

An Inventory Model for Weibull Deteriorating Items with Linear Demand, Shortages under Permissible Delay In Payments and Inflation

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ABSTRACT: An inventory model for Weibull deteriorating items with time varying demand under permissible delay in payments and inflation is considered. Holding cost is linear function of time. Shortages are permitted and are completely backlogged. Sensitivity with respect to parameters has been carried out. Numerical example is taken to support the model.

KEYWORDS: Deterioration, Inflation, Inventory, Linear demand, Permissible delay in payment, Shortages, Time varying holding cost

I. INTRODUCTION

The conventional inventory models did not take into account the effect of inflation. But during last two decades, the most of the countries economic condition has changed to such an extent due to inflation and there is change in the purchasing power of money. Therefore, it is not possible to ignore the effect of inflation. Buzacott [1] developed the first economic order quantity model by considering inflationary effects into account. A uniform inflation was assumed for all the associated costs and an expression for the EOQ was derived. Su et al. [2] developed model under inflation for stock dependent consumption rate and exponential decay. Chung et al. [3] developed the inventory replenishment policy over a finite planning horizon for a deteriorating item taking account of time value. Moon et al. [4] developed models for ameliorating / deteriorating items with time varying demand pattern over a finite planning horizon taking into account the effects of inflation and time value of money. Hou [5] developed an inflation model for deteriorating items with stock dependent consumption rate and shortages by assuming a constant length of replenishment cycles and a constant fraction of the shortage length with respect to the cycle length.

Datta and Pal [6] and Bose et al. [7] have developed the economic order quantity model incorporating the effects of time value of money and shortages. Demand was considered as linear function of time. Hariga [8] extended Datta and Pal's [6] model by relaxing the assumption of equal inventory carrying time during each replenishment cycle and modified their mathematical formulation. Hariga and Ben-Daya [9] extended Hariga's [8] model by removing the restriction of equal replenishment cycle and provided two solution procedures with and without shortages. Ray and Chaudhury [10] developed an EOQ model with stock dependent demand, shortages, inflation and time discounting. Chung and Lin [11] extended the inventory replenishment model of Chung et al. [3] for shortages allowed in each replenishment cycles. Mishra et al. [12] considered a model for deterministic perishable items that follows variable type demand rate with infinite time horizon, constant deterioration. An inventory model for stock dependent consumption and permissible delay in payment under inflationary conditions was developed by Liao et al. [13]. Singh [14] developed an EOQ model with linear demand and permissible delay in payments. The effect of inflation and time value of money were also taken into account.

In this paper we have developed EOQ model when demand rate is linear function of time, deterioration is two parameter Weibull distribution and inventory holding cost is linear function of time under inflationary conditions with permissible delay in payments. Shortages are allowed and are completely backlogged. Numerical example is taken and sensitivity analysis is also done.

II. NOTATIONS AND ASSUMPTIONS:

The following notations and assumptions are used here:

NOTATIONS:

- D(t) : $a + bt$, Demand is linear function of time, where $a > 0, 0 < b < 1$
- A : Ordering cost per order
- c : Unit purchasing cost per item
- p : Unit selling price of the item ($p > c$)
- h(t) : $x + yt$, Inventory variable holding cost per unit excluding interest charges
- c_2 : Shortage cost per unit
- Q_1 : Inventory level initially
- Q_2 : Shortage of inventory
- I_e : Interest earned per year
- I_p : Interest charged in stocks per year
- R : Inflation rate
- M : Permissible period of delay in settling the accounts with the supplier
- T : Time interval between two successive orders
- I(t) : Inventory level at any instant of time $t, 0 \leq t \leq T$
- Q : Order quantity
- α : Scale parametrs ($0 < \alpha < 1$)
- β : Shape parameter ($\beta > 0$)
- $\alpha\beta t^{\beta-1}$: the two parameter Weibull deterioration rate.

