

Sum and Product Connectivity Gourava Domination Indices of Graphs

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ABSTRACT: In this paper, we propose the sum and product connectivity Gourava domination indices and their corresponding exponentials of a graph. Furthermore, we compute these newly defined Gourava domination indices their corresponding exponentials of some standard graphs, French windmill graphs, friendship graphs, book graphs.

KEYWORDS: sum connectivity Gourava domination index, product connectivity Gourava domination index, windmill graphs, friendship graphs.

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I. Introduction

The graph $G = (V(G), E(G))$, where $V(G)$ be the vertex set and $E(G)$ be the edge set. $d_G(u)$ be the degree of a vertex u . For undefined term and notation, we refer the books [1, 2]. Graph indices have their applications in various disciplines of Science and Engineering. Recently some new graph indices were studied, for example, in [3, 4, 5].

The domination degree $d_d(u)$ of a vertex u [6] in a graph G is defined as the number of minimal dominating sets of G which contains u .

The first and second Gourava domination indices [7] of a graph are defined as

$$GOD_1(G) = \sum_{uv \in E(G)} [d_d(u) + d_d(v) + d_d(u)d_d(v)].$$

$$GOD_2(G) = \sum_{uv \in E(G)} (d_d(u) + d_d(v))(d_d(u)d_d(v)).$$

Recently some domination indices were studied in [8, 9, 10, 11, 12, 13, 14, 15, 16].

We propose the sum connectivity Gourava domination index of a graph and it is defined as

$$SGOD(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_d(u) + d_d(v) + d_d(u)d_d(v)}}.$$

We introduce the product connectivity Gourava domination index of a graph and it is defined as

$$PGOD(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{(d_d(u) + d_d(v))(d_d(u)d_d(v))}}$$

Considering the sum and product connectivity Gourava domination indices, we define the sum and product connectivity Gourava domination exponentials of a graph G as

$$SGOD(G, x) = \sum_{uv \in E(G)} x^{\frac{1}{\sqrt{d_d(u) + d_d(v) + d_d(u)d_d(v)}}}.$$

$$PGOD(G, x) = \sum_{uv \in E(G)} x^{\frac{1}{\sqrt{(d_d(u) + d_d(v))(d_d(u)d_d(v))}}}$$

Recently, some domination parameters were studied in [17, 18, 19, 20, 21, 22, 23, 24].

In this paper, we determine the sum and product connectivity Gourava domination indices of some standard graphs, French windmill graphs and friendship graphs.

II. Results for Some Standard Graphs

Proposition 1. If K_n is a complete graph with n vertices, then

$$(i) \quad SGOD(K_n) = \frac{n(n-1)}{2\sqrt{3}}.$$

$$(ii) \quad PGOD(K_n) = \frac{n(n-1)}{2\sqrt{2}}.$$

Proof: If K_n is a complete graph, then $d_d(u) = 1$. From definition, we have

$$(i) \quad SGOD(K_n) = \sum_{uv \in E(K_n)} \frac{1}{\sqrt{d_d(u) + d_d(v) + d_d(u)d_d(v)}} \\ = \frac{n(n-1)}{2\sqrt{1+1+(1 \times 1)}} = \frac{n(n-1)}{2\sqrt{3}}.$$

$$(ii) \quad PGOD(K_n) = \sum_{uv \in E(K_n)} \frac{1}{\sqrt{(d_d(u) + d_d(v))(d_d(u)d_d(v))}} \\ = \frac{n(n-1)}{2\sqrt{(1+1)(1 \times 1)}} = \frac{n(n-1)}{2\sqrt{2}}.$$

Proposition 2. If S_{n+1} is a star graph with $d_d(u) = 1$, then

$$(i) \quad SGOD(S_{n+1}) = \frac{n}{\sqrt{3}}.$$

$$(ii) \quad PGOD(S_{n+1}) = \frac{n}{\sqrt{2}}.$$

Proposition 3. If $S_{p+1,q+1}$ is a double star graph with $d_d(u) = 2$, then

$$(i) \quad SGOD(S_{p+1,q+1}) = \frac{p+q+1}{2\sqrt{2}}.$$

$$(ii) \quad PGOD(S_{p+1,q+1}) = \frac{p+q+1}{4}.$$

Proposition 4. Let $K_{m,n}$ be a complete bipartite graph with $2 \leq m \leq n$. Then

