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On Fuzzy Strongly g*p - Continuous Functions

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***ABSTRACT**: We define new kinds of fuzzy sets in this article, namely fuzzy strongly g*p - closed sets in fuzzy topological spaces (fts) and fuzzy strongly g*p - continuous functions. Compare the existing fuzzy closed sets with a fuzzy strongly g*p- closed set. We further study some of their properties.*
***KEYWORDS**:* fuzzy set, and fg-closed sets, fuzzy strongly g*p-closed set, and fuzzy strongly g*p-continuous functions.

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I. INTRODUCTION

The concept of fuzzy set theory was first proposed by L.A Zadeh in his classic article in 1965 [16] and it expanded the basic definition of the crisp or classical sets. Chang [7] developed the concept of fuzzy topological spaces (fts) in 1968 as a generalisation of topological spaces. In general topology, Veerakumar [14] introduced and investigated g*-closed sets. In 1970, Levine [9] introduced the notion of generalized closed sets in general topology. Since the introduction of the fuzzy set theory, subsequently several researchers have been worked on various basic concepts from general topology using fuzzy sets and various types of fuzzy sets were developed and studied by various researchers like, recently S.S. Benchalli and G.P. Siddapur, introduced fuzzy g*p-continuous maps[6], Parimezhagan and subramaniapillai, introduced strongly g*-closed sets in fuzzy topological spaces in 2012 [10].

In this study, we defined and investigated the concept of strongly g*p-closed sets in fuzzy topological spaces, fuzzy strongly g*p-continuous maps, and fuzzy irresolute maps, and we also investigated their relationship with other maps are investigated, and studied some properties.

Throughout this article (W, δ), (Z, η), or simply W, Z represent a fuzzy topological space (fts) in which no separation axioms are assumed unless otherwise stated.

II. PRELIMINARIES

This section includes some known definitions, they are useful in the sequel.

DEFINITION 2.1. A subset Q of W is said to be fuzzy pre closed set if cl(int(Q)) ≤ Q and fuzzy pre open if Q≤ int(cl(Q)) [5].

DEFINITION 2.2. A subset R of W is said to be fuzzy alpha closed if cl(int(R)) ≤ R and fuzzy alpha open if R ≤ int(cl(int(R)) [3].

DEFINITION 2.3. A subset K of a fts W is said to be fuzzy g-closed if cl(K) ≤ M whenever K≤M and M is a fuzzy open in W [4].

DEFINITION 2.4. A fuzzy set S of a fts W is said to be a fg*-closed if cl(S) ≤ T whatever S ≤ T and T is a fg-open in W [13].

DEFINITION 2.5. A fuzzy set D of fts W is said to be a fuzzy strongly g*-closed if cl(D) ≤ R whatever D ≤ R and R is fuzzy g-open in W [10,11].

DEFINITION 2.6. The fuzzy set H of a fts W is said to be a fwg –closed set if cl(int(H)) ≤ K whatever H≤ K and H is a fuzzy open in W[1].

DEFINITION 2.7. A fuzzy set M of a fts W is said to be a fuzzy gp-closed set if Pcl(M) ≤ H whenever M≤ H and H is a fuzzy open set in W[8].

DEFINITION 2.8. A fuzzy set R of a fts W is said to be a g*p-closed if pcl(R) ≤ S whatever R≤S & S is a fuzzy g-open in W[6].

III. SOME STRONGLY G*P-CLOSED SETS IN FUZZY TOPOLOGICAL SPACES

In this section, we define fuzzy strongly g*p closed sets and investigate some of their characteristics in fuzzy topological spaces.

DEFINITION 3.1: A fuzzy set H of an fts W is said to be a fuzzy strongly g*pre-closed set if int(pcl(H)) ≤ R whenever H ≤ R and R is a gp-open fuzzy set in W.

THEOREM 3.2: Every pre-closed fuzzy set is strongly g*p - closed, but the converse is not true.

PROOF. Allow N to be a fuzzy pre-closed set in X. Allow R to be a fgp-open in X such that N R. Since N is pre-closed, pcl(N) = N. As a result, pcl(N) = R ≤ N. Now we have int(pcl(N)) = cl(N) ≤ R. N is thus a strongly g*p-closed fuzzy set in X.

EXAMPLE 3.3: Allow Z = {l, m, n}. P(l) = 0.7, P(m) = 0.3, P(n) = 0.5; T(l) = 0.7, T(m) = 0.3, T(n) = 0.5. Let = {0, P, 1}. Then T is a strongly g*p - closed fuzzy set, but it is not a fuzzy closed set in (Z, η).