ASSUMPTIONS:

The following assumptions are used in the development of the model:

- The demand of the product is declining as a linear function of time.
- Replenishment rate is infinite and instantaneous.
- Lead time is zero.
- Shortages are allowed and are completely backlogged.
- The deteriorated units can neither be repaired nor replaced during the cycle time. During the time, the account is not settled; generated sales revenue is deposited in an interest bearing account. At the end of the credit period, the account is settled as well as the buyer pays off all units sold and starts paying for the interest charges on the items in stocks.

III. THE MATHEMATICAL MODEL AND ANALYSIS:

Let I(t) be the inventory at time t ($0 \leq t \leq T$) as shown in figure.

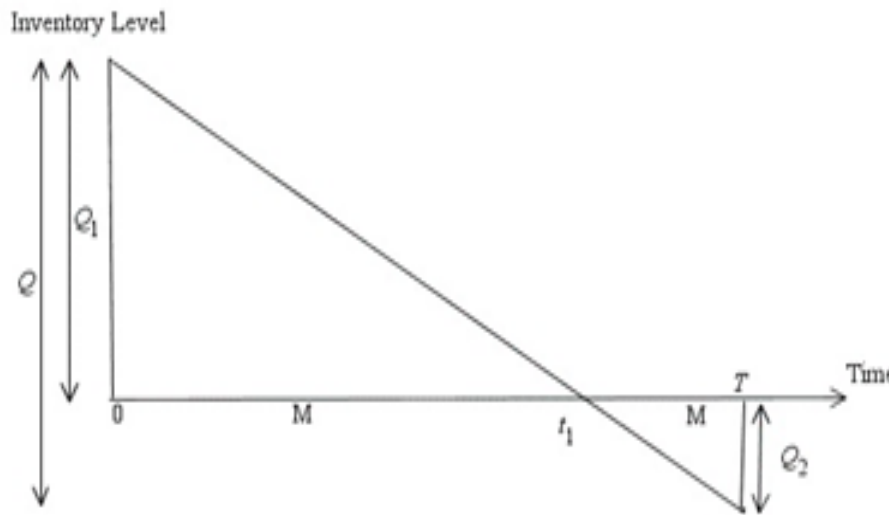


Figure 1

The differential equations which describes the instantaneous states of $I(t)$ over the period $(0, T)$ is given by

$$\frac{dI(t)}{dt} + \alpha\beta t^{\beta-1}I(t) = -(a+bt), \quad 0 \leq t \leq t_1, \quad (1)$$

$$\frac{dI(t)}{dt} = -(a+bt), \quad t_1 \leq t \leq T. \quad (2)$$

with the boundary conditions at $I(0) = Q_1$, $I(t_1) = 0$ and $I(T) = -Q_2$. The solution of equation (1) and (2) using boundary conditions are:

$$I(t) = \begin{bmatrix} a(t_1 - t) + \frac{b}{2}(t_1^2 - t^2) + \frac{a\alpha}{\beta+1}(t_1^{\beta+1} - t^{\beta+1}) + \frac{b\alpha}{\beta+2}(t_1^{\beta+2} - t^{\beta+2}) \\ - a\alpha t^\beta (t_1 - t) - \frac{b\alpha}{2} t^\beta (t_1^2 - t^2) \end{bmatrix}, \quad 0 \leq t \leq t_1, \quad (3)$$

$$I(t) = a(t_1 - t) + \frac{b}{2}(t_1^2 - t^2), \quad t_1 \leq t \leq T. \quad (4)$$

(by neglecting higher powers of α)

The initial order quantity at $t=0$ is obtained by putting $t=0$ in equation (3)

$$Q_1 = \left[at_1 + \frac{bt_1^2}{2} + \frac{a\alpha t_1^{\beta+1}}{\beta+1} + \frac{b\alpha t_1^{\beta+2}}{\beta+2} \right]. \quad (5)$$

For $t = T$, $I(T) = -Q_2$. So from equation (4), we have

$$Q_2 = -a(t_1 - T) - \frac{b}{2}(t_1^2 - T^2). \quad (6)$$

The associated costs are:

