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New Method of Solving a complicated polynomial expression a possible alternative to Lambert W Function

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Abstract
Lambert W function method is always used to solve equations of the form $A^x = Bx$ because series of trial to solve such equations by applying laws of indices failed.
Udeze Chigozie, a PhD student at university of Aberdeen introduced a pattern that can be used to solve equations of such order provided that A and B are integers. We are going to apply both the laws of indices and Udeze's comparison method (new) to generate a way of tackling such equations.
Series of examples of such equations are solved and the exact solution is gotten which made me to introduce this method as a new method in the mathematics world and I hope that Mathematicians all over the World will begin to think of a way of introducing new formulae to solve more complex problems.

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Equation generation/Solution
First, let us introduce the comparison theorem before going into the context of this work.
Given an equation, say $(A + B)^x = (C + D)^y (1)$
The comparison theorem states that the base in the left-hand side of (1) is equal to the base of the right-hand side of (1), if and only if theirpowers are equal.
In other words,$A + B = C + D, iff x = y$

To solve for the value of x in the equations of the type $A^x = Bx$, we need to use the already existing laws of indices and combine it with comparison theorem to get.

Examples
Find the exact value of x and y in the following problems
1. $4^x = 8x$
2. $9(3y^2) = 27^y$

Solutionsto Question (1)

$$4^x = 8x$$

Let $x = 2^p$

$$\therefore 4^x = 8x \Rightarrow 4^{2^p} = 8(2^p)$$
$$\Rightarrow 2^{2^{p+1}} = 2^3(2^p) = 2^{p+3}$$
$$\Rightarrow 2^{p+1} = p + 3$$

Multiplying both powers by $\frac{1}{p+1}$

$$\therefore 2 = (p + 3)^{\frac{1}{p+1}}(2)$$

Recall that $2 = (1 + 3)^{\frac{1}{2}}$

$$\therefore (2) \Rightarrow (1 + 3)^{\frac{1}{2}} = (p + 3)^{\frac{1}{p+1}}(3)$$

to get the value of p apply comparison theorem which state that the base in the left-hand side of (3) is equal to the base of the right-hand side of (3), if and only if their powers are equal.
In other words, $1 + 3 = p + 3, iff \frac{1}{2} = \frac{1}{p+1}$

$$\therefore p = 1, recall\ that\ x = 2^p = 2^1 = 2$$

Therefore $x = 2$ $(proved)$

Solutions to question (2)

$$9(3y^2) = 27^y$$
$$\therefore\ 27y^2 = 27^y$$
$$\Rightarrow\ y^2 = \frac{27^y}{27} = 27^{y-1}(4)$$

Let $y = 3^q$

$$(4) \Rightarrow (3^q)^2 = 27^{3^q-1} = 3^{3(3^q-1)}$$
$$\Rightarrow 3^{2q} = 3^{(3^{q+1}-3)}$$
$$2q = 3^{q+1} - 3\ \Rightarrow 3^{q+1} = 2q + 3$$

Multiplying both powers by $\frac{1}{q+1}$

$$\therefore 3 = (2q+3)^{\frac{1}{q+1}}(5)$$

Recall that $3 = (0+3)^{\frac{1}{0+1}}$

$$\therefore (5)\ \Rightarrow (0+3)^{\frac{1}{0+1}} = (2q+3)^{\frac{1}{q+1}}(6)$$

to get the value of p apply comparison theorem which state that the base in the left-hand side of (6) is equal to the base of the right-hand side of (6), if and only if their powers are equal.

In other words, $0 + 3 = 2q + 3,\ iff\ \frac{1}{0+1} = \frac{1}{q+1}$

$$\therefore q = 0, recall\ that\ y = 2^q = 2^0 = 1$$

Therefore $y = 1\ (proved)$

Conclusion

We have been able to introduce a theorem that enabled us to get the exact value of x in the equation of the type. $A^x = Bx$ if and only if $A\ and\ B$ are whole numbers.

Our next target is to investigate Fermat's last Theorem and get a general formula that can solve equations of the type $X^n + Y^n = Z^n$ since it is obvious that we cannot get an integer value for all $X, Y\ and\ Z\ if\ n\ is\ greater\ than$ 2 but if $n = 2$ we obtain a Pythagorean formula.

References

[1]. Nigel Boston (2003) "The proof of Fermat's last Theorem. University of Wisconsin-Madison. Springer.

[2]. https://en.wikipedia.org/wiki/Lambert_W_function

[3]. Abdelilah et al (2002). Numerical evaluation of Lambert W function and application to generation of generalized Gausian noise with exponent ½. IEEE Transactions of signal processing vol. 50, no. 9.