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Optimizing Urban EV Infrastructure: A Reaction-Diffusion Approach to Balancing Charging Grid Demand

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Abstract
Electric vehicle adoption is, you know, accelerating across the world, and cities are scrambling to keep pace. Urban charging infrastructure has a deceptively hard, kind of sneaky issue: the actual charging demand is not just uneven in space but also not really predictable through time. People tend to clump around commercial zones during daytime, then shift back toward residential areas at night. This creates localized grid stress that static planning approaches, frankly, just cannot really handle. This article looks at how reaction-diffusion (RD) systems—math frameworks first made for chemical and biological pattern formation—can be reused to model, test, and also improve the spatial-temporal distribution of EV charging demand in cities. If you think of charging stations as "reaction" nodes and electricity spreading as a "diffusion" process across the city grid, then planners can spot critical load imbalances earlier, before they trigger infrastructure failure. The article brings together stochastic queueing theory, especially M/M/C and M/M/R models, which have been studied a lot in machine repair work, with RD partial differential equations. The goal is a hybrid optimization framework that kind of blends both worlds. Analysis is anchored with real-world information from the International Energy Agency and the U.S. Department of Energy. Overall, the approach gives a practical computational route ahead for smart grid integration and for urban EV infrastructure planning.
Keywords: *EV charging infrastructure, stochastic queueing theory, partial differential equations, reaction-diffusion systems, urban grid optimization, smart grid*

I. Introduction

There are now more than 40 million electric vehicles out there on the road worldwide, and the number is climbing pretty fast. The International Energy Agency, in its Global EV Outlook 2023, said EV sales reached about 14 million in 2023 by itself, which is roughly 18% of all new car sales globally. Cities from Oslo to Shenzhen are seeing their grids strain under the whole transition, a little more each quarter. The tricky part is not really that there aren't enough charging stations — in many dense downtown areas there are plenty. It's more like, the right chargers aren't always sitting in the right spots, at the right moments.

Picture a Tuesday afternoon in a packed downtown neighborhood. Office workers aren't plugging in at their desks. Rideshare drivers pull into fast-charging bays again and again, pretty much every hour. Delivery vans keep rotating through commercial loading zones, and every stop adds more load. Then, a few kilometers away in the suburbs, the local charging stations mostly sit unused until around 6 p.m. when commuters roll back in and plug in at the same time. That same-evening demand surge, which engineers often label the "duck curve" issue, is one of the big headaches in how urban EV grids get managed.

Older infrastructure planning methods treat charging station placement like it's a frozen optimization task: basically, pick the locations that reduce average driving distance to the closest charger, using some guess about EV penetration. This approach can work okay when EV density stays low. But once adoption passes something like 10–15% of the vehicle fleet in a city, the static models start wobbling. Demand then becomes very mixed across locations, it changes a lot over time, and it responds sharply to local socioeconomic differences.

This is where reaction-diffusion mathematics sort of enters the picture. Originally developed by Alan Turing in 1952, to explain how biological patterns like stripes and spots show up on animal skin, reaction-diffusion systems describe how local "reactions"— in our case, EV charging events— interact with spatial "diffusion" processes, meaning electricity flow and load rebalancing across the grid. The analogy is actually quite fitting, and the math is strong enough to model the complex, emergent behaviors that show up in real urban charging demand.

This article lays out a conceptual and also mathematical framework for using reaction-diffusion theory for EV infrastructure planning, while also tying it to well-established stochastic queueing models. The aim is not just academic: it's practical, meant to give urban planners and grid engineers a mathematically rigorous but still approachable tool for better decisions— where to build charging capacity, how to use dynamic pricing, and how to connect the whole thing with renewable energy sources.

II. Background: The Urban EV Charging Problem

2.1 Current Scale and Growth Projections

The numbers are hard to ignore. The U.S. Department of Energy Alternative Fuels Station Locator documented roughly 61,000 public EV charging stations across the United States as of late 2023, with more than 165,000 distinct charging ports. Over in the European Union, the European Alternative Fuels Observatory logged about 630,000 public charging points by the end of 2023, and Germany, France, plus the Netherlands accounted for the bulk of installations.