THEOREM 3.4: Every closed (resp. α -closed) fuzzy set is a Strongly g*p-closed fuzzy set, but the reverse is not true.

PROOF. Since every fuzzy closed (resp. α -closed) set is a fuzzy pre closed set, and by **theorem 3.2**, the proof follows.

EXAMPLE 3.5. In the preceding example 3.3, the fuzzy set D is strongly g*p-closed but not a closed fuzzy set in W.

EXAMPLE 3.6. Let Z ={ m.n.t} and the fuzzy sets P, Q, and R be defined as follows: P = {(m,1), (n, 0), (t, 0)}, Q = {(m,1), (n,0), (t,1)}, and R = {(m,1), (n,1), (t,1)}. Then (Z, η) is an fts. where η = {0, P, Q, 1}. In this fts, the fuzzy set R is strongly g*p-closed but not α -closed in Z.

THEOREM 3.7: Every fuzzy g-closed fuzzy set in W is strongly g*p-closed, but the converse is not true.

PROOF: Let R be a fuzzy g-closed set in W. Allow H to be an open set in W such that R ≤ H. Since R is fuzzy g-closed, cl(R) ≤ H. Now we have int(pcl(R)) = cl(R) ≤ H. As a result, R is a fuzzy strongly g*s - closed set in W.

EXAMPLE 3.8: Allow W= {m.n}. Then, both D and S are fuzzy sets defined as D(m) =0.3, D(n) =0.3, and S(a) =0.5, S(b) =0.4. Let δ = {0, S, 1}. D is so f strly g*p-closed but not a fg-closed since cl(S) $\not\leq$ S.

THEOREM 3.9: In a fts Z, if a fuzzy set T is both open and gp-closed, then T is a fuzzy strongly g*p-closed set.

REMARK 3.10. The union of two g*p-closed fuzzy sets does not have to be a fstrly g*p-closed fuzzy set.

EXAMPLE 3.11. Let W={t, m, n} and S, T and R be the fuzzy sets defined as S = {(t, 0.7),(m, 0.8),(n, 0)}, T = {(t, 0.7), (m,0.6), (n, 0)}, and R ={(t,0.1),(m,0.8), (n,0)}. Consider δ = {0, S,1}. Then (W, δ) is an fts. Then T and R are two g*p-closed fuzzy sets, but their union S $\cup$ T is not a g*p-closed fuzzy set in W.

THEOREM 3.12: Every fuzzy strongly g*p-closed set in a fuzzy topological space W is a gp-closed fuzzy set.

PROOF. Assume that S is a strongly g*p-closed fuzzy set in Y. Let T be an open fuzzy set that is gp-open such that S ≤ T. Now, int(pcl(S)) ≤ pcl(S) ≤ T. Since S is a strongly g*p-closed fuzzy set, int(pcl(S)) ≤ T. As a result, S is a gp-closed fuzzy set in Y.

THEOREM 3.13. 12a Every strongly g*p-closed fuzzy set in a fuzzy topological space W is also a gsp-closed fuzzy set.

PROOF. Obvious

The following example illustrates that the converse is not true.

EXAMPLE 3.13. Let H ={p,q,r }and the fuzzy sets M and N be defined as follows. M = {(p,1), (q,0), (r, 0)}, and N = {(p,1), (q,1), (r,0)}. η ={ 0 M 1}. So (Z, η) is an fts, we can see, that fuzzy set N is gp-closed (or gsp-closed), but it is not f strly g*p-closed in Z.

THEOREM 3.14. In a fts Z, every g*-closed fuzzy set is fuzzy strly g*p-closed set. But the converse is not true.

PROOF. Allow K is a g*-closed set in fts Z. Let G be the fuzzy open set such that K ≤ G. Since each g-open set is gp-open, cl(K) ≤ G. It should be noted that int(pcl(K)) ≤ cl(K) is always true. So in(pcl(K) ≤ G. As a result, K is a fuzzy strongly g*p-closed in Z.

EXAMPLE 3.15. Allow Z ={m,n,t} and P,Q,R are the fuzzy sets in a fts (Z,)defined as P={(m,1),(n,0),(t,0)}; Q ={(m,0),(n,1),(t,1)}; R={(m,1),n,1),(t,0)}. The family, ={0,P,Q,1}. In this fts, the fuzzy set D is strongly g*p-closed but it is not a fuzzy g*-closed in Z.