$$(i) \quad SGOD(K_{m,n}) = \frac{mn}{\sqrt{mn + 2m + 2n + 3}}.$$

$$(ii) \quad PGOD(K_{m,n}) = \frac{mn}{\sqrt{(m+n+2)(m+1)(n+1)}}.$$

Proof: Let $G=K_{m,n}$, $m, n \geq 2$ with

$$d_d(u) = m+1$$

$$= n+1, \text{ for all } u \in V(G).$$

From definition, we have

$$\begin{aligned} (i) \quad SGOD(K_{m,n}) &= \sum_{uv \in E(K_{m,n})} \frac{1}{\sqrt{d_d(u) + d_d(v) + d_d(u)d_d(v)}} \\ &= \frac{mn}{\sqrt{(m+1+n+1) + (m+1)(n+1)}} \\ &= \frac{mn}{\sqrt{mn + 2m + 2n + 3}}. \end{aligned}$$

$$\begin{aligned} (ii) \quad PGOD(K_{m,n}) &= \sum_{uv \in E(K_{m,n})} \frac{1}{\sqrt{(d_d(u) + d_d(v))(d_d(u)d_d(v))}} \\ &= \frac{mn}{\sqrt{(m+1+n+1)(m+1)(n+1)}} \\ &= \frac{mn}{\sqrt{(m+n+2)(m+1)(n+1)}}. \end{aligned}$$

In the following proposition, by using definition, we obtain the sum and product connectivity Gourava domination exponentials of K_n , S_{n+1} , $S_{p+1,q+1}$ and $K_{m,n}$.

Proposition 5. The sum and product connectivity Gourava domination exponentials of K_n , S_{n+1} , $S_{p+1,q+1}$ and $K_{m,n}$ are given by

$$\begin{aligned} (i) \quad SGOD(K_n, x) &= \sum_{uv \in E(K_n)} x^{\frac{1}{\sqrt{d_d(u)+d_d(v)+d_d(u)d_d(v)}}} \\ &= \frac{n(n-1)}{2} x^{\frac{1}{\sqrt{1+1+(1 \times 1)}}} = \frac{n(n-1)}{2} x^{\frac{1}{\sqrt{3}}}. \end{aligned}$$

$$(ii) \quad SGOD(S_{n+1}, x) = nx^{\frac{1}{\sqrt{3}}}.$$

$$(iii) \quad SGOD(S_{p+1,q+1}, x) = (p+q+1)x^{\frac{1}{2\sqrt{2}}}.$$

$$(iv) \quad SGOD(K_{m,n}, x) = mnx^{\frac{1}{\sqrt{mn+2m+2n+3}}}.$$

$$(v) \quad PGOD(K_n, x) = \sum_{uv \in E(K_n)} \frac{1}{x \sqrt{(d_d(u) + d_d(v))(d_d(u)d_d(v))}}$$

$$= \frac{n(n-1)}{2} x^{\frac{1}{\sqrt{(1+1)(1 \times 1)}}} = \frac{n(n-1)}{2} x^{\frac{1}{\sqrt{2}}}.$$

$$(vi) \quad PGOD(S_{n+1}, x) = nx^{\frac{1}{\sqrt{2}}}.$$

$$(vii) \quad PGOD(S_{p+1, q+1}, x) = (p + q + 1)x^{\frac{1}{4}}.$$

$$(viii) \quad PGOD(K_{m,n}, x) = mnx^{\frac{1}{\sqrt{(m+n+2)(m+1)(n+1)}}}.$$

III. Results for French Windmill Graphs

The French windmill graph F_n^m is the graph obtained by taking $m \geq 3$ copies of K_n , $n \geq 3$ with a vertex in common, see Figure 1. The French windmill graph F_3^m is called a friendship graph.