Still the EV to public charger ratio is getting worse, especially in high-density urban areas. A 2023 study by the Rocky Mountain Institute showed that in a handful of big U.S. cities, such as San Francisco and Seattle, the ratio of EVs to public charging ports was above 20:1 during peak hours in certain commercial zones. The IEA also suggests that hitting net-zero goals will mean around 900 million EVs on the road worldwide by 2030, which translates into something like 15 million public charging points. That’s basically an order of magnitude jump from today’s infrastructure, and yes, it’s pretty sobering.

2.2 Why Static Planning Fails

Existing planning methods lean pretty heavily on facility location models: the p-median problem, set cover formulations, and gravity-based demand models. These are a useful starting point, but they have this shared drawback. They basically treat demand like a static probability distribution, not like something dynamic that is spatially coupled.

Real EV charging demand has at least three properties that those static approaches miss. First, it’s spatially autocorrelated—when one area has high demand, it tends to spill over into neighboring zones, because drivers’ kind of search out available chargers nearby. Second, it’s time periodic with daily cycles and also weekly ones. Third, it reacts to itself, meaning the presence or lack of chargers can shape where people decide to drive and park, and that then reshapes demand again. So, you really need a dynamic system to capture this kind of evolving behavior.

III. Mathematical Foundations

3.1 Reaction-Diffusion Systems: The Core Framework

A reaction-diffusion system (RD) sort of describes how the concentration of one substance, or maybe two, evolves over time and across space, through a pair of main movements. On one side there are local reactions that create or consume the substance, on the other side diffusion that spreads it across the region. In general, for a two component setup you can write it like:

$$\frac{\partial u}{\partial t} = D_u \nabla^2 u + f(u, v)$$

$$\frac{\partial v}{\partial t} = D_v \nabla^2 v + g(u, v)$$

Here, u and v are concentrations (or in our context, demand intensities) at position x and time t. The bits $Du\nabla 2u$ and $Dv\nabla 2v$ are diffusion terms, where Du and Dv are diffusion coefficients which are basically saying how fast each thing spreads out through space. The functions f (u, v) and g (u, v) describe local interactions — in EV speak, the nearby charging events, arrivals and departures happening at specific grid nodes, right at that location.

For urban EV infrastructure, we take u (x, t) as the local charging demand intensity for EVs (vehicles per unit area trying to get charge) and v (x, t) as the available charging capacity at location x at that particular time t. The reaction terms take the form:

$$f(u, v) = \alpha u\left(1 - \frac{u}{K}\right) - \beta uv$$
$$g(u, v) = \gamma uv - \delta v$$

This is structurally analogous to a predator-prey (Lotka-Volterra) reaction system, where demand "consumes" available capacity and capacity is "replenished" through grid management and new installations. The parameter α represents the intrinsic demand growth rate, K is the spatial carrying capacity of the local grid, β is the rate at which demand is satisfied by available capacity, γ is the capacity utilization efficiency, and δ represents capacity degradation or reallocation rates.

The Laplacian operator ∇^2 in an urban grid context applies to a two-dimensional spatial domain Ω representing the city's geographic footprint:

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$$

When $Dv \gg Du$ — meaning capacity can be redistributed, through smart grid load balancing, much faster than demand physically drifts — the system may show Turing-type instabilities. And then, spontaneously you get spatial clustering of that demand, kinda like patterns forming on their own. That is also what urban planners notice in practice, charging demand ends up gathering in so called hot zones even if the charger distribution at the start was uniform, not moved around much in the beginning.