1. Ordering cost (OC) = A (7)

2. Holding cost:

$$HC = \int_0^{t_1} h(t)I(t)e^{-Rt} dt = \int_0^{t_1} (x + yt) \begin{bmatrix} a(t_1 - t) + \frac{b}{2}(t_1^2 - t^2) + \frac{a\alpha}{\beta+1}(t_1^{\beta+1} - t^{\beta+1}) \\ + \frac{b\alpha}{\beta+2}(t_1^{\beta+2} - t^{\beta+2}) \\ - a\alpha t^\beta (t_1 - t) - \frac{b\alpha}{2} t^\beta (t_1^2 - t^2) \end{bmatrix} e^{-Rt} dt$$

The total cost per unit during a cycle $C_1(t_1, T)$ is consisted of the following:

$$C_1(t_1, T) = \frac{1}{T} [OC + HC + DC + SC + IP_1 - IE_1] \quad (13)$$

Putting values from equations (7) to (12) in equation (13) we get the total cost for case I. Differentiating equation (13) with respect to t_1 and T and equate it to zero, we have

$$\frac{\partial C_1(t_1, T)}{\partial T} = 0, \quad \frac{\partial C_1(t_1, T)}{\partial t_1} = 0. \quad (14)$$

By solving equation (14) for t_1 and T , we obtain the optimal cycle length $t_1 = t_1^*$ and $T = T^*$ provided it satisfies equation

$$\frac{\partial^2 C_1(t_1, T)}{\partial T^2} > 0, \quad \frac{\partial^2 C_1(t_1, T)}{\partial t_1^2} > 0 \text{ and } \left[\frac{\partial^2 C_1(t_1, T)}{\partial T^2} \right] \left[\frac{\partial^2 C_1(t_1, T)}{\partial t_1^2} \right] - \left[\frac{\partial^2 C_1(t_1, T)}{\partial T \partial t_1} \right]^2 > 0. \quad (15)$$

Case II: ($0 \leq t_1 \leq M$): In this case, the retailer earns interest on the sales revenue up to the permissible delay period and no interest is payable during this period. So

7. Interest earned up to the permissible delay period is:

$$IE_2 = pI_e \left[\int_0^{t_1} (a + bt)te^{-Rt} dt + (a + bt_1)t_1(M - t_1) \right] = pI_e \left[\frac{1}{10}bR^2t_1^5 + \frac{1}{4}\left(\frac{1}{2}aR^2 - bR\right)t_1^4 + \frac{1}{3}(-aR + b)t_1^3 + \frac{1}{2}at_1^2 + (a + bt_1)t_1(M - t_1) \right] \quad (16)$$

$$8. IP_2 = 0. \quad (17)$$

The total cost per unit during a cycle $C_2(T)$ is consisted of the following:

$$C_2(t_1, T) = \frac{1}{T} [OC + HC + DC + SC + IP_2 - IE_2] \quad (18)$$

Putting values from equations (7) to (10) and (16), (17) in equation (18) we get the total cost for case II. Differentiating equation (18) with respect to t_1 and T and equate it to zero, we have

$$\frac{\partial C_2(t_1, T)}{\partial T} = 0, \quad \frac{\partial C_2(t_1, T)}{\partial t_1} = 0. \quad (19)$$

By solving equation (19) for t_1 and T , we obtain the optimal cycle length $t_1 = t_1^*$ and $T = T^*$ provided it satisfies equation

$$\frac{\partial^2 C_2(t_1, T)}{\partial T^2} > 0, \quad \frac{\partial^2 C_2(t_1, T)}{\partial t_1^2} > 0 \text{ and } \left[\frac{\partial^2 C_2(t_1, T)}{\partial T^2} \right] \left[\frac{\partial^2 C_2(t_1, T)}{\partial t_1^2} \right] - \left[\frac{\partial^2 C_2(t_1, T)}{\partial T \partial t_1} \right]^2 > 0. \quad (20)$$

