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Lindley and Lomax Mixture distribution

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ABSTRACT*: Mixture distribution is a probability distribution of convex combination of distributions as components of the random variable, with the corresponding weights of each component, and called the mixture weights. In this paper, an attempt is made to propose a new mixture distribution using Lindley and Lomax probability distributions with different proportions and some of its properties are also derived.*
KEYWORDS*: Cumulative distribution function, Mixture distribution, Lindley distribution, Lomax distribution, Gamma and exponential distribution.*

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I. INTRODUCTION

In applied sciences, modelling and analysis help to explain the lifetime events in various aspects. These scenarios can be studied using various popular statistical distributions such as exponential, beta, gamma, pareto, weibull, lognormal etc. But each of these lifetime distributions have their own advantages and disadvantages over one another, due to the number of parameters involved, its shape, nature of hazard function and mean residual life function.

Let {Fj (x); j = 1, 2, …n}, be a finite set of family of probability distribution functions with *fj* (x) as the corresponding probability density functions and a_j's be mixture weights where $a_j \geq 0$; $\sum a_j = 1$. Let X be a random variable taking real values in a sample space W, then f(x) is said to be a finite mixture of density functions if

$$f(\mathrm{x}) = a_1 f_1(\mathrm{x}) + a_2 f_2(\mathrm{x}) + \cdots + a_n f_n(\mathrm{x}) \quad (1)$$

Lindley Distribution is a two-component mixture of an exponential distribution having scale parameter θ and gamma distribution having shape parameter 2 and scale parameter θ with mixing proportions $\frac{1}{\theta+1}$ and $\frac{\theta}{\theta+1}$. The probability density function of Lindley distribution is

$$f_1(\mathrm{x}) = \frac{\theta^2}{\theta+1}(1+\mathrm{x})\mathrm{e}^{-\theta \mathrm{x}}; \mathrm{x} > 0, \theta > 0 \quad (2)$$

Let X be a continuous random variable follow Lomax distribution, with the shape (α=1) and scale (θ) parameters with probability density function as

$$f_2(\mathrm{x}) = \theta(\mathrm{x}+\theta)^{-2}; \theta > 0, \mathrm{x} > 0 \quad (3)$$

II. LL MIXTURE DISTRIBUTION

In this section a new mixture distribution named as LL distribution is proposed using the convex combination of Lindley and Lomax distributions (2) and (3) with different proportions.

LL Distribution: The mixture of Lindley distribution with proportion $\frac{1}{\theta+1}$ (say C_1) and Lomax distribution with proportion $\frac{\theta}{\theta+1}$ (say C_2) generates a LL distribution with probability density function

$$f(\mathrm{x};\theta) = \frac{\theta^2}{(\theta+1)^2(\mathrm{x}+\theta)^2}\left[1+\theta+(1+\mathrm{x})(\mathrm{x}+\theta)^2\mathrm{e}^{-\theta \mathrm{x}}\right]; \mathrm{x} > 0, \theta > 0 \quad (4)$$

Theorem: The probability density function of mixture of Lindley and Lomax distribution with proportions $C_1$and C_2 is

$$f(\mathrm{x};\theta) = \frac{\theta^2}{(\theta+1)^2(\mathrm{x}+\theta)^2}\left[1+\theta+(1+\mathrm{x})(\mathrm{x}+\theta)^2\mathrm{e}^{-\theta \mathrm{x}}\right]; \mathrm{x} > 0, \theta > 0$$

Proof: The convex combination of Lindley and Lomax distribution with proportions $\frac{1}{\theta+1}$ (say C_1) and $\frac{\theta}{\theta+1}$ (say C_2) is $f(\mathrm{x};\theta) = C_1 f_1(\mathrm{x}) + C_2 f_2(\mathrm{x})$

$$= \frac{1}{\theta+1}\frac{\theta^2}{\theta+1}(1+\mathrm{x})\mathrm{e}^{-\theta \mathrm{x}} + \frac{\theta}{\theta+1}\theta(\mathrm{x}+\theta)^{-2}$$

$$= \frac{\theta^2}{(\theta+1)^2(x+\theta)^2}\left[1+\theta+(1+x)(x+\theta)^2 e^{-\theta x}\right]; x>0, \theta>0$$

Note:

1. $f(x;\theta) \geq 0$, for all $x > 0, \theta > 0$
2. $\int_0^\infty f(x;\theta)dx = 1$.
3. Cumulative distribution of LL is

$$F(x,\theta) = 1 - \frac{e^{-\theta x}}{\theta+1} - \frac{\theta x e^{-\theta x}}{(\theta+1)^2} - \frac{\theta^2}{(\theta+1)(x+\theta)}; \ x>0, \theta>0$$

III. PROPERTIES OF LL DISTRIBUTION

Theorem: The Characteristic function of Mixture of LL distribution is

$$\phi_X(t) = \frac{\theta^2 e^{-it\theta}}{\theta+1}\sum_{j\neq 1, j=0}^{\infty}\frac{(it)^j}{j!}\left[\frac{-\theta^{j-1}}{j-1}\right] + \frac{\theta-it+1}{(\theta-it)^2} \qquad (5)$$