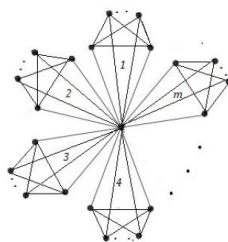


Figure 1. French windmill graph F_n^m

Let F be a French windmill graph F_n^m . Then

$$d_d(u) = 1, \quad \text{if } u \text{ is in center}$$

$$= (n-1)^{m-1}, \quad \text{otherwise.}$$

Theorem 1. Let F be a French windmill graph F_n^m . Then

$$SGOD(F) = \frac{m(n-1)}{\sqrt{1 + 2(n-1)^{(m-1)}}} + \frac{(mn(n-1)/2) - m(n-1)}{\sqrt{(n-1)^{(m-1)} [2 + (n-1)^{(m-1)}]}}.$$

Proof: In F , there are two sets of edges. Let E_1 be the set of all edges which are incident with the center vertex and E_2 be the set of all edges of the complete graph. Then

$$SGOD(F) = \sum_{uv \in E_1(F)} \frac{1}{\sqrt{d_d(u) + d_d(v) + d_d(u)d_d(v)}}$$

$$= \sum_{uv \in E_1(F)} \frac{1}{\sqrt{d_d(u) + d_d(v) + d_d(u)d_d(v)}} + \sum_{uv \in E_2(F)} \frac{1}{\sqrt{d_d(u) + d_d(v) + d_d(u)d_d(v)}}$$

$$= \frac{m(n-1)}{\sqrt{1 + (n-1)^{(m-1)} + 1(n-1)^{(m-1)}}} + \frac{(mn(n-1)/2) - m(n-1)}{\sqrt{(n-1)^{(m-1)} + (n-1)^{(m-1)} + (n-1)^{(m-1)}(n-1)^{(m-1)}}}$$

$$= \frac{m(n-1)}{\sqrt{1+2(n-1)^{(m-1)}}} + \frac{(mn(n-1)/2) - m(n-1)}{\sqrt{(n-1)^{(m-1)} [2+(n-1)^{(m-1)}]}}$$

Corollary 1.1. Let F_3^m be a friendship graph. Then

$$SGOD(F_3^m) = \frac{2m}{\sqrt{1+2^m}} + \frac{m}{\sqrt{2^{m-1}(2+2^{m-1})}}$$

In the following theorem, by using definitions, we obtain the sum connectivity Gourava domination exponentials of F_n^m and F_3^m .

Theorem 2 . The first Gourava domination polynomials of F_n^m and F_3^m are given by

$$\begin{aligned} \text{(i)} \quad SGOD(F_n^m, x) &= \sum_{uv \in E(F_n^m)} x^{\frac{1}{\sqrt{d_d(u)+d_d(v)+d_d(u)d_d(v)}}} \\ &= m(n-1)x^{\frac{1}{\sqrt{1+2(n-1)^{(m-1)}}}} + [(mn(n-1)/2) - m(n-1)]x^{\frac{1}{\sqrt{(n-1)^{(m-1)} [2+(n-1)^{(m-1)}]}}} \\ \text{(ii)} \quad SGOD(F_3^m, x) &= \sum_{uv \in E(F_3^m)} x^{\frac{1}{\sqrt{d_d(u)+d_d(v)+d_d(u)d_d(v)}}} = 2mx^{\frac{1}{\sqrt{1+2^m}}} + mx^{\frac{1}{\sqrt{2^{m-1}(2+2^{m-1})}}} \end{aligned}$$

Theorem 3. Let F be a French windmill graph F_n^m . Then

$$PGOD(F) = \frac{m(n-1)}{\sqrt{(1+(n-1)^{(m-1)})(n-1)^{(m-1)}}} + \frac{(mn(n-1)/2) - m(n-1)}{(n-1)^{m-1} \sqrt{2(n-1)^{m-1}}}$$

Proof: In F , there are two sets of edges. Let E_1 be the set of all edges which are incident with the center vertex and E_2 be the set of all edges of the complete graph. Then

$$\begin{aligned} PGOD(F) &= \sum_{uv \in E(F)} \frac{1}{\sqrt{(d_d(u)+d_d(v))(d_d(u)d_d(v))}} \\ &= \sum_{uv \in E_1(F)} \frac{1}{\sqrt{(d_d(u)+d_d(v))(d_d(u)d_d(v))}} + \sum_{uv \in E_2(F)} \frac{1}{\sqrt{(d_d(u)+d_d(v))(d_d(u)d_d(v))}} \\ &= \frac{m(n-1)}{\sqrt{(1+(n-1)^{(m-1)})1(n-1)^{(m-1)}}} + \frac{(mn(n-1)/2) - m(n-1)}{\sqrt{((n-1)^{m-1} + (n-1)^{m-1})((n-1)^{m-1} (n-1)^{m-1})}} \\ &= \frac{m(n-1)}{\sqrt{(1+(n-1)^{(m-1)})(n-1)^{(m-1)}}} + \frac{(mn(n-1)/2) - m(n-1)}{(n-1)^{m-1} \sqrt{2(n-1)^{m-1}}}. \end{aligned}$$