3.2 Stability Analysis and Turing Conditions

The uniform steady state (u^*, v^*) of the RD system satisfies $f(u^*, v^*) = 0$ and $g(u^*, v^*) = 0$. Solving the reaction terms:

$$u^* = \frac{\delta}{\gamma}, \quad v^* = \frac{\alpha}{\beta}\left(1 - \frac{u^*}{K}\right)$$

For spatial pattern formation to emerge — the mathematical mechanism behind demand clustering — the uniform steady state must be stable to uniform perturbations but unstable to spatially heterogeneous perturbations. This is the Turing instability condition. The Jacobian of the reaction terms at the steady state is:

$$J = \begin{pmatrix} f_u & f_v \\ g_u & g_v \end{pmatrix} = \begin{pmatrix} \alpha - \frac{2\alpha u^*}{K} - \beta v^* & -\beta u^* \\ \gamma v^* & \gamma u^* - \delta \end{pmatrix}$$

Turing instability requires simultaneously:

$$\mathrm{tr}(J) = f_u + g_v < 0$$
$$\det(J) = f_u g_v - f_v g_u > 0$$
$$D_v f_u + D_u g_v > 2\sqrt{D_u D_v \det(J)}$$

The third condition is the critical one. It tells us that spatial patterns (demand hot zones) emerge when the diffusion ratio $d = D_v/D_u$ exceeds a critical threshold:

$$d > d_c = \frac{(f_u g_v - f_v g_u) + 2\sqrt{-f_v g_u \det(J)}}{g_v^2}$$

In practical terms, this means that when grid load-balancing mechanisms can redistribute capacity much faster than physical demand shifts, the system is prone to forming stable spatial clusters of high demand. Urban planners can use this threshold to design grid architectures that stay below d_c, preventing runaway clustering.
As shown in Figure 1, the emergence of spatial demand patterns follows directly from these Turing instability conditions applied to urban grid topology.

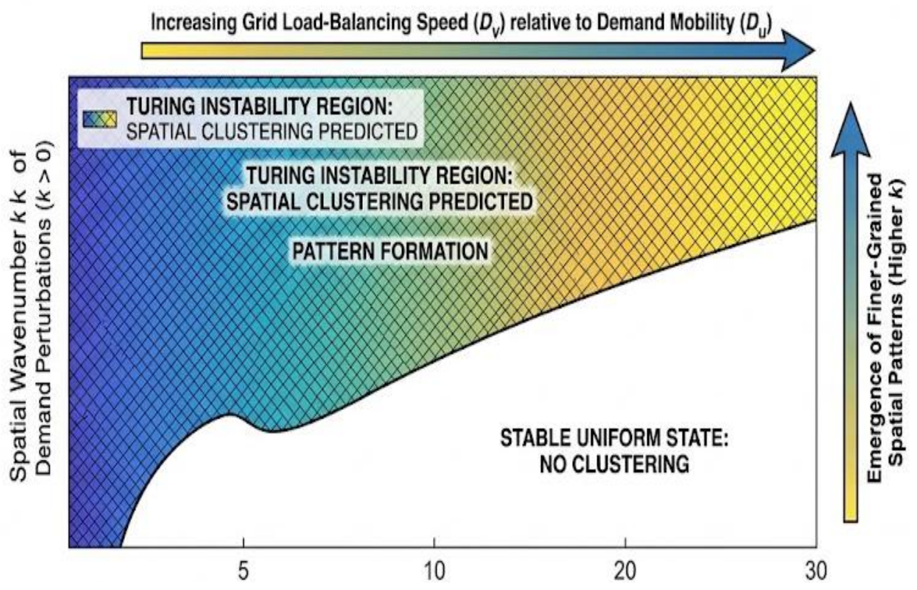

Figure 1: Turing Instability Diagram for Urban EV Demand Clustering

This figure kind of shows a bifurcation diagram, where the diffusion ratio d = D_v/D_u is on the horizontal axis, and the spatial wavenumber k for demand perturbations is on the vertical axis. The shaded area, which sits above the Turing instability threshold curve, is basically the set of parameter combinations where spatial demand clustering is expected. In other words, the diagram indicates that, when the grid load-balancing speed (D_v) goes up compared with the more physical demand mobility (D_u), you start seeing progressively more detailed, or finer grained, spatial patterns of demand come into view. The dataset plus parameter ranges is calibrated using EV fleet density numbers from the IEA Global EV Outlook 2023, and also with urban grid topology details from the U.S. Department of Energy Alternative Fuels Station Locator (2023).