Proof: The characteristic function,

$$\phi_X(t) = E(e^{itX})$$
$$= \int_0^\infty e^{itx} f(x;\theta)dx$$
$$= \int_0^\infty e^{itx}\frac{\theta^2}{(\theta+1)^2(x+\theta)^2}\left[1+\theta+(1+x)(x+\theta)^2 e^{-\theta x}\right]dx$$
$$= \frac{\theta^2}{(\theta+1)}\int_0^\infty e^{itx}(x+\theta)^{-2}dx + \frac{\theta^2}{(\theta+1)^2}\int_0^\infty (1+x)\, e^{-x(\theta-it)}\,dx$$
$$= \frac{\theta^2 e^{-it\theta}}{\theta+1}\sum_{j\neq 1, j=0}^{\infty}\frac{(it)^j}{j!}\left[\frac{-\theta^{j-1}}{j-1}\right] + \frac{\theta-it+1}{(\theta-it)^2}$$

The Moments of the new distribution can be obtained from the Characteristic function as

$$\mu_1{}' = \frac{\theta+2}{\theta(\theta+1)^2} - \frac{\theta^2}{\theta+1} = \text{Mean}$$
$$\mu_2{}' = \frac{2(\theta+3)}{\theta^2(\theta+1)^2}$$
$$\mu_3{}' = \frac{6(\theta+4)}{\theta^3(\theta+1)^2} + \frac{3\theta^4}{2(\theta+1)}$$
$$\mu_4{}' = \frac{24(\theta+5)}{\theta^4(\theta+1)^2} - \frac{5\theta^5(\theta+3)}{6(\theta+1)}$$

Its central Moments are:

$$\mu_2 = \frac{1}{\theta^2(\theta+1)^4} M_1$$

where $M_1 = [-\theta^8 - 2\theta^7 - \theta^6 + 2\theta^5 + 6\theta^3 - 9\theta^2 + 10\theta + 12]$

$$\mu_3 = \frac{1}{2\theta^3(\theta+1)^6} M_2,$$

where $M_2 = \frac{1}{2\theta^3(\theta+1)^6}[3\theta^{12} + 12\theta^{11} + 18\theta^{10} + 52\theta^9 + 67\theta^8 + 112\theta^7 - 12\theta^6 - 92\theta^5 - 129\theta4-112\theta3+174\theta2+48\theta+95$

$$\mu_4 = \frac{1}{6\theta^4(\theta+1)^8} M_3$$

where $M_3 = [-5\theta^{17} - 32\theta^{16} - 166\theta^{15} - 58\theta^{14} - 16\theta^{13} - 350\theta^{11} - 1262\theta^{10} - 2175\theta^9 - 2238\theta8-48\theta7+2640\theta6+1188\theta5+1023\theta4+1168\theta3+823\theta2+6144\theta-2064$

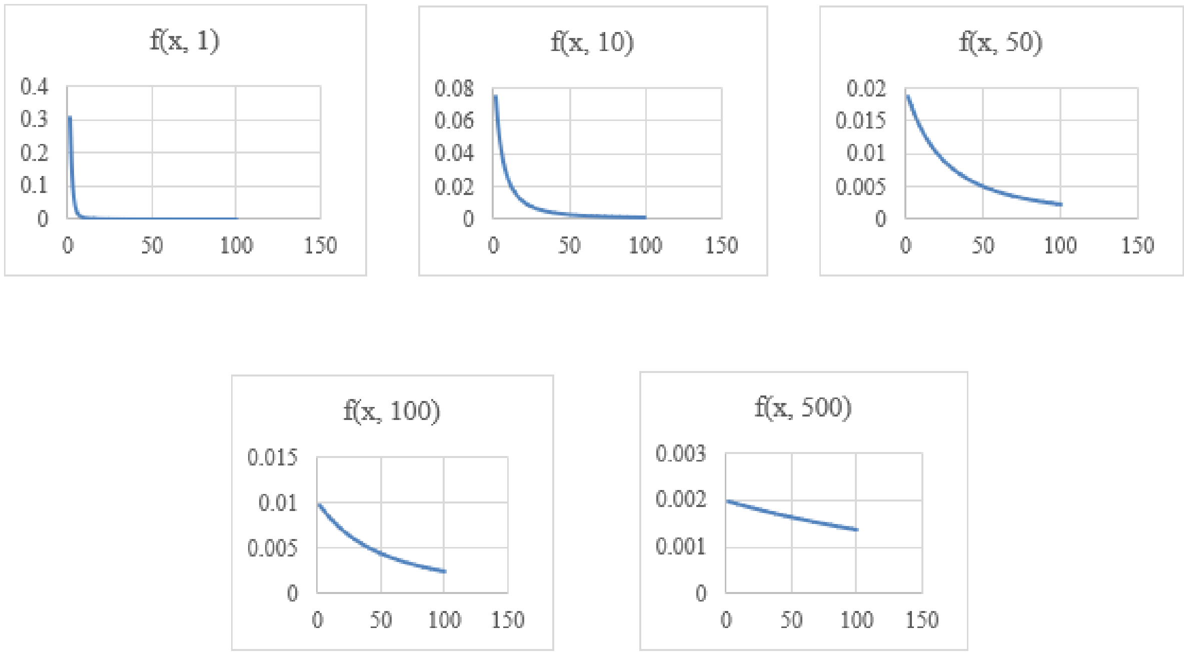
Then we have $\beta_1 = \frac{{M_2}^2}{4{M_1}^3}$; $\beta_2 = \frac{M_3}{6{M_1}^2}$.

Note: The moment generating function of LL mixture distribution is divergent.

IV. GRAPHICAL REPRESENTATION

Here presenting the graphical representation of probability density function of Mixture of Lindley and Lomax for $\theta = 1, 10, 50, 100$ and 500.

Figure 1: LL Distribution



Remark: One can observe from the probability density function graphs that the Mixture of Lindley and Lomax Curves are exactly same as Lomax Curves.

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