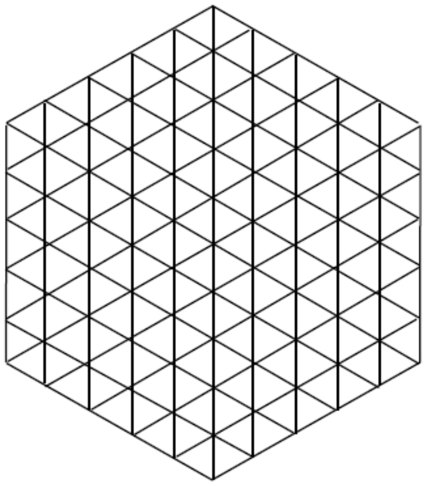

Figure 5. Hexagonal network of dimension six

Let G be the graph of a hexagonal network HX_n. We obtain that G has $3n^2-3n+1$ vertices and $9n^2-15n+6$ edges.

In G, there are five types of edges based on degrees of end vertices of each edge as follows:

$E_1 = \{uv \in E(G) \mid d_G(u) = 3, d_G(v) = 4\}$, $|E_1| = 12$.
$E_2 = \{uv \in E(G) \mid d_G(u) = 3, d_G(v) = 6\}$, $|E_2| = 6$.
$E_3 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 4\}$, $|E_3| = 6n - 18$.
$E_4 = \{uv \in E(G) \mid d_G(u) = 4, d_G(v) = 6\}$, $|E_4| = 12n - 24$.
$E_5 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 6\}$, $|E_5| = 9n^2 - 33n + 30$.

Thus $\delta(G)=3$ and hence $\delta_u = d_G(u) - \delta(G) + 1 = d_G(u) - 2$.

There are five types of δ-edges as given in Table 5.

$\delta_u, \delta_v \setminus uv \in E(G)$	(1, 2)	(1, 4)	(2, 2)	(2, 4)	(4, 4)
Number of edges	12	6	$6n-18$	$12n-24$	$9n^2-33n+30$

Table 5. δ-edge partition of HX_n

Theorem 9. Let G be the graph of a nanotube HX_n. Then

(i) $\delta H(G) = \frac{9}{4}n^2 - \frac{5}{4}n + \frac{29}{10}$.

(ii) $\delta H(G,x) = 12x^{\frac{2}{3}} + 6x^{\frac{2}{5}} + (6n-18)x^{\frac{1}{2}} + (12n-24)x^{\frac{1}{3}} + (9n^2-33n+30)x^{\frac{1}{4}}$.

Proof: From definitions and by using Table 5, we deduce

(i) $$\delta H(G)=\sum_{uv\in E(G)}\frac{2}{\delta_u+\delta_v}$$

$$=12\frac{2}{1+2}+6\frac{2}{1+4}+(6n-18)\frac{2}{2+2}+(12n-24)\frac{2}{2+4}+(9n^2-33n+30)\frac{2}{4+4}$$

$$=\frac{9}{4}n^2-\frac{5}{4}n+\frac{29}{10}.$$

(ii) $$\delta H(G,x)=\sum_{uv\in E(G)}x^{\frac{2}{\delta_u+\delta_v}}$$

$$=12x^{\frac{2}{1+2}}+6x^{\frac{2}{1+4}}+(6n-18)x^{\frac{2}{2+2}}+(12n-24)x^{\frac{2}{2+4}}+(9n^2-33n+30)x^{\frac{2}{4+4}}$$

$$=12x^{\frac{2}{3}}+6x^{\frac{2}{5}}+(6n-18)x^{\frac{1}{2}}+(12n-24)x^{\frac{1}{3}}+(9n^2-33n+30)x^{\frac{1}{4}}.$$

Theorem 10. Let G be the graph of a nanotube HX_n. Then

$$\delta HII(G)=\left(\frac{2}{3}\right)^{12}\times\left(\frac{2}{5}\right)^{6}\times\left(\frac{1}{2}\right)^{6n-18}\times\left(\frac{1}{3}\right)^{12n-24}\times\left(\frac{1}{4}\right)^{9n^2-33n+30}.$$

Proof: From definition and by using Table 5, we deduce

$$\delta HII(G)=\prod_{uv\in E(G)}\frac{2}{\delta_u+\delta_v}$$

$$=\left(\frac{2}{1+2}\right)^{12}\times\left(\frac{2}{1+4}\right)^{6}\times\left(\frac{2}{2+2}\right)^{6n-18}\times\left(\frac{2}{2+4}\right)^{12n-24}\times\left(\frac{2}{4+4}\right)^{9n^2-33n+30}$$

$$=\left(\frac{2}{3}\right)^{12}\times\left(\frac{2}{5}\right)^{6}\times\left(\frac{1}{2}\right)^{6n-18}\times\left(\frac{1}{3}\right)^{12n-24}\times\left(\frac{1}{4}\right)^{9n^2-33n+30}.$$

VII. CONCLUSION

In this study, we have determined the delta harmonic index and its corresponding polynomial for certain nanostructures. Also we have computed the multiplicative delta harmonic index for some nanostructures.

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