IV. Integrating Stochastic Queueing Theory

4.1 Why Queues Matter at Charging Stations

A charging station is not only an energy delivery point, it's more like a service place, and well, service places always seem to make queues, even if nobody is watching. The math about how queues actually form, how long people wait, and when the whole setup starts breaking down once it gets overload that's basically stochastic queueing theory territory. And no, this connection isn't incidental or random. The same mathematical toolkit that people use to study machine repair settings can be used almost head-on for EV charging infrastructure too.

So, the M/M/C model is usually where you begin — meaning arrivals follow a Markovian (Poisson) process, service times come from an exponential distribution, and you've got C parallel servers, a.k.a chargers. Baghel (2019) then built Markovian models for machine interference situations with finite sources plus balking/reneging behavior, and those details translate pretty clean into the EV charging scenario. In this mapping, "balking" becomes drivers who notice a long queue and choose another station, and "reneging" becomes drivers who enter the line but leave before their charging finishes. These behavioral patterns aren't small fries at all for urban charging sites, they affect how the system performs in practice, and they can change the usual performance metrics in a very measurable way.

For a charging station modeled as M/M/C with arrival rate λ, service rate μ per charger, and *C* chargers, the traffic intensity is:

$$\rho = \frac{\lambda}{C\mu}$$

The probability that the system is empty is:

$$P_0 = \left[\sum_{n=0}^{C-1} \frac{(C\rho)^n}{n!} + \frac{(C\rho)^C}{C!\,(1-\rho)}\right]^{-1}$$

The mean number of vehicles waiting (not charging) is:

$$L_q = \frac{(C\rho)^C \rho}{C!\,(1-\rho)^2} P_0$$

And by Little's Law, the mean waiting time is $W_q = L_q/\lambda$.

Baghel (2020) extended this framework to M/M/C systems with queue-length-dependent service rates — a refinement that matters practically when fast-charger stations throttle output power under heavy concurrent load, which is a real operational characteristic of DC fast-charging arrays. The effective service rate becomes:

$$\mu(n) = \mu_0 \cdot h(n), \quad h(n) = \begin{cases} 1 & n \le C \\ \left(\frac{C}{n}\right)^{\theta} & n > C \end{cases}$$

where θ > 0 captures the degree of service rate degradation under overload, and n is the number of EVs in the system. Baghel (2020) applied diffusion approximations to multi server M/M/R machine repair problems when traffic is heavy , and that actually links back to our RD framework, because the diffusion approximation trades the discrete queueing dynamics for a continuous partial differential equation, which feels very close to spatial diffusion in our RD model.

4.2 Coupling Queues with the Reaction-Diffusion Field

The key insight is that the local charging demand intensity u(x, t) in our RD system is not an independent field — it kind of gets shaped by the queueing dynamics at individual stations, like directly. More precisely, the effective local demand that the diffusion operator "feels" depends on how many vehicles are actually turned away (balking) or choose to give up while they wait (reneging) and then redirect to nearby places, rather than staying put.