IV. NUMERICAL EXAMPLE:

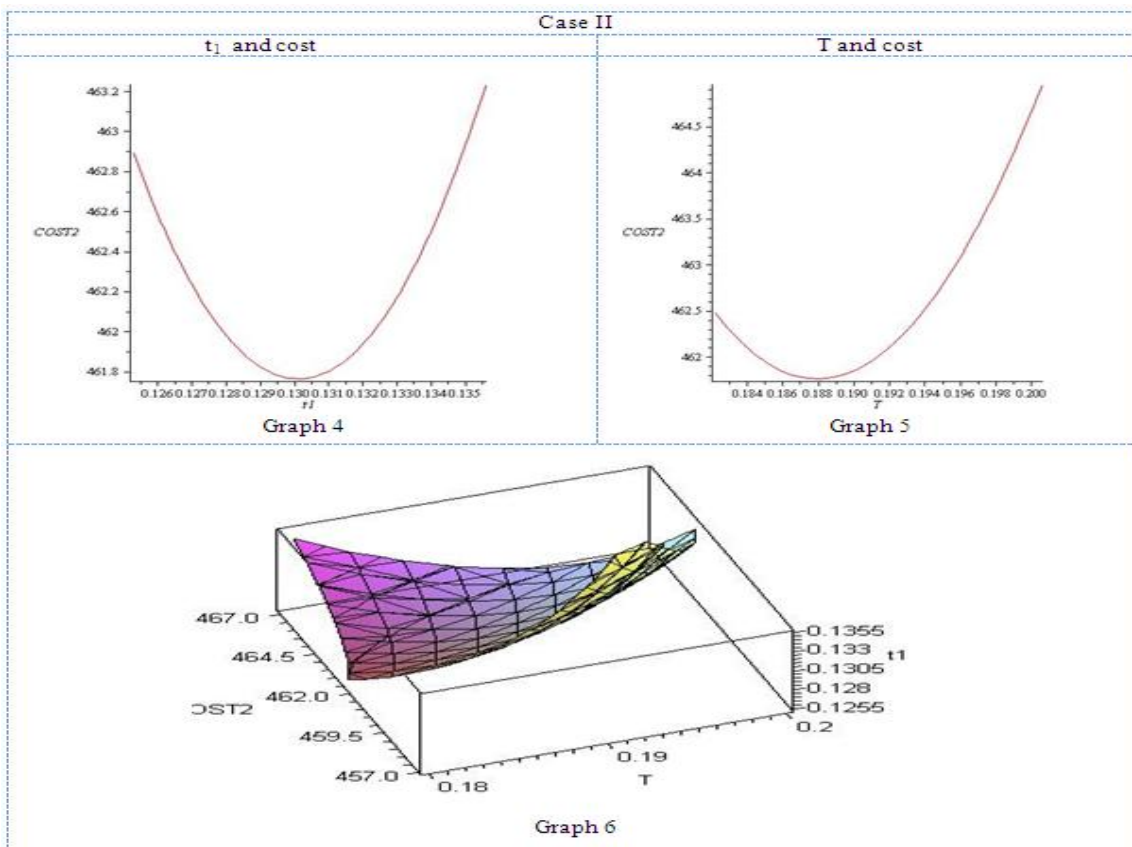
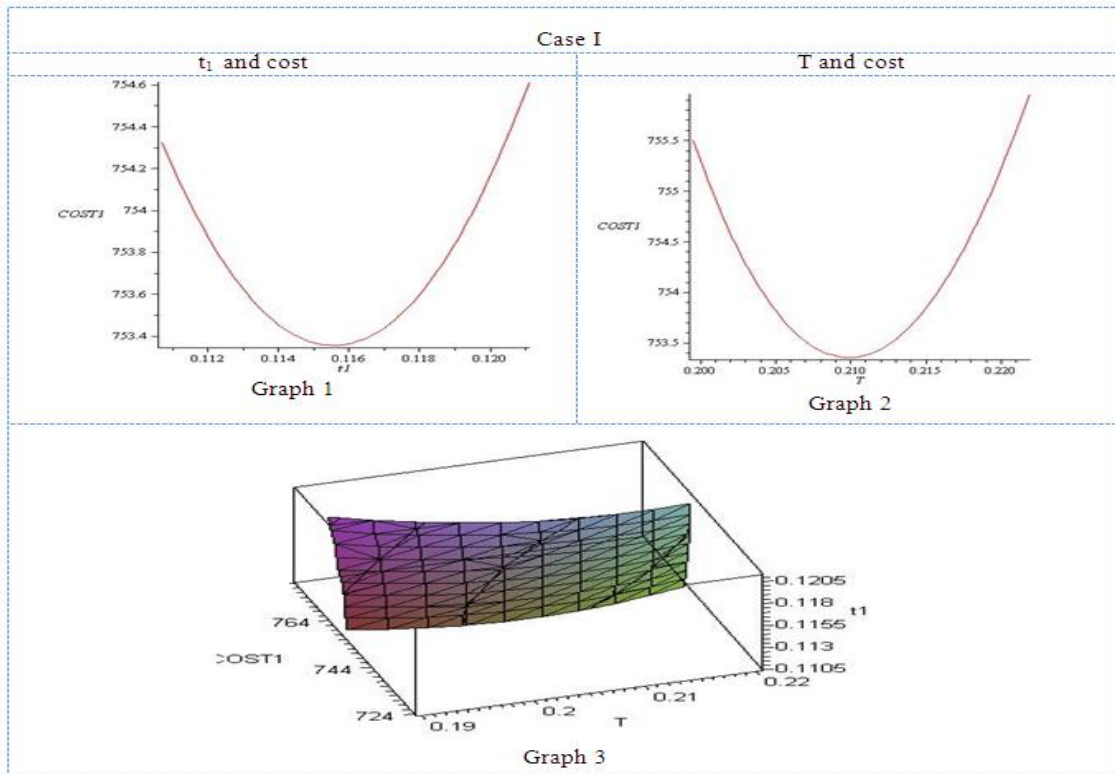
Case I:

Considering $A = \text{Rs } 100$, $c = \text{Rs. } 25$, $p = \text{Rs } 40$, $I_p = \text{Rs } 0.15$, $I_e = 0.12$, $M = 0.08$ years, $\alpha = 0.04$, $\beta = 2$, $a = 1000$, $b = 0.05$, $x = 5$, $y = 0.05$, $c_2 = \text{Rs. } 8$, $R = 0.01$ in appropriate units. Then we obtained the optimal value of $t_1^* = 0.1155$, $T^* = 0.2099$ and the optimal total cost $C_1^* = \text{Rs. } 753.3544$ and the optimum order quantity $Q^* = 209.9216$.

Case II:

Considering $A = \text{Rs } 100$, $c = \text{Rs. } 25$, $p = \text{Rs } 40$, $I_p = \text{Rs } 0.15$, $I_e = 0.12$, $M = 0.18$ years, $\alpha = 0.04$, $\beta = 2$, $a = 1000$, $b = 0.05$, $x = 5$, $y = 0.05$, $c_2 = \text{Rs. } 8$, $R = 0.01$ in appropriate units. Then we obtained the optimal value of $t_1^* = 0.1301$, $T^* = 0.1879$ and the optimal total cost $C_2^* = \text{Rs. } 461.7638$ and the optimum order quantity $Q^* = 187.9302$.

The second order conditions given in equations (15) and (20) are also satisfied. The graphical representation of the convexity of the cost function for the two cases is also given.



V. SENSITIVITY ANALYSIS:

On the basis of the data given in example above we have studied the sensitivity analysis by changing the following parameters one at a time and keeping the rest fixed.

