International Journal of Mathematics and Statistics Invention (IJMSI)
E-ISSN: 2321 – 4767 P-ISSN: 2321 - 4759
www.ijmsi.org Volume 10 Issue 4 || Jul.-Aug., 2022 || PP-12-13

A New Method for the Construction of Super-Saturated Design

VaasanthiAlugoluand Bhatracharyulu,N.Ch.
Department of Statistics, University College of Science, Osmania University, Hyderabad, TelanganaState, India.

Abstract
Super-saturated design is a fractional factorial design having the more number of factors when compared with number of design points. Several authors made attempts on the construction of super-saturated designs. In this paper, an attempt is made to propose a new method for the construction of super-saturated design. The method is illustrated with suitable example.
Key words: *Saturated design, $E(S^2)$ criteria, BIBD and PBIBD.*

Date of Submission: 29-08-2022 Date of Acceptance: 24-09-2022

I. INTRODUCTION

A supersaturated design is a fractional factorial design in which the number of factors is more than the number of design points and the degrees of freedom for all its main effects and interaction terms exceed the number of design points. The advantage of these designs is that they reduce the experimental cost and time significantly due to their run size.These designs used to identify active factor main effects when experimentation is expensive and the number of potential factors is large. These designs are more economical and flexible because of their run size.

Satterthwaite (1959) initially made an attempt to construct saturated designs randomly and suggested the random balance designs. Booth and Cox (1962) initially proposed a systematic method for the construction of super-saturated designs, which are factorial designs in which the number of factors exceeds the number of design points and also computed $E(s^2)$ criterion.

DEFINITION-1.1: A design X is said to be ‘saturated’ if the number of design points ‘n’ is equal to the number of factors ‘f’ plus one i.e. n=f+1.

DEFINITION-1.2: A design X is said to be a super-saturated design, if the number of factors ‘’v’’ is more than the number of design points ‘n’ i.e. f > n.

DEFINITION-1.3: A design X is said to be $E(s^2)$-optimal super-saturated if a super-saturated design processing the property that the mean of S_{ij}^2 of all pairs (i,j) for ($i \neq j$) is minimum.It is the measure of non-orthogonality under the strong assumption that only two out of ‘m’ factors whose value of s_{ij}^2 is minimum, then such design is better to select. The designs that are near orthogonal are preferable if it is not possible to conduct the experiment with orthogonal designs. The lack of orthogonality measured based on the dispersion matrix of the design. If $E(s^2) = 0$, then these designs are leads toorthogonal designs.

To study the effects of active factors, which are in fewer, then these designs, used to reduce the experimental cost and time significantly. Several authors made attempts on the construction of super-saturated design.

II. NEW METHOD FOR THE CONSTRUCTION OF SUPER-SATURATED DESIGN

In this section, an attempt is made to propose a new method for the construction of super-saturated design using incomplete block designs. The method is also illustrated with suitable example.

METHOD: Consider the incidence matrix of a variance balanced design with parameters v′=v+1, b′= 2b, r′ = (2b, 2r) k_j'= (2b, k+1). (2b, 2r) stands for ‘v’ treatments each replicated ‘2r’ times and the remaining one treatment replicated 2b times. Let N be the incidence matrix of a Balanced Incomplete Block Designs with parameters v, b, r, k. Arrange the incidence matrix N in the form.

$$N' = \begin{bmatrix} N & N \\ J & J \end{bmatrix}$$

Replace zero’s with -1 then the resulting design is a Super- saturated design

EXAMPLE: Let us consider N be the incidence matrix of a Balanced Incomplete Block Design with parameters v = 6, b= 11, r= 4 or 5, k= 2, 4. N=

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

The resulting design is the incidence matrix of a variance-balanced design with parameters: $v'=7$, $b'= 22$, $r' = 8$ or 22, $k'= 3$ or 5.

$$N' = \begin{bmatrix} N & N \\ J & J \end{bmatrix}$$

$$\begin{bmatrix} 1&1&1&1&0&0&0&0&0&0&0&1&1&1&1&0&0&0&0&0&0&0 \\ 1&1&0&0&1&1&0&0&0&0&0&1&1&0&0&1&1&0&0&0&0&0 \\ 1&1&0&0&0&0&1&1&0&0&0&1&1&0&0&0&0&1&1&0&0&0 \\ 1&1&0&0&0&0&0&0&1&1&0&1&1&0&0&0&0&0&0&1&1&0 \\ 0&0&1&0&1&0&1&0&1&0&1&0&0&1&0&1&0&1&0&1&0&1 \\ 0&0&0&1&0&1&0&1&0&1&1&0&0&0&1&0&1&0&1&0&1&1 \\ 1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1 \end{bmatrix}$$

Then the resulting Super-saturated design is

$$\begin{bmatrix} 1&1&1&1&-1&-1&-1&-1&-1&-1&-1&1&1&1&1&-1&-1&-1&-1&-1&-1&-1 \\ 1&1&-1&-1&1&1&-1&-1&-1&-1&-1&1&1&-1&-1&1&1&-1&-1&-1&-1&-1 \\ 1&1&-1&-1&-1&-1&1&1&-1&-1&-1&1&1&-1&-1&-1&-1&1&1&-1&-1&-1 \\ 1&1&-1&-1&-1&-1&-1&-1&1&1&-1&1&1&-1&-1&-1&-1&-1&-1&1&1&-1 \\ -1&-1&1&-1&1&-1&1&-1&1&-1&1&-1&-1&1&-1&1&-1&1&-1&1&-1&1 \\ -1&-1&-1&1&-1&1&-1&1&-1&1&1&-1&-1&-1&1&-1&1&-1&1&-1&1&1 \\ 1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1 \end{bmatrix}$$

The mean value of S^2 of the Super saturated design is $E(S^2) = 0.5839$.

References

[1]. Booth and Cox(1962): “Some systematic Supersaturated Designs”, Technometrics, vol.4, pp 489-495.
[2]. Lin (1993): “A new class of Supersaturated Designs”, Technometrics, vol.35,pp 28-31.
[3]. Lin(1995):“Generating Systematic Supersaturated Designs”, Technometrics, vol.37,pp 213-225.
[4]. Nguyen(1996): “An algorithm approach to constructing Supersaturated Designs”, Technometrics, vol.38,pp 69-73.
[5]. Satterthwaite (1959): “Random Balance Experimentation”, Technometrics, vol.1, pp 111-137.