International Journal of Mathematics and Statistics Invention (IJMSI)
E-ISSN: 2321 – 4767 P-ISSN: 2321 - 4759
www.ijmsi.org Volume 10 Issue 4 || Jul.-Aug., 2022 || PP-01-02

Limiting Distribution for Poissono-Poisson Derived Power Series Distribution

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ABSTRACT: *Ranjitha, et al (2022) proposed a new distribution named as Poissono-Poisson derived power series distribution and also studied its properties and its applications in real life. In this paper, an attempt is made to derive its asymptotic distribution.*

KEYWORDS: *Derived power series distribution, Poisson distribution and Asymptotic distribution.*

Date of Submission: 06-08-2022 Date of Acceptance: 22-08-2022

I. INTRODUCTION

A large class of random variable with discrete probability distributions is derived from power series distributions. Several authors made attempts in expressing and on deriving Power Series distributions and also in studying their properties like, probability generating function, moment generating function, characteristic function, recurrence relations between the probabilities etc., Some of the well-known theoretical models of this family are: Binomial, Poisson, Negative-binomial and Geometric etc. Power series distributions have several applications in real life.

Jayasree and Swamy (2006), derived a new class of derived power series distributions by considering the ratio of two power series probability distributions. A discrete random variable X is said to follow Derived Power series distribution, if it is expressed in the form,

$$\left[\frac{a_0 g(\eta_2)}{b_0 f(\eta_1)}\right] \frac{1+\sum_{i=1}^{\infty} a_i s^i}{1+\sum_{i=1}^{\infty} b_i s^i} = \sum_{i=1}^{\infty} p_i s^i \quad (1)$$

where, $p_i = \left[\frac{a_0 g(\eta_2)}{b_0 f(\eta_1)}\right] d_i$; with $d_i = a_i - \sum_{j=1}^{i} b_j d_{i-j}$; i = 1, 2, 3, ...; d_0 =1 and

$$a_i = \left(\frac{a_i^*}{a_0}\right)(\eta_1)^i \text{ and } b_j = \left(\frac{b_j^*}{b_0}\right)(\eta_2)^j.$$

Ranjitha et al (2022) derived a new power series distribution named as Poissono-Poisson derived power series distribution. Let $X_1 \sim$ Poisson (λ_1) and $X_2 \sim$ Poisson (λ_2) where $\lambda_1 > \lambda_2$, the ratio of the probability generating functions of the two distributions provides a Poissono-Poisson derived power series distribution with probability function

$$P_n = P[X = n] = e^{-(\lambda_1 - \lambda_2)} \frac{(\lambda_1 - \lambda_2)^n}{n!} \text{; for n = 1,2,3,… and } \lambda_1 > \lambda_2. \quad (2)$$

Note:

i. $E(X) = (\lambda_1 - \lambda_2)$ and $V(X) = (\lambda_1 - \lambda_2)$.
ii. $M_X(t) = e^{-(\lambda_1 - \lambda_2)(e^t - 1)}$.
iii. $P_X[s] = e^{(\lambda_1 - \lambda_2)(s-1)}$; $|s| \le 1$.

II. Limiting Distribution of Poissono-Poisson

In this section an attempt is made to derive the limiting distribution of the Poissono-Poisson derived power series distribution.

Theorem: If X follows Poissono-Poisson derived power series random variable with parameter $\lambda = \lambda_1 - \lambda_2 > 0$, then $Z = \frac{X-(\lambda_1 - \lambda_2)}{\sqrt{(\lambda_1 - \lambda_2)}}$ follows asymptotically Normal.

Proof: Let X follows Poissono-Poisson derived power series distribution with parameter λ, where $\lambda = \lambda_1 - \lambda_2 > 0$. The standardization of the variate is

$$Z = \frac{X-(\lambda_1 - \lambda_2)}{\sqrt{(\lambda_1 - \lambda_2)}}.$$

The moment generating function of the standard variate Z is

$$M_Z(t) = E\left(e^{t\left(\frac{X-(\lambda_1-\lambda_2)}{\sqrt{(\lambda_1-\lambda_2)}}\right)}\right)$$

$$= E\left(e^{\frac{Xt}{\sqrt{(\lambda_1-\lambda_2)}}} . e^{\frac{-(\lambda_1-\lambda_2)t}{\sqrt{(\lambda_1-\lambda_2)}}}\right)$$

$$= e^{\frac{-(\lambda_1-\lambda_2)t}{\sqrt{(\lambda_1-\lambda_2)}}} E\left(e^{\frac{Xt}{\sqrt{(\lambda_1-\lambda_2)}}}\right),$$

$$= e^{-t\sqrt{(\lambda_1-\lambda_2)}} e^{(\lambda_1-\lambda_2)\left(e^{\frac{t}{\sqrt{(\lambda_1-\lambda_2)}}}-1\right)} \quad \text{(from note (ii))}$$

$$= e^{-t\sqrt{(\lambda_1-\lambda_2)}+(\lambda_1-\lambda_2)\left[\left(1+\frac{t}{\sqrt{(\lambda_1-\lambda_2)}}+\frac{t^2}{2!\,(\lambda_1-\lambda_2)}+\frac{t^3}{3!\,(\lambda_1-\lambda_2)^{3/2}}+\cdots\right)-1\right]}$$

$$= e^{-t\sqrt{(\lambda_1-\lambda_2)}+(\lambda_1-\lambda_2)\left[\left(\frac{t}{\sqrt{(\lambda_1-\lambda_2)}}+\frac{t^2}{2!\,(\lambda_1-\lambda_2)}+\frac{t^3}{3!\,(\lambda_1-\lambda_2)^{3/2}}+\cdots\right)\right]}$$

$$= e^{-t\sqrt{(\lambda_1-\lambda_2)}+\left(\frac{(\lambda_1-\lambda_2)t}{\sqrt{(\lambda_1-\lambda_2)}}+\frac{t^2}{2!}+O(\lambda_1-\lambda_2)^{-1/2}\right)}$$

$$= e^{\frac{t^2}{2!}+O(\lambda_1-\lambda_2)^{-1/2}} \qquad (3)$$

where $O(\lambda_1-\lambda_2)^{-1/2}$ is negligible. When $n\to\infty$,

$$\lim_{n\to\infty} Mz(t) = e^{\frac{t^2}{2!}} \qquad (4)$$

which is the moment generating function of a standard normal variate. Hence, standard variate of Poissono-Poisson derived power series distribution follows asymptotically Normal.

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Ranjitha Chul, et. al. "Limiting Distribution for Poissono-Poisson Derived Power Series Distribution." *International Journal of Mathematics and Statistics Invention (IJMSI)*, vol. 10(04), 2022, pp. 01-02.