**Table 1 Sensitivity Analysis
Case I: ($0 \leq M \leq t_1$)**

| Parameter | % | t_1 | T | Cost | Q |
|-----------|------|--------|--------|----------|----------|
| a | +10% | 0.1107 | 0.1995 | 779.8513 | 219.4708 |
| | +5% | 0.1130 | 0.2045 | 766.8981 | 210.1712 |
| | -5% | 0.1182 | 0.2157 | 739.1783 | 209.8120 |
| | -10% | 0.1211 | 0.2219 | 724.3227 | 199.7325 |
| α | +10% | 0.1154 | 0.2098 | 753.6021 | 209.8236 |
| | +5% | 0.1155 | 0.2099 | 753.4782 | 209.9126 |
| | -5% | 0.1156 | 0.2099 | 753.2302 | 209.9206 |
| | -10% | 0.1157 | 0.2100 | 753.1058 | 210.0196 |
| x | +10% | 0.1112 | 0.2075 | 768.7488 | 207.5194 |
| | +5% | 0.1133 | 0.2087 | 761.1771 | 208.7204 |
| | -5% | 0.1178 | 0.2112 | 745.2679 | 211.2229 |
| | -10% | 0.1203 | 0.2126 | 736.9034 | 212.6213 |
| M | +10% | 0.1166 | 0.2085 | 733.3487 | 208.5222 |
| | +5% | 0.1161 | 0.2092 | 743.4277 | 209.2219 |
| | -5% | 0.1150 | 0.2106 | 763.1297 | 210.6213 |
| | -10% | 0.1144 | 0.2112 | 772.7556 | 211.2210 |
| R | +10% | 0.1153 | 0.2098 | 754.1148 | 209.8215 |
| | +5% | 0.1154 | 0.2098 | 753.7349 | 209.8215 |
| | -5% | 0.1156 | 0.2100 | 752.9733 | 210.0217 |
| | -10% | 0.1158 | 0.2101 | 752.5911 | 210.1218 |

**Table 2 Sensitivity Analysis
Case II: ($0 \leq t_1 \leq M$)**

| Parameter | % | t_1 | T | Cost | Q |
|-----------|------|--------|--------|----------|----------|
| a | +10% | 0.1253 | 0.1768 | 453.1241 | 194.5096 |
| | +5% | 0.1276 | 0.1822 | 457.8406 | 191.3399 |
| | -5% | 0.1328 | 0.1940 | 464.8500 | 184.3306 |
| | -10% | 0.1356 | 0.2006 | 467.0494 | 180.5709 |
| α | +10% | 0.1300 | 0.1878 | 462.1593 | 187.8331 |
| | +5% | 0.1300 | 0.1879 | 461.9618 | 187.9316 |
| | -5% | 0.1302 | 0.1880 | 461.5657 | 188.0288 |
| | -10% | 0.1303 | 0.1880 | 461.3675 | 188.0274 |
| x | +10% | 0.1261 | 0.1867 | 483.6670 | 186.7276 |
| | +5% | 0.1281 | 0.1873 | 472.8674 | 187.3289 |
| | -5% | 0.1322 | 0.1886 | 450.3427 | 188.6376 |
| | -10% | 0.1343 | 0.1893 | 438.5905 | 189.3331 |
| M | +10% | 0.1325 | 0.1827 | 400.5326 | 182.7318 |
| | +5% | 0.1313 | 0.1854 | 431.5064 | 185.4310 |
| | -5% | 0.1288 | 0.1903 | 491.3341 | 190.3293 |
| | -10% | 0.1274 | 0.1925 | 520.2425 | 192.5284 |
| R | +10% | 0.1299 | 0.1879 | 462.8843 | 187.9301 |
| | +5% | 0.1300 | 0.1879 | 462.3245 | 187.9301 |
| | -5% | 0.1302 | 0.1879 | 461.2023 | 187.9303 |
| | -10% | 0.1303 | 0.1880 | 460.6398 | 188.0303 |

From the table we observe that as parameter a increases/ decreases, order quantity and average total cost increases/ decreases for case I and average total cost decrease/ increase and order quantity increase/ decrease for case II. We observe that with increase and decrease in parameter α , there is corresponding very slight increase/ decrease in total cost for both cases.

Also we observe that with increase and decrease in parameters x, there is corresponding increase/ decrease in total cost and decrease/ increase in total quantity for both case I and case II. Moreover, we observe that with increase and decrease in the value of M, there is corresponding decrease/ increase in total cost in both case I and case II, but there is slight decrease/ increase for case I and case II in quantity. Also, we observe that

with increase and decrease in the value of R, there is corresponding increase/ decrease in total cost for both case I and case II, where as there is little change in quantity for both case I and case II.

VI. CONCLUSION

In this paper we have developed an EOQ model for deteriorating items with linear demand, time varying holding cost and complete backordering under inflationary conditions and permissible delay in payments. Sensitivity with respect to parameters has been carried out. The results show that with the increase/ decrease in the parameter values there is corresponding increase/ decrease in the value of cost.

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