THEOREM. 3.16. The fuzzy set N is both g-open and fstrly g*p-closed in the fts W, and then N is a pre closed fuzzy set.

PROOF. Let H Q, where Q is g-open and hence it is gp-open s.t H Q. Now int(pcl(H) ≤ pcl(H) H, but H a fuzzy strongly g*p-closed. Since H, int(pcl(H) ≤ pcl(H)) is always true. Thus, pc(H) = H. As a result, H is a pre-closed fuzzy set.

THEOREM 3.17. Every fstrly g*p-closed set is wg-closed in Z, but not conversely.

PROOF. Suppose D is a fstrly g*p-closed set in Z, and the fuzzy set T is open in Z s.t $D \leq T$. But every open set is gp –open and hence D is fstrly g*p-closed, we have $int(pcl(D) \leq T$. Thus, D is fwg-closed in Z.
EXAMPLE 3.18. Allow Z ={x,y}, η ={0,1,S,T,R} . Then S, T, R and H be the fuzzy sets defined as
S(x)= 0.4; S(y) = 0.7, T(x)=0.7; T(y) = 0.6, R(x)=0.6; R(y)=0.4, H(x)=0.5;H(y)=0.4. Then H is not fwg-closed but it is fstrly g*p-closed in Z.

IV. FUZZY STRONGLY G*P - CONTINUOUS FUNCTIONS IN FTS

In this section, we introduce fuzzy strongly g*p-continuous maps and fuzzy strongly g*p-irresolute maps in fuzzy topological spaces along with some of their characteristics.
DEFINITION 4.1: Assuming that W and Z are fuzzy topological spaces, a function. $f : W \rightarrow Z$ is said to be fuzzy strly g*p-continuous, if the inverse image of each open (or closed) set in Z is fstrly g*p-open (or closed) in W.
DEFINITION 4.2: A function $h : W \rightarrow Z$ is said to be a fuzzy strongly g*p-irresolute if it is fstrly g*p-closed in W for any fstrly g*p-closed set in Z.
THEOREM 4.3. The function $h:W \rightarrow Z$ being fstrly g*p-irresolute iff the inverse image of each fstrly g*p-open set in Z is a fstrly g*p-open in W.
THEOREM 4.4. Every f-continuous map is fstrly g*p-continuous but not conversely.
PROOF. The proof follows from the definitions**.**
EXAMPLE 4.5. Allow W = Z = {p, q, r}. Then S, T, M, N and K are fuzzy sets defined as
S = {(p,1), (q,0), (r,0)}; T = {(p,0), (q,1), (r,1)}; M={(p,0),(q,1),(r,0)}, N={(p,1),(q,1),(r,0)}, and K={(p,1)(q,0),(r,1)}. Now consider, δ ={1,S,T,0}, η ={1,0,S,M,N}, where (W, δ), (Z, η) be fts. Then identity function h: $W \rightarrow Z$ is fstrly g*p-continuous but not h-continuous, since fuzzy closed set, K in Z, but $f^{-1}(K)$ = K is not f closed in Z but it is fstrly g*p-closed in Z.
THEOREM 4.6. Each fuzzy strongly g*p- continuous map being a fgp-continuous map but not conversely.
PROOF. Follows from theorem 3.12
EXAMPLE 4.7. Suppose W = Z = {p, q, r}. Now M, N, P, Q, and R are fuzzy sets defined as
M={(p,1), (q, 0), (r, 0)}; N={(p,0), (q, 1), (r, 1)}, P={(p, 0), (q, 1),(r, 0)}, Q={(p, 1),(q, 1),(r, 0)}, and R={(p,1), (q, 0), (r,1)}. Then consider the families, δ ={0, M, N, 1}; η ={0, 1, M, P, Q}. Where W, Z are fts, define a map $h:W \rightarrow Z$ as h(p) = p, h(q) = r and h(r) = q. Then h is fgp-continuous but not fstrly g*p-continuous, since for fuzzy closed set R in Z, Then $f^{-1}(R)$ is not fstrly g*p-closed in W, but it is fgp-closed in Z.
THEOREM 4.8 Each fuzzy strongly g*p-irresolute map is fstrly g*p continuous, but not conversely.
PROOF. Obvious.

V. CONCLUSION

In the present work, a new class of sets called fuzzy strongly g*p - closed sets in fuzzy topological spaces (fts) is introduced and some of their properties are studied. This new class of sets widens the scope to do further research in the areas like Nano topological Spaces, Mico topological Spaces and Fuzzy Soft Topological Spaces.

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