Corollary 3.1. Let F_3^m be a friendship graph. Then

$$PGOD(F_3^m) = \frac{2m}{\sqrt{2^{m-1}(1+2^{(m-1)})}} + \frac{m}{2^{m-1} \sqrt{2^m}}$$

In the following theorem, by using definitions, we obtain the product connectivity Gourava domination exponentials of F_n^m and F_3^m .

Theorem 4. The second Gourava domination polynomials of F_n^m and F_3^m are given by

$$\begin{aligned}
 \text{(i) } PGOD(F_n^m, x) &= \sum_{uv \in E(F_n^m)} x^{\frac{1}{\sqrt{(d_d(u)+d_d(v))(d_d(u)d_d(v))}}} \\
 &= m(n-1)x^{\frac{1}{\sqrt{(1+(n-1)^{(m-1)})(n-1)^{(m-1)}}}} + [(mn(n-1)/2) - m(n-1)]x^{\frac{1}{(n-1)^{m-1}\sqrt{2(n-1)^{m-1}}}}. \\
 \text{(ii) } PGOD(F_3^m, x) &= \sum_{uv \in E(F_3^m)} x^{\frac{1}{\sqrt{(d_d(u)+d_d(v))(d_d(u)d_d(v))}}} \\
 &= 2mx^{\frac{1}{\sqrt{2^{m-1}(1+2^{(m-1)})}}} + mx^{\frac{1}{2^{m-1}\sqrt{2^m}}}.
 \end{aligned}$$

IV. Results for GoK_p

Theorem 5. Let $H=GoK_p$, where G is a connected graph with n vertices and m edges; and K_p is a complete graph. Then

$$\begin{aligned}
 \text{(i) } SGOD(H) &= \frac{(2m + np^2 + np)}{2\sqrt{(p+1)^{n-1} [2 + (p+1)^{n-1}]}}. \\
 \text{(ii) } PGOD(H) &= \frac{(2m + np^2 + np)}{2\sqrt{2(p+1)^{3(n-1)}}}.
 \end{aligned}$$

Proof: If $H=GoK_p$, then $d_d(u) = (p+1)^{n-1}$. In F , there are $\frac{p(p-1)}{2}$ edges. Thus H has $\frac{1}{2}(2m + np^2 + np)$ edges. Thus

$$\begin{aligned}
 \text{(i) } SGOD(H) &= \sum_{uv \in E(H)} \frac{1}{\sqrt{d_d(u) + d_d(v) + d_d(u)d_d(v)}} \\
 &= \frac{1}{2}(2m + np^2 + np) + \frac{1}{\sqrt{(p+1)^{n-1} + (p+1)^{n-1} + (p+1)^{n-1}(p+1)^{n-1}}} \\
 &= \frac{(2m + np^2 + np)}{2\sqrt{(p+1)^{n-1} [2 + (p+1)^{n-1}]}}. \\
 \text{(ii) } PGOD(H) &= \sum_{uv \in E(H)} \frac{1}{\sqrt{(d_d(u) + d_d(v))(d_d(u)d_d(v))}} \\
 &= \frac{1}{2}(2m + np^2 + np) \frac{1}{\sqrt{[(p+1)^{n-1} + (p+1)^{n-1}](p+1)^{n-1}(p+1)^{n-1}}}
 \end{aligned}$$

$$= \frac{(2m + np^2 + np)}{2\sqrt{2(p+1)^{3(n-1)}}}$$

In the following theorem, by using definitions, we obtain the sum and product connectivity Gourava domination exponentials of H .

Theorem 6. The sum and product connectivity Gourava domination exponentials of H are given by

$$(i) \quad SGOD(H, x) = \frac{1}{2}(2m + np^2 + np)x^{\frac{1}{\sqrt{(p+1)^{n-1}[2+(p+1)^{n-1}]}}}$$

$$(ii) \quad PGOD(H, x) = \frac{1}{2}(2m + np^2 + np)x^{\frac{1}{\sqrt{2(p+1)^{3(n-1)}}}}$$

V. Results for B_n

The book graph $B_n, n \geq 3$, is a cartesian product of star S_{n+1} and path P_2 .