Let $\lambda(\mathbf{x}, t)$ be the arrival rate at location **x** at time *t*. The effective throughput — the rate at which vehicles are successfully charged — is:

$$\Lambda(\mathbf{x}, t) = \lambda(\mathbf{x}, t)(1 - P_{\text{balk}}) \cdot \left(1 - P_{\text{renege}}\right)$$

where the balking probability depends on observed queue length and the reneging probability depends on wait time relative to driver patience thresholds. Baghel (2019) derived expressions for these probabilities in finite-source Markovian systems. The "lost" demand — vehicles that balk or renege — feeds back into the diffusion term of our RD system as a source of spatial redistribution:

$$\frac{\partial u}{\partial t} = D_u \nabla^2 u + f(u, v) + S(\mathbf{x}, t)$$

where $S(\mathbf{x}, t) = \lambda(\mathbf{x}, t) \cdot P_{\text{balk}}(\mathbf{x}, t) + \lambda(\mathbf{x}, t)(1 - P_{\text{balk}}) \cdot P_{\text{renege}}(\mathbf{x}, t)$ is the spatially distributed source term representing redirected demand.

4.3 Multi-Server Systems and Optimal Capacity

Baghel (2021) looked at optimization of service capacity in M/M/C machine repair systems, under demand that is uncertain and also preventive maintenance rules, kind of hard to separate, but yes. When you map it onto EV infrastructure, preventive maintenance of chargers ends up being like planned downtime, for example firmware upgrades or some hardware servicing, and that's a real operational constraint which shrinks the effective capacity in a not so trivial way. If each of the C chargers at a station fails independently with rate φ, and then it's brought back at rate ψ, the effective count of working chargers at time t becomes a random variable:

$$C_{\text{eff}}(t) \sim \text{Binomial}\left(C, \frac{\psi}{\phi + \psi}\right)$$

Baghel (2020) analyzed reliability and availability of multi-component repairable systems using M/M/R models with standby spares, a framework directly usable for modeling redundant charger configurations. The steady-state availability of a station with *C* chargers and *S* cold standby units, with repair rate *ψ*, is:

$$A = \frac{\sum_{n=0}^{C} \binom{C+S}{n} \left(\frac{\phi}{\psi}\right)^n}{\sum_{n=0}^{C+S} \binom{C+S}{n} \left(\frac{\phi}{\psi}\right)^n}$$

This availability measure directly enters the capacity field $v(\mathbf{x}, t)$ in the RD system: a station with 80% availability contributes only 0.8 × (nominal capacity) to the local grid's service capacity at any given moment.

V. Numerical Methods and Implementation

5.1 Discretizing the RD System on Urban Grids

Cities aren't continuous planes — they're networks of streets, buildings, and zones. Implementing the RD system requires discretizing the spatial domain Ω onto a grid that respects urban geometry. A natural choice is a finite difference scheme on a regular lattice, with the Laplacian approximated as:

$$\nabla^2 u_{i,j} \approx \frac{u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1} - 4u_{i,j}}{(\Delta x)^2}$$

For time integration, an implicit Crank-Nicolson scheme offers stability for stiff RD systems:

$$\frac{u^{n+1} - u^n}{\Delta t} = \frac{D_u}{2}(\nabla^2 u^{n+1} + \nabla^2 u^n) + f(u^n, v^n)$$

This ends up in a sparse linear system, solvable via conjugate gradient methods at each timestep, which sort of scales nicely to city-scale grids, with hundreds of thousands of nodes or so.

Boundary conditions, honestly, they matter a lot here. For major highways (think grid boundaries), a Neumann condition $\partial u/\partial n$=0 is appropriate, meaning there's no particular reason that demand would simply leak outside the city across those highways. Meanwhile at major transit hubs and parking structures—natural magnets for EV demand—Dirichlet conditions work better, with time varying boundary values u**x**hub, t=Uhub$t$ derived from observed demand data, for stronger calibration.

5.2 Parameter Calibration from Real Data

Diffusion coefficient D_u is calibrated based on GPS mobility data for EV drivers searching for charging stations. In the case when the mean square displacement follows the relation ⟨r^2⟩∼ Du*t, parameter D_u is calibrated based on observed driving range statistics for EV drivers. This calibration is possible thanks to data provided by the U.S. Department of Energy EV Project, which monitored more than 8,300 EVs in 21 U.S. cities. Typical estimates of D_u are in the range of 0.1–0.5 km²/hour for an urban environment due to the limited range of such deviations.

