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Super-Saturated Design using PBIBD

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Abstract
In this paper, a new series supersaturated design is proposed using Partially Balanced Incomplete Block Design through a combinatorial arrangement of the incidence matrix of a Balanced Incomplete Block Design. The method was illustrated with suitable example.
Keywords: *Super-Saturated design, E(S^2) criteria, BIBD.*

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I. INTRODUCTION

A design matrix 'X' is said to be saturated if the number of design points is equal to the number of factors plus one and is the design matrix 'X' is said to be a super-saturated, if the number of factors is more than the number of design points.

Super-saturated design is a fractional factorial design and the degrees of freedom for all its main effects and interaction terms exceed the number of design points. These designs reduce the experimental cost and time significantly due to a smaller number of experimental runs.These designs used to identify active factor main effects when experimentation is expensive and the number of potential factors is large. These designs are more economical and flexible due to their run size.

Satterthwaite (1959) initially made an attempt to construct saturated designs randomly and suggested the random balance designs. Booth and Cox (1962) proposed a systematic method for the construction of super-saturated designs and also computed E(s^2) criterion.If E(s^2) = 0, then these designs are leads toorthogonal designs.The designs that are near orthogonal are preferable if it is not possible to conduct the experiment with orthogonal designs. The lack of orthogonality measured based on the dispersion matrix of the design. the property that the mean of S_{ij}^2 of all pairs (i,j) for (i≠j) is minimum, such design is said to be E(s^2)-optimal super-saturated.

II. NEW SERIES OF SUPER-SATURATED DESIGNS

In this section, two new series of super-saturated design using three associate partially balanced incomplete block design and the method is also illustrated with a suitable example.

SERIES-1:

Step-1: Let N be the incidence matrix of a Partially Balanced Incomplete Block Design with parameters v, b(>v), r, k, λ_1, λ_2, λ_3, n_1, n_2,n_3 and $\overline{N}$ be the dual of N (i.e. replace zeros with ones and one with zeros).
Step-2: The combinatorial arrangement of the incidence matrix N and $\overline{N}$in the form of N′ as

$$N' = \begin{bmatrix} N & \overline{N} \\ \overline{N} & N \end{bmatrix}$$

Step-3: Replace the 0's with -1's and treat each column is cooresponding to a factor and each row as combination different factors with different levels..
Step-4: The resulting design is super saturated design with 2b factors, in 2v design points.

EXAMPLE 2.1: Consider a three associate class Partial Balanced Incomplete BlockDesign wit parameters v = 8, b= 6, r = 3, k = 4,λ_1 = 0, λ_2 = 1, λ_3 = 2, n_1 = 3, n_2 = 3, n_3 = 1.

$$N = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \quad \overline{N} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The arrange the incidence matrices N and $\overline{N}$ in N′ as

$$N' = \left[\begin{array}{cccccccc|cccccccc} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{array}\right]$$

The resulting super-saturated design X in 12 design points with 16 factors is presented below.

$$X_{bxv} = \begin{bmatrix} +1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & +1 & +1 & +1 & +1 \\ +1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 & +1 \\ +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 \\ -1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 \\ -1 & -1 & +1 & +1 & -1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 \\ -1 & -1 & -1 & -1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 \\ -1 & -1 & +1 & +1 & -1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 \\ -1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 \\ +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & -1 & +1 & -1 & +1 & -1 & +1 & -0 & +1 \\ +1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 & +1 \\ +1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & +1 & +1 & +1 & +1 \end{bmatrix}$$

The (X′X) matrix of the design is

$$\begin{bmatrix} +12 & +4 & +4 & -4 & +4 & -4 & -4 & -12 & -12 & -4 & -4 & +4 & -4 & +4 & +4 & +12 \\ +4 & +12 & -4 & +4 & -4 & +4 & -12 & -4 & -4 & -12 & +4 & -4 & +4 & -4 & +12 & +4 \\ +4 & -4 & +12 & +4 & -4 & -12 & +4 & -4 & -4 & +4 & -12 & -4 & +4 & +12 & -4 & +4 \\ -4 & +4 & +4 & +12 & -12 & -4 & -4 & +4 & +4 & -4 & -4 & -12 & +12 & +4 & +4 & -4 \\ +4 & -4 & -4 & -12 & +12 & +4 & +4 & -4 & -4 & +4 & +4 & +12 & -12 & -4 & -4 & +4 \\ -4 & +4 & -12 & -4 & +4 & +12 & -4 & +4 & +4 & -4 & +12 & +4 & -4 & -12 & +4 & -4 \\ -4 & -12 & +4 & -4 & +4 & -4 & +12 & +4 & +4 & +12 & -4 & +4 & -4 & +4 & -12 & -4 \\ -12 & -4 & -4 & +4 & -4 & +4 & +4 & +12 & +12 & +4 & +4 & -4 & +4 & -4 & -4 & -12 \\ -12 & -4 & -4 & +4 & -4 & +4 & +4 & +12 & +12 & +4 & +4 & -4 & +4 & -4 & -4 & -12 \\ -4 & -12 & +4 & -4 & +4 & -4 & +12 & +4 & +4 & +12 & -4 & +4 & -4 & +4 & -12 & -4 \\ -4 & +4 & -12 & -4 & +4 & +12 & -4 & +4 & +4 & -4 & +12 & +4 & -4 & -12 & +4 & -4 \\ +4 & -4 & -4 & -12 & +12 & +4 & +4 & -4 & -4 & +4 & +4 & +12 & -12 & -4 & -4 & +4 \\ -4 & +4 & +4 & +12 & -12 & -4 & -4 & +4 & +4 & -4 & -4 & -12 & +12 & +4 & +4 & -4 \\ +4 & -4 & +12 & +4 & -4 & -12 & +4 & -4 & -4 & +4 & -12 & -4 & +4 & +12 & -4 & +4 \\ +4 & +12 & -4 & +4 & -4 & +4 & -12 & -4 & -4 & -12 & +4 & -4 & +4 & -4 & +12 & +4 \\ +12 & +4 & +4 & -4 & +4 & -4 & -4 & -12 & -12 & -4 & -4 & +4 & -4 & +4 & +4 & +12 \end{bmatrix}$$

$E(s^2) = 41.6$.

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