For $B_n, n \geq 3$, we have

$$d_d(u) = 3, \text{ if } u \text{ is the center vertex,} \\ = 2^{n-1} + 1, \text{ otherwise.}$$

Theorem 7. If $B_n, n \geq 3$, is a book graph, then

$$(i) \quad SGOD(B_n) = \frac{1}{\sqrt{15}} + \frac{2n}{\sqrt{(7 + 4 \times 2^{n-1})}} + \frac{n}{\sqrt{(2^{n-1} + 1)(2^{n-1} + 3)}}$$

$$(ii) \quad PGOD(B_n) = \frac{1}{\sqrt{54}} + \frac{2n}{\sqrt{3(2^{n-1} + 4)(2^{n-1} + 1)}} + \frac{n}{\sqrt{2(2^{n-1} + 1)^3}}$$

Proof: In B_n , there are three types of edges as follow:

$$E_1 = \{uv \in E(B_n) \mid d_d(u) = d_d(v) = 3\}, \quad |E_1| = 1.$$

$$E_2 = \{uv \in E(B_n) \mid d_d(u) = 3, d_d(v) = 2^{n-1} + 1\}, |E_2| = 2r.$$

$$E_3 = \{uv \in E(B_n) \mid d_d(u) = d_d(v) = 2^{n-1} + 1\}, \quad |E_3| = r.$$

(i) By definition, we have

$$SGOD(B_n) = \sum_{uv \in E(B_n)} \frac{1}{\sqrt{d_d(u) + d_d(v) + d_d(u)d_d(v)}} \\ = \frac{1}{\sqrt{[3 + 3 + (3 \times 3)]}} + \frac{2n}{\sqrt{[3 + (2^{n-1} + 1) + 3(2^{n-1} + 1)]}} \\ + \frac{n}{\sqrt{[(2^{n-1} + 1) + (2^{n-1} + 1) + (2^{n-1} + 1)(2^{n-1} + 1)]}}$$

$$= \frac{1}{\sqrt{15}} + \frac{2n}{\sqrt{(7+4 \times 2^{n-1})}} + \frac{n}{\sqrt{(2^{n-1}+1)(2^{n-1}+3)}}.$$

(ii) By definition, we have

$$\begin{aligned} PGOD(B_n) &= \sum_{uv \in E(B_n)} \frac{1}{\sqrt{(d_d(u) + d_d(v))(d_d(u)d_d(v))}} \\ &= \frac{1}{\sqrt{[(3+3)(3 \times 3)]}} + \frac{2n}{\sqrt{[(3+(2^{n-1}+1))3(2^{n-1}+1)]}} \\ &\quad + \frac{n}{\sqrt{((2^{n-1}+1)+(2^{n-1}+1))(2^{n-1}+1)(2^{n-1}+1)}} \\ &= \frac{1}{\sqrt{54}} + \frac{2n}{\sqrt{3(2^{n-1}+4)(2^{n-1}+1)}} + \frac{n}{\sqrt{2(2^{n-1}+1)^3}}. \end{aligned}$$

In the following theorem, by using definitions, we obtain the sum and product connectivity Gourava domination exponentials of B_n .

Theorem 8. The sum and product connectivity Gourava domination exponentials of B_n are given by

$$(i) \quad SGOD(B_n, x) = x^{\frac{1}{\sqrt{15}}} + 2nx^{\frac{1}{\sqrt{(7+4 \times 2^{n-1})}}} + nx^{\frac{1}{\sqrt{(2^{n-1}+1)(2^{n-1}+3)}}}.$$

$$(ii) \quad PGOD(B_n, x) = x^{\frac{1}{\sqrt{54}}} + 2nx^{\frac{1}{\sqrt{3(2^{n-1}+4)(2^{n-1}+1)}}} + nx^{\frac{1}{\sqrt{2(2^{n-1}+1)^3}}}.$$

VI. CONCLUSION

In this study, we have defined the sum and product connectivity Gourava domination indices and their corresponding exponentials of a graph. Also the sum and product connectivity Gourava domination indices and their corresponding exponentials of some standard graphs, windmill graphs, book graphs are computed.

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