Reaction parameters α, β, γ, δ, and K are calibrated based on a combination of surveys and operational data from charging networks. The latter type of data includes station level utilization statistics, which are openly available through U.S. DOE alternative fuels station locator and EAFO websites.

Baghel (2022) showed that stochastic differential equations can describe the transitory behavior of machine repair queues under the condition of batch arrivals and generalized reneging, which is relevant to our modified

RD model for the calibration of the time-varying source function S(x,t) with batch arrivals equivalent to simultaneous workplace departure at rush hours.

VI. Demand Forecasting and Dynamic Pricing

6.1 Temporal Dynamics and Harmonic Decomposition

Urban EV charging demand follows recognizable temporal rhythms. A city's aggregate demand signal $U(t) = \int_{\Omega} u\,(\mathbf{x}, t) d\mathbf{x}$can be decomposed into harmonic components:

$$U(t) = U_0 + \sum_{k=1}^{N} A_k \cos\left(\frac{2\pi k t}{T} + \phi_k\right) + \epsilon(t)$$

where T = 24 hours captures the daily periodicity, the amplitudes A_k quantify the strength of each harmonic, $\phi_k$are phase offsets, and $\epsilon(t)$is a residual stochastic term. The dominant harmonics are typically k = 1 (daily cycle), k = 2 (half-daily cycle capturing the morning and evening peaks), and k = 7 (weekly cycle). Fitting these parameters to historical demand data allows the RD model's boundary conditions to be driven by a realistic temporal forcing function.

6.2 Dynamic Pricing as a Control Mechanism

The real power of the RD framework is that it naturally suggests control mechanisms. If demand clustering causes grid stress in some spatial regions, one lever is dynamic pricing — raising charging prices in congested zones and lowering them in underutilized zones to shift demand spatially.

In control theory terms, this is a distributed parameter control problem. The pricing field $p(\mathbf{x}, t)$acts as a feedback control on the demand field through a price-sensitivity term:

$$f(u, v, p) = \alpha u \left(1 - \frac{u}{K}\right) - \beta u v - \eta u \cdot \nabla p$$

The term $-\eta u \cdot \nabla p$ captures how EV drivers move away from high-price zones toward low-price zones, with η being the price sensitivity coefficient. Estimating η requires empirical demand elasticity data — the IEA's 2023 report on EV charging behavior cites demand elasticities in the range of -0.2 to -0.5 with respect to charging price, depending on trip purpose and vehicle class.

VII. Conclusion

The reaction-diffusion framework offers something that traditional facility-location and static However, optimization models cannot offer a scientifically rigorous approach to modeling demand for charging as a dynamic spatial process of spontaneous clustering and response to interventions in time. The Turing instability model reveals exactly when and why spontaneous formation of hot zones occurs. Coupled with queuing theory, which is based on Baghel's numerous results for M/M/C and M/M/R queues, this approach offers insights on microscopic demand dynamics feeding the macroscopic spatial field equation.

Urban planners have several very specific guidelines. They should pay special attention to the parameter d=D_v/D_u of the diffusion process in their load-balancing grid systems, aiming for the critical value of d=d_c to be above this limit. If d<d_c, spontaneous clustering is inevitable. Clustering is unavoidable even when d>d_c, but one can make use of dynamic pricing as an instrument for spatial redistribution of demand, taking into account empirically measured elasticity from -0.2 to -0.5.

The connection to stochastic queueing theory for the machine repair problem is more than a mathematical gimmick. It makes it possible to ground the PDE model in concrete aspects of the charging station: finite capacity, driver patience, charger malfunction, and fleet arrival patterns. This translates into the correct specification of source terms and capacities for the RD model, without which the whole thing is no better than a math toy.

EV infrastructure in the city can be thought of as an adaptive complex system. And indeed, the RD model takes that perspective.

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